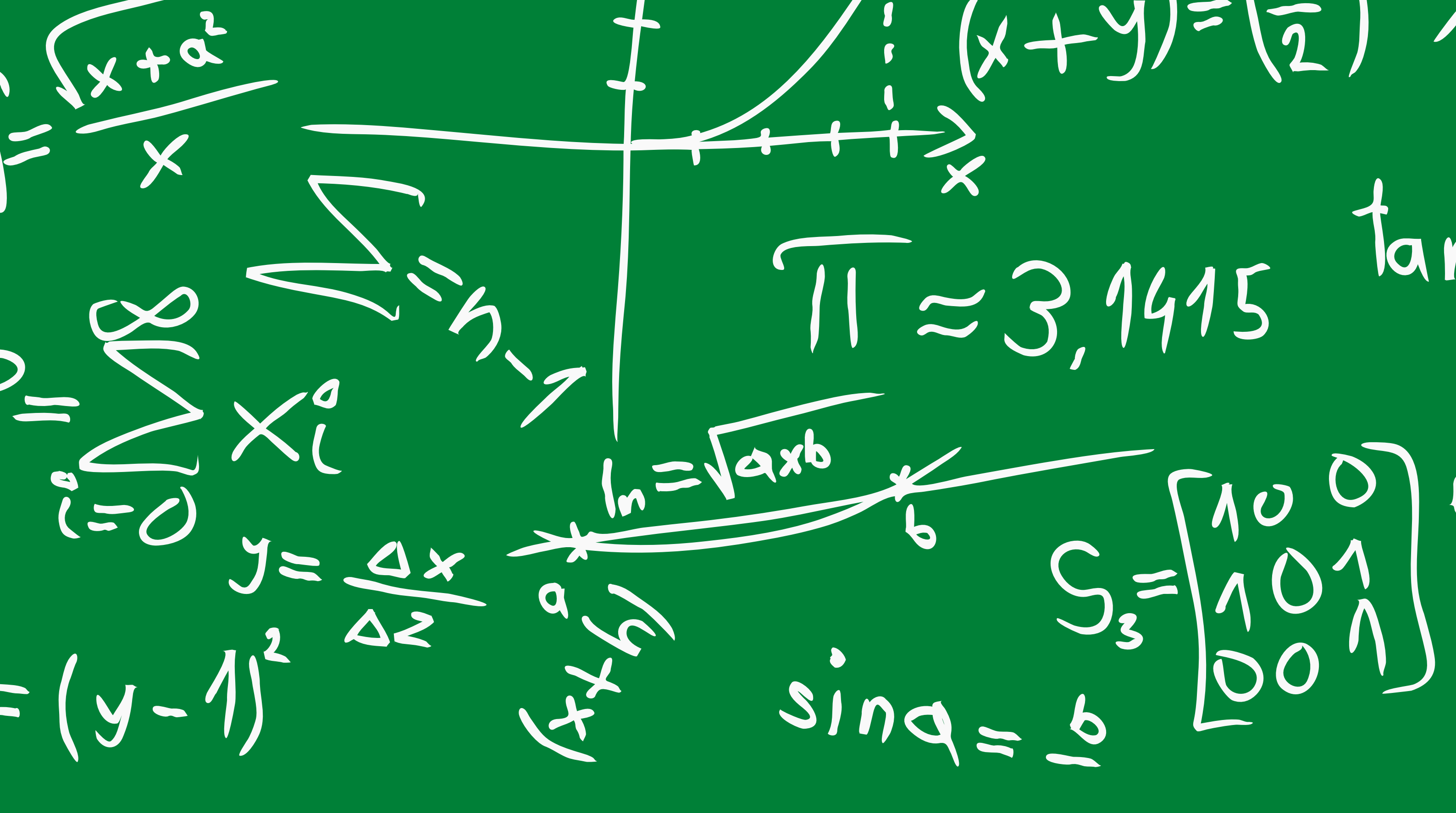
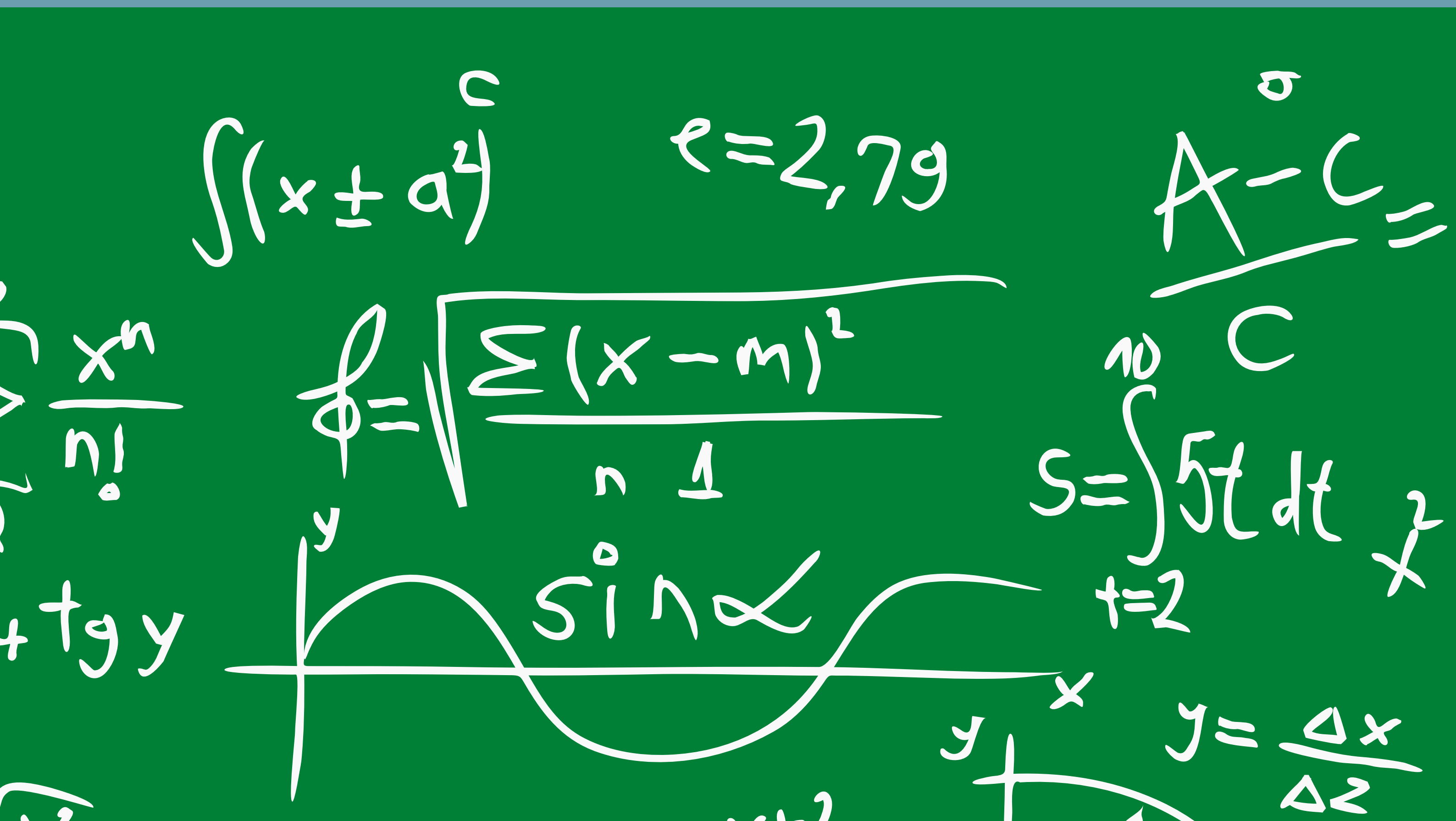


