Universidad del Valle de Guatemala
Departamento de Matemática
Licenciatura en Matemática Aplicada

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Parcial 3 - JEFE CASI FINAL

Definition 15.3.1 Fourier Integral

The **Fourier integral** of a function f defined on the interval $(-\infty, \infty)$ is given by

$$f(x) = \frac{1}{\pi}\int_0^\infty [A(\alpha)\cos\alpha x + B(\alpha)\sin\alpha x]\,d\alpha, \tag{4}$$

where

$$A(\alpha) = \int_{-\infty}^{\infty} f(x)\cos\alpha x\,dx \tag{5}$$

$$B(\alpha) = \int_{-\infty}^{\infty} f(x)\sin\alpha x\,dx. \tag{6}$$

Theorem 15.3.1 Conditions for Convergence

Let f and f' be piecewise continuous on every finite interval, and let f be absolutely integrable on $(-\infty, \infty)$.* Then the Fourier integral of f on the interval converges to $f(x)$ at a point of continuity. At a point of discontinuity, the Fourier integral will converge to the average

$$\frac{f(x+) + f(x-)}{2},$$

where $f(x+)$ and $f(x-)$ denote the limit of f at x from the right and from the left, respectively.

Definition 15.3.2 Fourier Cosine and Sine Integrals

(*i*) The Fourier integral of an even function on the interval $(-\infty, \infty)$ is the **cosine integral**

$$f(x) = \frac{2}{\pi}\int_0^\infty A(\alpha)\cos\alpha x\, d\alpha, \tag{8}$$

where
$$A(\alpha) = \int_0^\infty f(x)\cos\alpha x\, dx. \tag{9}$$

(*ii*) The Fourier integral of an odd function on the interval $(-\infty, \infty)$ is the **sine integral**

$$f(x) = \frac{2}{\pi}\int_0^\infty B(\alpha)\sin\alpha x\, d\alpha, \tag{10}$$

where
$$B(\alpha) = \int_0^\infty f(x)\sin\alpha x\, dx. \tag{11}$$

Definition 15.4.1 Fourier Transform Pairs

(*i*) Fourier transform:
$$\mathscr{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{i\alpha x}\, dx = F(\alpha) \tag{5}$$

Inverse Fourier transform:
$$\mathscr{F}^{-1}\{F(\alpha)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\alpha)e^{-i\alpha x}\, d\alpha = f(x) \tag{6}$$

(*ii*) Fourier sine transform:
$$\mathscr{F}_s\{f(x)\} = \int_0^\infty f(x)\sin\alpha x\, dx = F(\alpha) \tag{7}$$

Inverse Fourier sine transform:
$$\mathscr{F}_s^{-1}\{F(\alpha)\} = \frac{2}{\pi}\int_0^\infty F(\alpha)\sin\alpha x\, d\alpha = f(x) \tag{8}$$

(*iii*) Fourier cosine transform:
$$\mathscr{F}_c\{f(x)\} = \int_0^\infty f(x)\cos\alpha x\, dx = F(\alpha) \tag{9}$$

Inverse Fourier cosine transform:
$$\mathscr{F}_c^{-1}\{F(\alpha)\} = \frac{2}{\pi}\int_0^\infty F(\alpha)\cos\alpha x\, d\alpha = f(x) \tag{10}$$