Problema 2 $\quad \dfrac{\partial^2 y}{\partial x^2} = \dfrac{\partial^2 y}{\partial t^2}$

$$\frac{\partial y}{\partial t}(x,0) = 0$$

$$y(x,0) = \begin{cases} 2-|x| , & -2 \le x \le 2 \\ 0 & |x|>2 \end{cases}$$

$2-x$ y $2+x$

SV:

$$X''T = T''X \Rightarrow \frac{X''}{X} = \frac{T''}{T} = -w^2$$

$X'' + w^2 X = 0$	$T'' + w^2 T = 0$
$X(x) = A\cos wx + B\,\text{sen}\,wx$	$T(t) = C\cos wt + D\,\text{sen}\,wt$
	$T'(t) = -Cw\,\text{sen}\,wt + Dw\cos wt$
	$T'(0) = 0 + Dw = 0$
	$D = 0$
	$T_w = C_w \cos wt$

$$X(x)T(t) = \cos wt\,[A\cos wx + B\,\text{sen}\,wx]$$

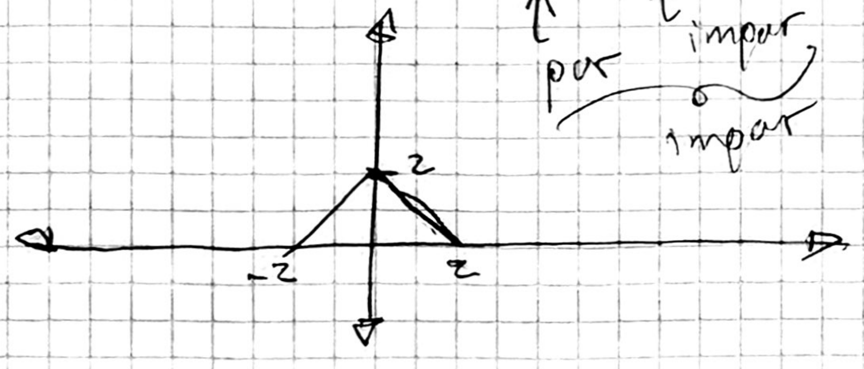
Por superposición:

$$* \; u(x,y) = \int_0^\infty \cos wt\,[A\cos wx + B\,\text{sen}\,wx]\,dw$$

$$u(x,0) = \int_0^\infty [A\cos wx + \overset{0}{\cancel{B\,\text{sen}\,wx}}]\,dw = \begin{cases} 2-|x|, & -2\le x\le 2 \\ 0, & |x|>2 \end{cases}$$

par $\quad$ impar

impar



$$\begin{cases} 2-|x| , & -2 \le x \le 2 \\ 0 & |x|>2 \end{cases} \quad \text{es par}$$

$f(x) = \int_0^\infty [A\cos wx + \cancel{B \operatorname{sen} wx}]\, dw$

$= \frac{2}{\pi}\int_0^\infty A_w \cos wx\, dw$

$\Rightarrow$

$A_w = \int_0^\infty f(x)\cos\alpha x\, dx = \int_0^2 (2-x)\cos\alpha x\, dx$

+	$2-x$	$\cos\alpha x$
−	-1	$\frac{1}{\alpha}\operatorname{sen}\alpha x$
+	0	$-\frac{1}{\alpha^2}\cos\alpha x$

$\Rightarrow \cancel{\frac{2-x}{\alpha}\operatorname{sen}\alpha x\Big|_0^2} - \frac{1}{\alpha^2}\cos\alpha x\Big|_0^2 =$

$= -\frac{1}{\alpha^2}[\cos 2x - 1]$

$f(x) = \int_0^\infty \frac{2}{\pi w^2}[1-\cos 2x] \cdot \cos wx\, dw$

$\Rightarrow$ Reemplazando en $*$

$* \quad u(x,y) = \int_0^\infty \cos wt \left[\left(\frac{2}{\pi w^2}[1-\cos 2x]\right)\cos wx\right] dw$

$\cancel{= \int_0^\infty \frac{2}{\pi w^2}}$

posición

Problema 3

$$\frac{\partial^2 u}{\partial x^2} = k\frac{\partial u}{\partial t} \qquad -\infty < x < \infty,\ t > 0$$

$$u(x,0) = \begin{cases} \operatorname{sen} x, & -\pi \le x \le \pi \\ 0, & |x| > \pi \end{cases}$$

$\Rightarrow$ SPV:

$$X''T = kT'X \qquad (\sqrt{k})^2 w^2 \quad (\sqrt{k}w)^2$$

$$\Rightarrow \quad \frac{X''}{kX} = \frac{T'}{T} = -w^2$$

$\Rightarrow$

$X'' + kw^2 X = 0$ | $T' + w^2 T = 0$

$X(x) = A\operatorname{sen}(\sqrt{k}wx)$ ~~+Bcos~~ $+ B\cos(\sqrt{k}wx)$ | $T(t) = Ce^{-w^2 t}$

$$X(x)\cdot T(t) = e^{-w^2 t}\left[A\operatorname{sen}(\sqrt{k}wx) + B\cos(\sqrt{k}wx)\right]$$

Por superposición:

$$u(x,t) = \int_0^\infty e^{-wt}\left[A\operatorname{sen}\sqrt{k}wx + B\cos\sqrt{k}wx\right]dw$$

Condición:

$$u(x,0) = f(x) = \frac{2}{\pi}\int_0^\infty \left[A\operatorname{sen}\sqrt{k}wx + \cancel{B\cos\sqrt{k}wx}^{\,0}\right]dw$$

$f(x)$ ~~impar~~ $A(w) = \int_0^\infty f(x)\operatorname{sen}\sqrt{k}wx\,dx =$

$$= \int_0^\pi \operatorname{sen} x \operatorname{sen}\sqrt{k}w x\,dx =$$

$$= \frac{1}{2}\int_0^\pi \left[\cos((1-\sqrt{k}w)x) - \cos((1+\sqrt{k}w)x)\right]dx$$

$$\frac{1}{2}\left[\frac{1}{1-\sqrt{kw}}\,\cancel{\operatorname{sen}}^{0}((1-\sqrt{kw})x)\Big|_0^{\pi} - \frac{1}{1+\sqrt{kw}}\operatorname{sen}((1+\sqrt{kw})x)\Big|_0^{\pi}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\sqrt{kw}}\operatorname{sen}(1-\sqrt{kw})\pi - \frac{1}{1+\sqrt{kw}}\operatorname{sen}(1+\sqrt{kw})\pi\right]$$

$$= \frac{1}{2}\operatorname{sen}(1-\sqrt{kw})\pi\left[\frac{1}{1-\sqrt{kw}} - \frac{1}{1+\sqrt{kw}}\right]$$

$$\Rightarrow f(x) = \int_0^{\infty}\frac{1}{\pi}\operatorname{sen}((1-\sqrt{kw})\pi)\left[\frac{1}{1-\sqrt{kw}} - \frac{1}{1+\sqrt{kw}}\right]\operatorname{sen}\sqrt{kw}\,dw$$

$$u(x,t) = \int_0^{\infty} e^{-w^2 t}\left(\frac{1}{\pi}\operatorname{sen}((1-\sqrt{kw})\pi)\right)\left[\frac{1}{1-\sqrt{kw}} - \frac{1}{1+\sqrt{kw}}\right]\operatorname{sen}\sqrt{kw}\,x\,dw$$