Cálculo 3

Notas de clases

Rudik Roberto Rompich



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Índice general

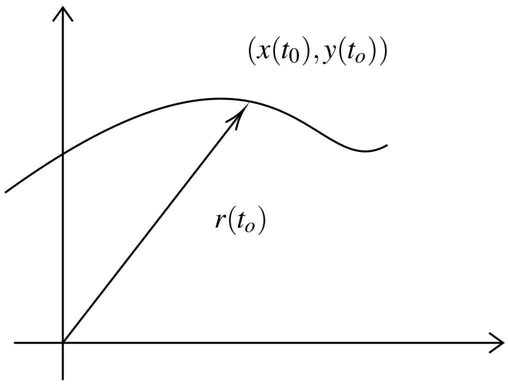
I Contenido Parcial 3

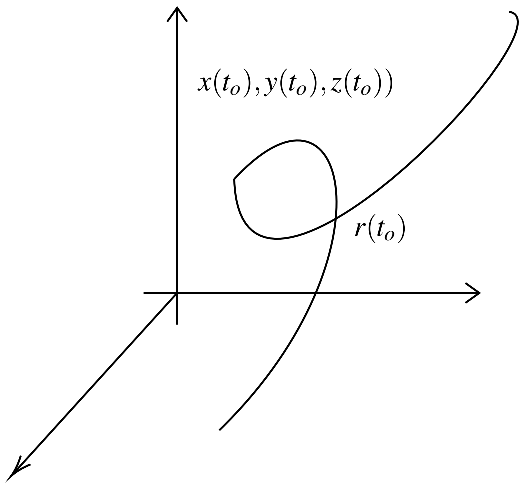
Contenido Parcial 3

1. Cálculo vectorial

1.1 28 de septiembre de 2020

1.1.1 Cálculo diferencial en varias variables - Funciones vectoriales





Definición 1.1.1

$$x = f(t)y = g(t) \qquad a \leq t \leq b \tag{1.1}$$

$$r(t) = \langle f(t), g(t) \rangle = f(t)\hat{\imath} + y(t)\hat{\jmath} \tag{1.2}$$

$$r_o = \langle x_o, x_o, z_o \rangle r_1 = \langle x_1, y_1, z_1 \rangle \tag{1.3}$$

$$t = (1-t)r_o + tr_1 \tag{1.4}$$

Ejercicio 1.1

¿Función vectorial? Para recta que va $P_o(3,2,-1)$ al punto $P_1(1,4,5)$

$$r(t) = (1-t)\langle 3,2,-1\rangle + t\langle 1,4,5\rangle \tag{1.1}$$
$$r(t) = \langle 3-2t, 2+2t, -1+6t\rangle \tag{1.2}$$

■

$$4\cos t\hat{\mathbf{i}} + t\hat{\mathbf{j}} + 2\sin t\hat{\mathbf{k}} \qquad \text{c. helicoloides} \tag{1.1}$$

$$t\cos t\hat{\mathbf{i}} + t\sin t\hat{\mathbf{j}} + t\hat{\mathbf{k}} \tag{1.1}$$

$$t^2\hat{\mathbf{i}} + t^4\hat{\mathbf{j}} + t^6\hat{\mathbf{k}} \tag{1.1}$$

$x,y; y = x^3$ y $x,z; z = x^3$

$$\tag{1.2}$$

1.1.2 Campos vectoriales

Definición 1.1.2

Sobre $\mathbf{R}^2$ una función F que asigna a cada punto (x,y) en D (conjunto en $\mathbf{R}^2$) un vector bidimensional.

