Lecture 10

Row Operations to Transform Matrix

15 December 2021 — Thota Rudhra Kumar
Lectures on Machine Learning by Rakesh Nigam

10.1 Introduction

What is Matrix ?

It is an arrangement of elements, especially numbers, in a particular way. A matrix is a mathematical structure having rows and columns. The horizontal and vertical lines of the matrix are represented as rows and columns.The numbers in the matrices are called entries or elements.

- A matrix is a rectangular array of numbers or symbols which are generally arranged in rows and columns.
- The order of the matrix is defined as the number of rows and columns.
- The entries are the numbers in the matrix and each number is known as an element.
- The plural of matrix is matrices.
- The size of a matrix is referred to as 'n by m' matrix and is written as m×n, where n is the number of rows and m is the number of columns.

- For example, we have a 3×2 matrix, that's because the number of rows here is equal to 3 and the number of columns is equal to 2.

What is Rows ?

A matrix having a single row is called a row matrix. But the number of columns could be more than one.

Row Matrix,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_n \\ \end{bmatrix}$$

What is column ?

A column matrix is a rectangular array of elements, arranged in a vertical line. The general representation of the column matrix.

$$ColumnMatrix, A = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ . \\ . \\ . \\ a_m \end{bmatrix}$$

10.2 A **matrix** is a rectangular arrangement of numbers into rows and columns.For example, matrix ***A*** has two *rows* and three *columns*.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ a & b & c \end{bmatrix}$$

Matrix dimensions

The *dimensions* of a matrix tells its size the number of rows and columns of the matrix, in that order.Since matrix ***A*** has two rows and three columns, we write its dimensions as 2 x 3 matrix.

In contrast, matrix ***B*** has three rows and two columns, so 3 x 2 matrix.

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ & \end{bmatrix}$$

10.3 Row Operations to Transform Matrix

The elementary operation of a matrix, also know ass elementary transformation are the operations performed on rows and columns of a matrix to transform the given matrix into a different form in order to make the calculation simpler. Transformation row operations are the operations that are performed on rows of a matrix. Similarly,transformation column operations are the performed on columns of a matrix.

For Example:

$$x + 2y + z = 2 \rightarrow (1)$$

$$3x + 8y + z = 12 \rightarrow (2)$$

$$0x + 4y + z = 2 \rightarrow (3)$$

$$[A|b] = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \\ & & & \end{bmatrix}$$

$$\overrightarrow{r}_1 \leftarrow \overrightarrow{r}_1$$

$$\overrightarrow{r}_2 \leftarrow \overrightarrow{r}_2 - 3$$

$$[A|b] = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 \\ \end{bmatrix}$$

$$\overrightarrow{r}_3 \leftarrow \overrightarrow{r}_3 - 2$$

$$[A|b] = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & -10 \\ \end{bmatrix}$$

$$x + 2y + z = 2$$

$$2y - 2z = 6$$

$$5z = -10$$

- Pattern in this matrix is upper Triangle in a matrix

Now, (3) equation in the given matrix form

$$Z = -10/5$$

$$Z = -2$$

Now, Substitute

$$(2^{nd})$$

equation

$$2y = 6 + 2z$$

$$2y = 6 + 2(-2)$$

$$6 - 4 = 2$$

$$y = 2/2$$

$$y = +1$$

Now, Substitute

$$(1^{st})$$

equation

$$x = 2 - 2y - 3$$

$$2 - 2(1) - (-2)$$

$$2 - 2 + 2$$

$$x = 2$$

$$\overrightarrow{x} = \begin{bmatrix} x \to 2 \\ y \to 1 \\ z \to -2 \\ \\ \end{bmatrix}$$

10.4 Relative of the Identity matrix

An identity matrix refers to a type of the square matrix in which its diagonal entries are equal to 1 and the off-diagonal entries are equal to 0.Identity matrices play a vital role in the linear algebra. In particular, their role in the matrix multiplication is similar to the role that is played by the number 1 when it comes to the multiplication of the real numbers:

1. The real number remains unchanged if it is multiplied by 1.
2. The matrix remains unchanged if it is multiplied by an identity matrix.

10.4.1 Identity Matrix Properties

- Identity matrix is always in the form of a square matrix.The identity matrix is called a square matrix because it has the same number of the rows and the columns. For any given whole number n, the identity matrix is given by n x n.
- Multiplying a given matrix with the identity matrix would result in the matrix itself.Since the multiplication is not always defined, the size of the matrix matters when you work on the matrix multiplication.
- When multiplying two inverse matrices, you would get an identity matrix.

For Example :

$$\overrightarrow{A}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A_2\overrightarrow{X} = x, \overrightarrow{A}x_2 = A = I_2A, I_2I_2 = I_2$$

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\overrightarrow{X} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\overrightarrow{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\overrightarrow{P}X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\overrightarrow{P}A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\overrightarrow{P}A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Note:

- Multiplication from left effects rows.
- Multiplication from right effects columns.

Write an identity matrix of the order 4.

The identity matrix of the order 4 x 4 is given as

$$\overrightarrow{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The product of any square matrix and the appropriate identity matrix is always the original matrix, regardless of the order in which the multiplication was performed, In other words, A.I = I.A = A

10.5 Matrix multiplication

What is Matrix Multiplication

Matrix multiplication, also known as matrix product and the multiplication of two matrices, produces a single matrix.

If A and B are the two matrices, then the product of the two matrices A and B are denoted by:

$$\overrightarrow{X} = AB$$

1. Matrix multiplication also known as matrix product .
2. It is a binary operation that produces a single matrix by taking two or more different matrices. We know that a matrix can be defined as an array of numbers.
3. When we multiply a matrix by a scalar value, then the process is known as scalar multiplication.

For Example:

Multiply the matrix

$$\overrightarrow{A} = \begin{bmatrix} 3 & 4 & -1 \\ 0 & 9 & 5 \end{bmatrix}$$

by 4

$$\overrightarrow{A} = \begin{bmatrix} 3 & 4 & -1 \\ 0 & 9 & 5 \end{bmatrix}$$

$$4XA = 4$$

$$\overrightarrow{A} = \begin{bmatrix} 3 & 4 & -1 \\ 0 & 9 & 5 \end{bmatrix}$$

Now, we have to multiply each element of the matrix A by 4.

$$\overrightarrow{A} = \begin{bmatrix} 12 & 16 & -4 \\ 0 & 36 & 20 \end{bmatrix}$$

Practice problems

1) Show that matrices A and B are row equivalent if ,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$$

2)Show that matrices by 8.

$$\begin{bmatrix} 1 & 2 & 3 \\ a & b & c \end{bmatrix}$$

3) Solve Matrix Multiplication

$$\begin{bmatrix} a & b & c \\ c & d & d \\ e & f & g \end{bmatrix}$$