Parcial 3

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$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad , \text{ sujeta: } u(x,0) = f(x)$$

$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

Separación de variable:

$$X''T = XT'' \Rightarrow \frac{X''}{X} = \frac{T''}{T} = -w^2$$

① $\frac{X''}{X} = -w^2 \Rightarrow X'' + w^2 X = 0$

$\Rightarrow X(x) = A_w \cos wx + B_w \operatorname{sen} wx$

② $T'' + w^2 T = 0$

$T(t) = C_w \cos wt + D_w \operatorname{sen} wt$

$T'(t) = C_w w \cancel{\operatorname{sen} wt} + D_w w \cos wt$

$T'(0) = B_w w = g(x)$

$\Rightarrow B_w = \frac{g(t)}{w}$

$$\Rightarrow X(x)T(t) = [A_w \cos wx + B_w \operatorname{sen} wx]\left[C_w \cos wt + \frac{g(x)}{w} \operatorname{sen} wt\right]$$

Por superposición:

$$* \; U(x,t) = \int_0^\infty [A_w \cos wx + B_w \operatorname{sen} wx]\left[C_w \cos wt + \frac{g(t)}{w} \operatorname{sen} wt\right] dw$$

Aplicando la condición

$$u(x,0) = f(x) = \int_0^\infty [A_w \cos wx + B_w \operatorname{sen} wx][C_w]\, dw$$

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Fourier $\Rightarrow A_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x) \cos \psi x \, d\psi \; ; \; B_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x) \operatorname{sen} \psi x \, d\psi$

$\Rightarrow$ Solo se reemplazan A_w y B_w en $*$

(2) $\dfrac{\partial^2 u}{\partial t \partial x} = \dfrac{\partial^2 u}{\partial x^2}$, sujeta a $u(x,0) = f(x)$

Por separación de variables

$\Rightarrow u(x,t) = X \cdot T$

$\Rightarrow X'T' = X''T \Rightarrow \dfrac{T'}{T} = \dfrac{X''}{X'} = -w^2$

(1) $T' + w^2 T = 0$

$\Rightarrow T(t) = A_w e^{-w^2 t}$

$X'' + w^2 X' = 0$

$X(x) = B_w \cos wx + C_w \operatorname{sen} wx$

$\Rightarrow$ ~~Por superp~~

$X(x) \cdot T(t) = [B_w \cos wx + C_w \operatorname{sen} wx] A_w e^{-w^2 t}$

Por superposición:

$* \; u(x,t) = \int_0^{\infty} A_w e^{-w^2 t} [B_w \cos wx + C_w \operatorname{sen} wx] \, dw$

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Aplicando la condición:

$u(x,0) = \int_0^{\infty} [B_w \cos wx + C_w \operatorname{sen} wx] \, dw$

$B_w = \dfrac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos w\psi \, d\psi$; $C_w = \dfrac{1}{\pi} \int_{-\infty}^{\infty} f(x) \operatorname{sen} w\psi \, d\psi$

$\Rightarrow$ Reemplazar B_w y C_w en $*$.

③ $-c^2 \frac{\partial^4 u}{\partial x^4} = \frac{\partial^2 u}{\partial t^2}$

sujeta a : $u(x,0) = f(x)$, $\frac{\partial u}{\partial t}(x,0) = g(x)$

Por separación de variables: 18

$\Rightarrow -c^2 X^{(4)} T = X T'' \Rightarrow -\frac{X^{(4)}}{X} = \frac{T''}{c^2 T} = -w^2$

$\Rightarrow$ ① $X^{(4)} - w^2 X = 0$

$\Rightarrow m^4 - w^2 = 0$

$\Rightarrow (m^2 - w)(m^2 + w) = 0$

$\Rightarrow m = \pm\sqrt{w}$, $m = \pm i\sqrt{w}$

$\Rightarrow X(x) = A_w e^{\sqrt{w}x} + B_w e^{-\sqrt{w}x} + C_w \cos(\sqrt{w}x) + D_w \operatorname{sen}(\sqrt{w}x)$

② $T'' + w^2 c^2 T = 0$

$T'' + (wc)^2 T = 0$

$T(t) = E_w \cos wct + F_w \operatorname{sen} wct$

$T'(t) = -E_w wc \operatorname{sen} wct + F_w wc \cos wct$

$T'(0) = g(x) = F_w wc =$

$\Rightarrow F_w = \frac{g(x)}{wc}$

$\Rightarrow$ ~~$X(x)T(t) = [A_w e^{\sqrt{w}x} + B_w$~~ $u(x,t) = X(x)T(t)$

Por superposición:

$\ast\ u(x,t) = \int_0^\infty \left[A_w e^{\sqrt{w}x} + B_w e^{-\sqrt{w}x} + C_w \cos(\sqrt{w}x) + D_w \operatorname{sen}(\sqrt{w}x)\right] \cdot \left[E_w \cos(wct) + \frac{g(x)}{wc} \operatorname{sen}(wct)\right] dw$

Aplicando la condición:

$u(x,0) = f(x) = \int_0^\infty E_w \left[A_w e^{\sqrt{w}x} + B_w e^{-\sqrt{w}x} + C_w \cos(\sqrt{w}x) - D_w \operatorname{sen}(\sqrt{w}x)\right] \cdot dw$

??

$\Rightarrow A_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(\sqrt{w}\,\psi)\,d\psi$; $D_w = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\operatorname{sen}(\sqrt{w}\,\psi)\,d\psi$

Reemplazando en $\ast$

(4) $t\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0$ sujeta a:
$u(x,0) = f(x)$

Por separación de variables:

$tX'T + XT' = 0 \Rightarrow tX'T = -XT'$

$\Rightarrow -\frac{X'}{X} = \frac{T'}{tT} = -w^2$

(1) $\Rightarrow X' = w^2 X \Rightarrow X' - w^2 X = 0$

$X(x) = A_w e^{w^2 x}$

(2) $T' = -w^2 t T \Rightarrow$

$\Rightarrow T' + w^2 t T = 0$

$\Rightarrow T(t) = B_w e^{-\frac{(w^2 t^2)}{2}}$

$X(x)T(t) = A_w e^{w^2 x} \cdot B_w e^{-\frac{(w^2 t^2)}{2}}$

Por superposición:

$u(x,t) = \int_0^\infty F_w e^{w^2 x} \cdot e^{-\frac{(w^2 t^2)}{2}} dw$

Aplicando la condición ~~de~~:

$u(x,0) = f(x) = \int_0^\infty F_w e^{w^2 x} dw$

en donde F_w es:

$F_w = \frac{1}{w^2 + iw}$

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