$$F(x,y) = P(x,y)\hat{\mathbf{i}} + Q(x,y)\hat{\mathbf{j}} \tag{1.1}$$
$$\langle P(x,y), Q(x,y)\rangle \tag{1.2}$$
$$F = P_i + Q_j \tag{1.3}$$

También es posible expresarlas:

$$F(x,y,z) = P\hat{\mathbf{i}} + Q\hat{\mathbf{j}} + R\hat{\mathbf{k}} \tag{1.4}$$

■ **Ejemplo 1.1**

$$F(x,y) = -y\hat{\mathbf{i}} + x\hat{\mathbf{j}} \tag{1.1}$$
$$||F|| = C \tag{1.2}$$
$$\sqrt{y^2+x^2} = C \qquad x^2+y^2 = C^2 \tag{1.3}$$
$$(1,0) \mapsto -(0)\hat{\mathbf{i}} + (1)\hat{\mathbf{j}} = j \tag{1.4}$$
$$(0,1) \mapsto -(1)\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = -i \tag{1.5}$$
$$(1{,}5, 1{,}5) \mapsto -1{,}5\hat{\mathbf{i}} + 1{,}5\hat{\mathbf{j}} \tag{1.6}$$

■

■ **Ejemplo 1.2**

$$F(x,y) = 2x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \tag{1.1}$$
$$||F|| = C \tag{1.2}$$
$$\sqrt{(2x)^2 + y^2} = C \tag{1.3}$$
$$4x^2 + y^2 = C^2 \qquad \text{elipse} \tag{1.4}$$
$$(0{,}5, 0) \implies 1i \tag{1.5}$$
$$(0,1) \implies 1j \tag{1.6}$$
$$(0{,}25, 0{,}25) \implies 0{,}5\hat{\mathbf{i}} + 0{,}25\hat{\mathbf{j}} \tag{1.7}$$
$$(-2,-2) \implies -4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} \tag{1.8}$$

■

■ **Ejemplo 1.3**

$$F(x,y,z) = 1\hat{\mathbf{i}} - 1\hat{\mathbf{j}}0\hat{\mathbf{k}} \tag{1.1}$$
$$||F|| = C \tag{1.2}$$
$$\sqrt{(1)^2 + (1)^2 + (0)^2} = \sqrt{2} \tag{1.3}$$

■

1.1.3 Gradiente

Definición 1.1.3

$$f(x,y) = x^2y + 3xy^3 \tag{1.1}$$
$$\nabla f(x,y) = f_x(x,y)\hat{\mathbf{i}} + f_y(x,y)\hat{\mathbf{j}}) \tag{1.2}$$
$$\nabla f(x,y) = (2xy + 3y^3)\hat{\mathbf{i}} + (x^2 + 9xy^2)\hat{\mathbf{j}} \tag{1.3}$$

Si $z = f(x,y)$

$$\tag{1.4}$$

Ejercicio 1.2

$$f(x,y,z) = \sqrt{x^2 + y^2 + z^2} \tag{1.1}$$
$$\nabla(x,y,z) = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{i}}\frac{y}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{j}}\frac{z}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{k}} \tag{1.2}$$
$$\tag{1.3}$$

■

Ejercicio 1.3

Una partícula se mueve en un campo de velocidad $V(x,y) = \langle x^2, x+y^2\rangle$. Si su posición es (2,1) en un tiempo t=3, estime su posición en t=3.01

$$t = 3 \mapsto (2,1) \tag{1.1}$$
$$v(2,1) = \langle 2^2, 2+1^2\rangle = \langle 4,3\rangle \tag{1.2}$$
$$t = 3 \mapsto 3{,}01 \implies 0{,}001 \tag{1.3}$$
$$0{,}01V(2,1) = 0{,}01\langle 4,3\rangle = \langle 0{,}04, 0{,}03\rangle \tag{1.4}$$
$$\text{Posición } (2.04,1.03) \tag{1.5}$$

■

1.1.4 Integral de Línea

Definición 1.1.4 De f a lo largo de C.

$$\int_c f(x,y)ds = \lim_{n\mapsto\infty} \sum_{i=1}^{n} f(x_i^*, y_i^*)\Delta S \tag{1.1}$$

$L = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dS$

$$\int_c f(x,y)dS = \int_a^b f(x(t),y(t)) * \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dS \tag{1.2}$$

Ejercicio 1.4

$$\int_c (x^2 - y + 3z)\, dS \tag{1.1}$$

x = t, y = 2t, z = t. En donde $0 \leq t \leq$

x'=1,y=2,z=1.

$$\sqrt{(1)^2 + (2)^2 + (1)^2} = \sqrt{6} \tag{1.2}$$
$$\int_c (x^2 - y + 3z)\, dS = \int_0^1 (t^2 - 2t + 3t)\sqrt{6}\, dt \tag{1.3}$$
$$\sqrt{6}\int_0^1 (t^2 + t)\, dt \tag{1.4}$$
$$\sqrt{6}[\frac{t^3}{3} + \frac{t^2}{2}]_0^1 \tag{1.5}$$
$$= \frac{5\sqrt{6}}{6} \tag{1.6}$$

■

Ejercicio 1.5

$$\int_c (x+2)\,ds \tag{1.1}$$

donde C es

$$r(t) = t\hat{\mathbf{i}} + \frac{4t^{3/2}}{3}\hat{\mathbf{j}} + \frac{t^2}{2}\hat{\mathbf{k}} \qquad 0 \leq t \leq 2 \tag{1.2}$$

$$||r'(t)|| = \sqrt{(1)^2 + (2t^{1/2})^2 + (t)^2} = \sqrt{1+4t+t^2} \tag{1.3}$$

$$\int_c (x+2)\,ds = \int_0^2 (t+2)\sqrt{1+4t+t^2}\,dt \tag{1.4}$$

$$= 15{,}29 \qquad x = t, y = \frac{4}{t^{2/2}}, z = \frac{t^2}{2} \tag{1.5}$$

■

1.2 29 de septiembre de 2020

Ejercicio 1.6 ¿Masa de un resorte?

$$r(t) = \frac{1}{\sqrt{2}}\cos t\hat{\mathbf{i}}\sin t\hat{\mathbf{j}}t \qquad 0 \leq t \leq 6\pi \tag{1.1}$$

$$||r'(t)|| = \frac{1}{\sqrt{2}}\sqrt{(-sent)^2 + (cost)^2 + (1)^2} = 1 \tag{1.2}$$

$$\frac{1}{\sqrt{2}}\sqrt{2} = \frac{\sqrt{2}}{\sqrt{2}} \tag{1.3}$$

$$\int_c (1+z)\,dS = \int_0^{6\pi} (1+\frac{t}{\sqrt{2}})(1)\,dt \tag{1.4}$$

$$[t + \frac{t^2}{2\sqrt{2}}]_0^{6\pi} = 6\pi(1+\frac{3\pi}{\sqrt{2}}) = 144{,}47 \tag{1.5}$$

■

Ejercicio 1.7

$$\int_c f(x,y)\,dx = \int_a^b f(x(t),y(t))x'(t)\,dt \tag{1.1}$$

$$\int_c xy^2\,dx \quad x = 4cost, y = 4sent \quad 0 \leq t \leq \frac{\pi}{2} \tag{1.2}$$

$$\int_c xy^2\,dx = \int_0^{\pi/2} (4cost)(16\sin^2 t)(-4sent)\,dt \tag{1.3}$$

$$-256\int_0^{\pi/2} \sin^2 t * cost\,dt = -256[\frac{\sin^4 t}{4}]_0^{\pi/2} = -64 \tag{1.4}$$

■

Ejercicio 1.8

$$\int_c f(x,y)\,dx = \int_a^b f(x(t),y(t))y'(t)\,dt \tag{1.1}$$

$$\int_c xy^2\,dxx = 4\cos t, y = 4\sin t \quad 0 \le t \le \frac{\pi}{2} \tag{1.2}$$

$$\int_c xy^2\,dx = \int_0^{\pi/2} (4cost)(16\sin^2 t)(4cost)\,dt \tag{1.3}$$

$$256\int_0^{\pi/2} \sin^2 t * cost\,dt = 256\int_0^{\pi/2} \frac{\sin^2 2t}{4}\,dt \tag{1.4}$$

$$= 64\int_0^{\pi/2} \frac{1}{2}(1 - cos(4t)\,dt \tag{1.5}$$

$$= 16\pi \tag{1.6}$$

■

Ejercicio 1.9

$$\int_c f(x,y)\,dx = \int_a^b f(x(t),y(t))||f||\,dt \tag{1.1}$$

$$\int_c xy^2\,dxx = 4\cos t, y = 4\sin t \quad 0 \le t \le \frac{\pi}{2} \tag{1.2}$$

$$\int_c xy^2\,dx = \int_0^{\pi/2} (4cost)(16\sin^2 t)(\sqrt{16(\cos^2 t + \sin^2 t)})\,dt \tag{1.3}$$

$$256/3 \tag{1.4}$$

■

Ejercicio 1.10

$$\int_c xy\,dx + x^2dyc : y = x^3 \qquad -1 \le x \le 2 \tag{1.1}$$

$$\int_{-1}^2 x(x^3)\,dx + x^2(3x^2dx) \qquad dy = 3x^2dx \tag{1.2}$$

$$\int_{-1}^2 4x^4\,dx = \frac{4x^5}{5}|_{-1}^2 = \frac{132}{5} \tag{1.3}$$

■

Ejercicio 1.11

$$\int_c y^2\,dx - x^2dy \tag{1.1}$$

$\int_c = \int_{c_1} + \int_{c_2} + \int_{c_3}$

$$\int_{c_1} y^2\,dx - x^2 dyy = 0, dy = 0dx \tag{1.2}$$

$$= \int_0^2 (0)^2\,dx - x^2(0dx) = 0 \tag{1.3}$$

$$\int_{c_2} y^2\,dx - x^2 dy = \int_0^4 y^2(0\,dy) - 4dy \qquad x = 2, dx = 0dy \tag{1.4}$$

$$[-4y]_0^4 = -16 \tag{1.5}$$

$$\int_{c_3} y^2\,dx - x^2 dy = \int_2^0 x^4\,dx - x^2(2xdx) \tag{1.6}$$

$$\int_2^0 (x^4 - 2x^3)\,dx = [(\frac{x^5}{5} - \frac{x^4}{2})]_2^0 = 8/5 \tag{1.7}$$

$$\int_c y^2\,dx - x^2 dy = 0 + (-16) + 8/5 = -72/5 \tag{1.8}$$

■

1.2.1 Definición de trabajo

Definición 1.2.1

$$W = \int_a^b f(x)\,dx \tag{1.1}$$

$$\text{Fuerza constante.} \qquad F \mapsto W = F \cdot D \qquad D = \vec{PQ} \tag{1.2}$$

$$\int_c F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\,dt \tag{1.3}$$

Ejercicio 1.12

$$F(x,y,z) = -\frac{1}{2}x\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}} + \frac{1}{4} \tag{1.1}$$

$$r(t) = cost\hat{\mathbf{i}} + sent\hat{\mathbf{j}} + t\hat{\mathbf{k}} \tag{1.2}$$

$$(1,0,0) \mapsto (-1,0,3\pi) \quad x = cost, y = sent, z = t \tag{1.3}$$

$$F(x(t),y(t),z(t)) = -\frac{cost}{2}\hat{\mathbf{i}} - \frac{sent}{2}\hat{\mathbf{j}}\frac{1}{4} \tag{1.4}$$

$$r'(t) = -sent\hat{\mathbf{i}} + cost\hat{\mathbf{j}} + 1\hat{\mathbf{k}} \tag{1.5}$$

$$\int_c F \cdot dr = \int_0^{3\pi} (-\frac{cost}{2}\hat{\mathbf{i}} - \frac{sent}{+2}\hat{\mathbf{j}} + \frac{1}{4}) \cdot (-sent\hat{\mathbf{i}} + cost\hat{\mathbf{j}} + 1\hat{\mathbf{k}})\,dt \tag{1.6}$$

$$\int_0^{3\pi} (\frac{cost sint}{2}) - \frac{cost sent}{2} + \frac{1}{4}\,dt \tag{1.7}$$

$$\int_0^{3\pi} \frac{1}{4}\,dt = [\frac{1}{4}t]_0^{3\pi} = \frac{3\pi}{4} \tag{1.8}$$

■

Ejercicio 1.13

$F(x,y) = y\hat{\mathbf{i}} + x\hat{\mathbf{j}}$ a lo largo de $y = ln(x)$ desde (1,0) hasta (e,1). ¿Cuál es el w?

$$w = \int_c F \cdot dr \quad r(t) = x\hat{\mathbf{i}} + lnx\hat{\mathbf{j}} \tag{1.1}$$

Parametrización de x

$$F = lnx\hat{\mathbf{i}} + x\hat{\mathbf{j}} \tag{1.2}$$

$$dr = (1\hat{\mathbf{i}} + \frac{1}{x}\hat{\mathbf{j}})dx \tag{1.3}$$

$$w = \int_1^e lnx\,dx + \int_1^e 1\,ddx \qquad xln(x) - \int_1^e 1\,dx + \int_1^e 1\,dx = [xln(x)]_1^e = e \tag{1.4}$$

■

Ejercicio 1.14

¿w? $F(x,y) = a\hat{\mathbf{i}}b\hat{\mathbf{j}}$ alrededor de $x^2 + y^2 = 9 \longleftarrow r = 3\cos t\hat{\mathbf{i}} + 3\sin t\hat{\mathbf{j}}$ en donde $0 \leq t \leq 2\pi$

$dr = -3\sin t\hat{\mathbf{i}} + \cos t\hat{\mathbf{j}}$

$$w = \int_0^{2\pi} (-3asent + 3bcost)\,dt = [3acost + 3bsent]_0^{2\pi} = 0 \tag{1.1}$$

■

Ejercicio 1.15

$$\tag{1.1}$$

■

1.3 5 de octubre de 2020

Definición 1.3.1

$$\text{Campo vectorial conservativo} \tag{1.1}$$

F si, existe una función f tal que $\nabla f = F$.$f \mapsto$ función de potencial para F.

$$\tag{1.2}$$

Ejercicio 1.16

$$F(x,y) = y\hat{\mathbf{i}} + x\hat{\mathbf{j}} \qquad \nabla f = F \tag{1.1}$$

$$\nabla f = \frac{df}{dx}\hat{\mathbf{i}} + \frac{df}{dy}\hat{\mathbf{j}} = y\hat{\mathbf{i}} + x\hat{\mathbf{j}} \qquad \text{F es conservativo} \tag{1.2}$$

$$f(x,y) = xy \tag{1.3}$$

Ejercicio 1.17

$$F(x,y) = cosx\hat{\mathbf{i}} + (a + seny)\hat{\mathbf{j}} \tag{1.1}$$
$$f(x,y) = senx + y + cosy \qquad \nabla f = F \tag{1.2}$$
$$\frac{df}{dx} = cosx\frac{dd}{dy} = (1 - seny) \tag{1.3}$$

Ejercicio 1.18

$$F(x,y,z) = y^2z^3\hat{\mathbf{i}}2xyz^3\hat{\mathbf{j}}3xy^2z^2\hat{\mathbf{k}} \tag{1.1}$$
$$f(x,y,z) = xy^2z^3 \qquad \nabla f = F \tag{1.2}$$
$$\frac{df}{dx} = y^2z^3 \tag{1.3}$$

Teorema 1.3.1

Teorema fundamental de las integrales de línea

$$\int_a^b F'(x)\,dx = F(b) - F(a) \qquad \text{antiderivada} \tag{1.1}$$

Función vectorial $r(t)$ en donde $a \leq t \leq b$ tal que f es derivable ∇f es continuo

$$\tag{1.2}$$
$$\int_c \nabla f \cdot dr = f(r(b) - f(r(a)) \tag{1.3}$$
$$\tag{1.4}$$

Ejercicio 1.19

$$yz\hat{\mathbf{i}}xz\hat{\mathbf{j}}xy\hat{\mathbf{k}} \qquad \text{donde: } f(x,y,z) = xyz \tag{1.1}$$

A(-1,3,) hasta el punto B(1,6,-4)

$$\tag{1.2}$$
$$w = \int_c F \cdot dr = \int_A^B \nabla f \cdot dr = f(B) - f(A) \tag{1.3}$$
$$xyz|_(1,6,-4) - xyz|_(-1,3,9) \tag{1.4}$$
$$(1)(-4)(6) - (-1)(3)(9) = 3 \tag{1.5}$$

Teorema 1.3.2

Teorema $\int_c F \cdot dr$ es independiente de su trayectoria en D ssi $\int_c F \cdot dr$ es igual para toda trayectoria cerrada.

(1.1)

Ejercicio 1.20

$$F(x,y) = P(x,y)\hat{\mathbf{i}} + Q(x,y)\hat{\mathbf{j}} \quad (1.1)$$

$$\frac{dP}{dy} = \frac{dQ}{dx} \quad (1.2)$$

■

Ejercicio 1.21

$$F(x,y) = -ye^{-xy}\hat{\mathbf{i}} - xe^{-xy}\hat{\mathbf{j}} \qquad P = -ye^{-xy} \qquad Q = -xe^{-xuy} \quad (1.1)$$

$$\frac{dP}{dy} = xye^{-xy} - e^{-xy} \quad (1.2)$$

$$\frac{dQ}{dx} = xye^{-xy} - e^{-xy} \quad (1.3)$$

$$\frac{dP}{dy} = \frac{dQ}{dx} \quad (1.4)$$

F es conservativo. (1.5)

■

Ejercicio 1.22

$$F(x,y) = (x^2 - 2y^3)\hat{\mathbf{i}}(x+5y)\hat{\mathbf{j}} \quad (1.1)$$

$$P = x^2 - 2y^3 Q = x + 5y \quad (1.2)$$

$$\frac{dP}{dy} = -6y^2 \neq \frac{dQ}{dx} = 1 \quad (1.3)$$

■

Definición 1.3.2

$$F = P(x,y,z)\hat{\mathbf{i}} + Q(x,y,z)\hat{\mathbf{j}} + R(x,y,z)\hat{\mathbf{k}} \quad (1.1)$$

$$\frac{dP}{dy} = \frac{dQ}{dx}\frac{dP}{dz} = \frac{dR}{dx} \qquad \frac{dQ}{dz} = \frac{dR}{dy} \quad (1.2)$$

(1.3)

Ejercicio 1.23

$$F = (e^x cosy + yz)\hat{\mathbf{i}} + (xz - e^x seny)\hat{\mathbf{j}}(xy+z)\hat{\mathbf{k}} \tag{1.1}$$

$$\frac{dP}{dy} = \frac{dQ}{dx} = -e^x seny + z \tag{1.2}$$

$$\frac{dP}{dz} = \frac{dR}{dx} = y \tag{1.3}$$

$$\frac{dQ}{dz} = \frac{dR}{dy} = x \tag{1.4}$$

F sí es conservativo. (1.5)

$$\frac{df}{dx} = (e^x cosy + yz)\frac{df}{dy} = xz - e^x seny \quad \frac{df}{dz} = xy + z \tag{1.6}$$

$$f(x,y,z) = e^x cosy + xyz + g(y,z) \tag{1.7}$$

$\frac{dg}{dy} = xz - xz - e^x seny + e^x seny = 0$

$$\frac{df}{dy} = -e^x seny + xz + \frac{dg}{dy} = xz - e^x seny \tag{1.8}$$

$$f(x,y,z) = e^x cosy + xyz + h(z) \tag{1.9}$$

$$\frac{df}{dz} = xy + \frac{dh}{dz} = xy + z \qquad \frac{dh}{dz} = z \tag{1.10}$$

$$h(z) = \frac{z^2}{2} + c \tag{1.11}$$

$$f(x,y,z) = e^x cosy + xyz + \frac{z^2}{2} + c \tag{1.12}$$

■

Ejercicio 1.24

$$F = -\frac{y}{x^2+y^2}\hat{\mathbf{i}} + \frac{x}{x^2+y^2}\hat{\mathbf{j}}0\hat{\mathbf{k}} \tag{1.1}$$

$$\frac{dP}{dy} = \frac{dQ}{dx} = \frac{-x^2+y^2}{(x^2+y^2)^2} \tag{1.2}$$

$$\frac{dP}{dz} = \frac{dR}{dx} = 0 \tag{1.3}$$

$$\frac{dQ}{dz} = \frac{dR}{dy} = 0 \tag{1.4}$$

F no es conservativo (1.5)

■

Ejercicio 1.25

$$r(t) = cost\hat{\mathbf{i}} + sent\hat{\mathbf{j}} \qquad 0 \leq t \leq 2\pi \tag{1.1}$$

$$F = -\frac{sent}{(\sin^2) + (\cos^2 t)}\hat{\mathbf{i}} + \frac{\cos t}{(\sin^2) + (\cos^2 t)}\hat{\mathbf{j}} \tag{1.2}$$

$$= -sent\hat{\mathbf{i}} + cost\hat{\mathbf{j}} \tag{1.3}$$

$$dr = (-sent\hat{\mathbf{i}} + cost\hat{\mathbf{j}} \tag{1.4}$$

$$\int_c F \cdot ddr = \int_c F \cdot \frac{dr}{dt} dt = \int_0^{2\pi} (sen^2 t + cos^2 t)\, dt = 2\pi \tag{1.5}$$

■

Ejercicio 1.26

$$\int_c (y + yz)\, dx + (x + 3z^2 + xz)dy + (9yz^2 + xy - 1)dz) \tag{1.1}$$

Demostrar que es independiente de la trayectoria (1,1,1) y (2,1,4)

$$\frac{dP}{dy} = \frac{dQ}{dz} = 1 + z \tag{1.2}$$

$$\frac{dP}{dz} = \frac{dR}{dx} = y \tag{1.3}$$

$$\frac{dQ}{dz} = \frac{dR}{dy} = 9z^2 + x \tag{1.4}$$

$$\int_{(1,1,1)}^{(2,1,4)} F \cdot dr \tag{1.5}$$

$$f = xy + xyz + g(y,z) \tag{1.6}$$

$$\frac{df}{dy} = x + xz + \frac{dg}{dy} = x + 3z^2 + xz \qquad \frac{dg}{dy} = 3z^2 \tag{1.7}$$

$$g(y \quad = 3yz^3 + h(z) \tag{1.8}$$

$$f = xy + xyz + 3yz^3 + h(z) \tag{1.9}$$

$$\frac{df}{dz} = xy + 9yz^2 + \frac{dh}{dz} = 9yz^2 + xy - 1 \qquad h(z) = -z + c \tag{1.10}$$

$$f = xy + xyz + 3yz^3 + (-z) \tag{1.11}$$

$$\int_{(1,1,1)}^{(2,1,3)} F \cdot dr = [xy + xyz + 3yz^3 - z]_{(1,1,1)}^{(2,1,4)} = 198 - 40 = 194 \tag{1.12}$$

■

1.4 13 de octubre de 2020

Ejercicio 1.27

$$\int_{(0,0,0)}^{(2,3,-6)} 2x\,dx + 3ydy + 2zdz \tag{1.1}$$

$$\int_{(0,0,0,0)}^{(2,3,6)} F \cdot dr \tag{1.2}$$

$$F(x,y,z) = 2x\hat{\mathbf{i}} + 2y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}} \quad \frac{dP}{dy} = 0 = \frac{dQ}{dx} \tag{1.3}$$

$$\frac{dQ}{dz} = 0 = \frac{dR}{dy} \quad \frac{dP}{dz} = 0 = \frac{dR}{dx} \qquad F = \nabla f \tag{1.4}$$

$$\frac{df}{dx} = 2x \quad f(x,y,z) = x^2 + g(y,z) \tag{1.5}$$

$$\frac{df}{dy} = \frac{dq}{dy} = 2y \mapsto g(y) = y^2 + h(z) \tag{1.6}$$

$$\frac{df}{dz} = \frac{dh}{dz} = 2z \mapsto h(z) = z^2 \tag{1.7}$$

$$\int_{(0,0,0)}^{(2,3,-6)} 2x\,dx + 3ydy + 2zdz \tag{1.8}$$

$$= [(x^2 + y^2 + z^2)]_{0,0,0}^{2,3,-6} = z^2 + 3^2 + (-6)^2 = 49 \tag{1.9}$$

■

Ejercicio 1.28

$$\int_{(1,0,0)}^{(0,1,1)} senycosx\,dx + cosysenxdy + dz \tag{1.1}$$

$$\frac{dP}{dy} = \frac{dQ}{dx} = cosycosx \quad \frac{dQ}{dz} = \frac{dR}{dy} = 0 \qquad \frac{dP}{dz} = \frac{dR}{dx} = 0 \tag{1.2}$$

$$\frac{df}{dx} = senycosx \quad f(x,y,z) = senysenx + g(y,z) \tag{1.3}$$

$$\frac{df}{dy} = cosysenx + \frac{dg}{dy} = cosysenx \quad g(y) = h(z) \tag{1.4}$$

$$\frac{df}{dz} = \frac{dh}{dz} = 1 \quad h(z) = z \qquad f(x,y,z) = senysenx + z \tag{1.5}$$

$$\int_{(1,0,0)}^{(0,1,1)} F \cdot dr = [senysenx + z]_{(1,0,0)}^{(0,1,1)} = 1 \tag{1.6}$$

■

Ejercicio 1.29

$F = \nabla(x^3y^2)$ y C: trayectoria en plano xy que va desde (-1,1) a (1,1) y que consiste en los segmentos. (-1,1) a (0,0) y (0,0) a (1,1)

Evalúe $\int_c F \cdot dr$ para los parámetros.

$$F = \nabla(x^3y^2) \implies F = 3x^2y^2\hat{\mathbf{i}} + 2x^3y\hat{\mathbf{j}} \tag{1.1}$$

$$C_1 : (-1,1) \implies (0,0) \tag{1.2}$$

$$x = t-1 y = -t+1 \quad 0 \le t \le 1 \tag{1.3}$$

$$F = 3(t-1)^2(-t+1)^2\hat{\mathbf{i}}2(t-1)^3(-t+1)\hat{\mathbf{j}} \tag{1.4}$$

$$r = (t-1)\hat{\mathbf{i}}(-t+1)\hat{\mathbf{j}} \tag{1.5}$$

$$dr = dt\hat{\mathbf{i}} - dt\hat{\mathbf{j}} \tag{1.6}$$

$$\int_0^1 (3(t-1)^2(-t+1)^2\,dt - 2(t-1)^3(-t+1)dt = 1 \tag{1.7}$$

$$C_2 : (0,0) \mapsto (1,1) \tag{1.8}$$

$$x = ty = t \qquad 0 \le t \le 1 \tag{1.9}$$

$$F = 3t^4\hat{\mathbf{i}} + 2t^4\hat{\mathbf{j}} \tag{1.10}$$

$$r = t\hat{\mathbf{i}}t\hat{\mathbf{j}}dr = dt\hat{\mathbf{i}}dt\hat{\mathbf{j}} \tag{1.11}$$

$$\int_0^1 (3t^4 + 2t^4)\,dt = 1 \tag{1.12}$$

$$\int_{c_1} F \cdot dr + \int_{c_2} F \cdot dr = 1+1 = 2 \tag{1.13}$$

Use $f(x,y) = x^3y^2$ como una función potencial para F.

$$\tag{1.14}$$

$$f(x,y) = x^3y^2 = \int_{(-1,1)}^{(1,1)} F \cdot dr = [x^3y^2]_{(-1,1)}^{(1,1)} = 1-(-1) = 2 \tag{1.15}$$

■

1.5 12 de octubre de 2020

1.5.1 Teorema de Green

R **Orientación positiva:** Sentida antihorario.
Orientación negativa: Sentida horario.

Ejercicio 1.30

$$\oint_C (x^2-y^2)dx+(2y-x)dy \tag{1.1}$$
$$c: y=x^2, y=x^3 \tag{1.2}$$
$$P=x^2-y^2 \tag{1.3}$$
$$Q=2y-x \tag{1.4}$$
$$\frac{dQ}{dx}=-1 \qquad \frac{dP}{dy}=-2y \tag{1.5}$$
$$\iint_D(-1-(-2y))dA=\int_0^1\int_{x^3} x^2(-1+y)dydx \tag{1.6}$$
$$\int_0^1[-y+y^2]_{x^3}^{x^2}\,dx=\int_0^1(-x^6+x^4+x^3-x^2)\,dx \tag{1.7}$$
$$\frac{-x^7}{7}+\frac{x^5}{5}+\frac{x^4}{4}-\frac{x^3}{4}]_0^1=\frac{-11}{420} \tag{1.8}$$

■

Ejercicio 1.31

$$\oint_C (x^5+3y)\,dx+(2x-e^{y^2})dy \qquad c:(x-1)^2+(y-5)^2=4 \tag{1.1}$$
$$P=x^5+3y \qquad \frac{dP}{dy}=3 \tag{1.2}$$
$$Q=2x-e^{y^2} \qquad \frac{dQ}{dx}=2 \tag{1.3}$$
$$\iint_D(2-3)dA=-\iint_D 1dA \qquad A=\pi r^2=\pi(2)^2=4\pi \tag{1.4}$$
$$-\iint_P 1dA=-4\pi \tag{1.5}$$

■

Ejercicio 1.32

$$F=(-16y+sen(x^2)\hat{\mathbf{i}}+(4e^y+3x^2)\hat{\mathbf{j}} \tag{1.1}$$
$$w=\oint_c F\cdot dr \tag{1.2}$$
$$w=\oint_c(-16y+sen(x^2))\,dx+(4e^y+3x^2)dy \tag{1.3}$$
$$w=\iint_D(6x+16)dA=\int_{\pi/4}^{3\pi/4}\int_0^1(6rcos\theta+16)rdrd\theta \tag{1.4}$$
$$=\int_{\pi/4}^{3\pi/4}\int_0^1(6r^2cos\theta+16r)drd\theta \tag{1.5}$$
$$=\int_{\pi/4}^{3\pi/4}(2r^3cos\theta+8r^2)]_0^1\,d\theta \tag{1.6}$$
$$=\int_{\pi/4}^{3\pi/4}(2cos\theta+8)\,d\theta \qquad 4\pi \tag{1.7}$$

Definición 1.5.1

$$\iint_D (\frac{dQ}{dx} - \frac{dP}{dy})dA \tag{1.1}$$

$$\iint_{D_1} (\frac{dQ}{dx} - \frac{dP}{dy})dA + \iint_{D_2} (\frac{dQ}{dx} - \frac{dP}{dy})dA \tag{1.2}$$

$$= \oint_{C_1} Pdx + Qdy + \oint_{C_2} Pdx + Qdy \tag{1.3}$$

$$= \oint_C Pdx + Qdy \tag{1.4}$$

Ejercicio 1.33

$$\oint_c \frac{1}{3}y^3 dx + (xy + xy^2)dy \tag{1.1}$$

c: es la frontera de la región en el primer cuadrante de $y = 0, x = y^2, x = 1 - y^2$

$$P = \frac{1}{3}y^3 \qquad P_y = y^2 \tag{1.2}$$

$$Q = xy + xy^2 \qquad Q_x = y + y^2 \tag{1.3}$$

$$\iint_D [(y + y^2) - y^2]dA = \int_0^{\frac{1}{\sqrt{2}}} \int_{y^2}^{1-y^2} (y)dxdy \tag{1.4}$$

$$\int_0^{1/\sqrt{2}} xy]_{y^2}^{1-y^2} = \int_0^{1/\sqrt{2}} (y - 2y^3)\, dy = \frac{y^2}{2} - \frac{y^4}{2}]_0^{1/\sqrt{2}} = \frac{1}{8} \tag{1.5}$$

Ejercicio 1.34

$$\oint_c e^{x^2} dx + 2\tan^{-1} x dy \tag{1.1}$$

triángulo con vértices (0,0),(0,1),(-1,1)

$$P = e^{x^2} \qquad Q = 2\tan^{-1} x \tag{1.2}$$

$$P_y = 0 \qquad Q_x = \frac{2}{1 + x^2} \tag{1.3}$$

$$\oint_c e^{x^2} dx + 2\tan^{-1} x dy = \iint_D \frac{2}{1 + x^2} dA \tag{1.4}$$

$$\int_{-1}^0 \int_{-x}^1 \frac{2}{1 + x^2} dydx \tag{1.5}$$

$$\int_{-1}^0 \frac{2y}{1 + x^2}]_{-x}^1 dx = \int_{-1}^0 (\frac{2}{1 + x^2} + \frac{2x}{1 + x^2})\, dx \tag{1.6}$$

$$[2\tan^{-1} x + ln(1 + x^2)]_{-1}^0 = \frac{\pi}{2} - ln(2) = 0{,}87 \tag{1.7}$$

1.6 13 de octubre de 2020

Ejercicio 1.35

$$\oint_c (4x^2 - y^3)dx + (x^3 + y^2)dy \tag{1.1}$$

La gráfica: $x^2 + y^2 = 1$ y $x^2 + y^2 = 4$

$$P_y = -3y^2 \qquad Q_x = 3x^2 \tag{1.2}$$

$$\iint_d (3x^3 + 3y^2)dA = \int_0^{2\pi} \int_1^2 3r^2 rdrd\theta \tag{1.3}$$

$$\int_0^{2\pi} \frac{3r^4}{4} d\theta]_1^2 \tag{1.4}$$

$$\int_0^{2\pi} \frac{45}{4} d\theta = \frac{45\pi}{2} \tag{1.5}$$

$$\tag{1.6}$$

■