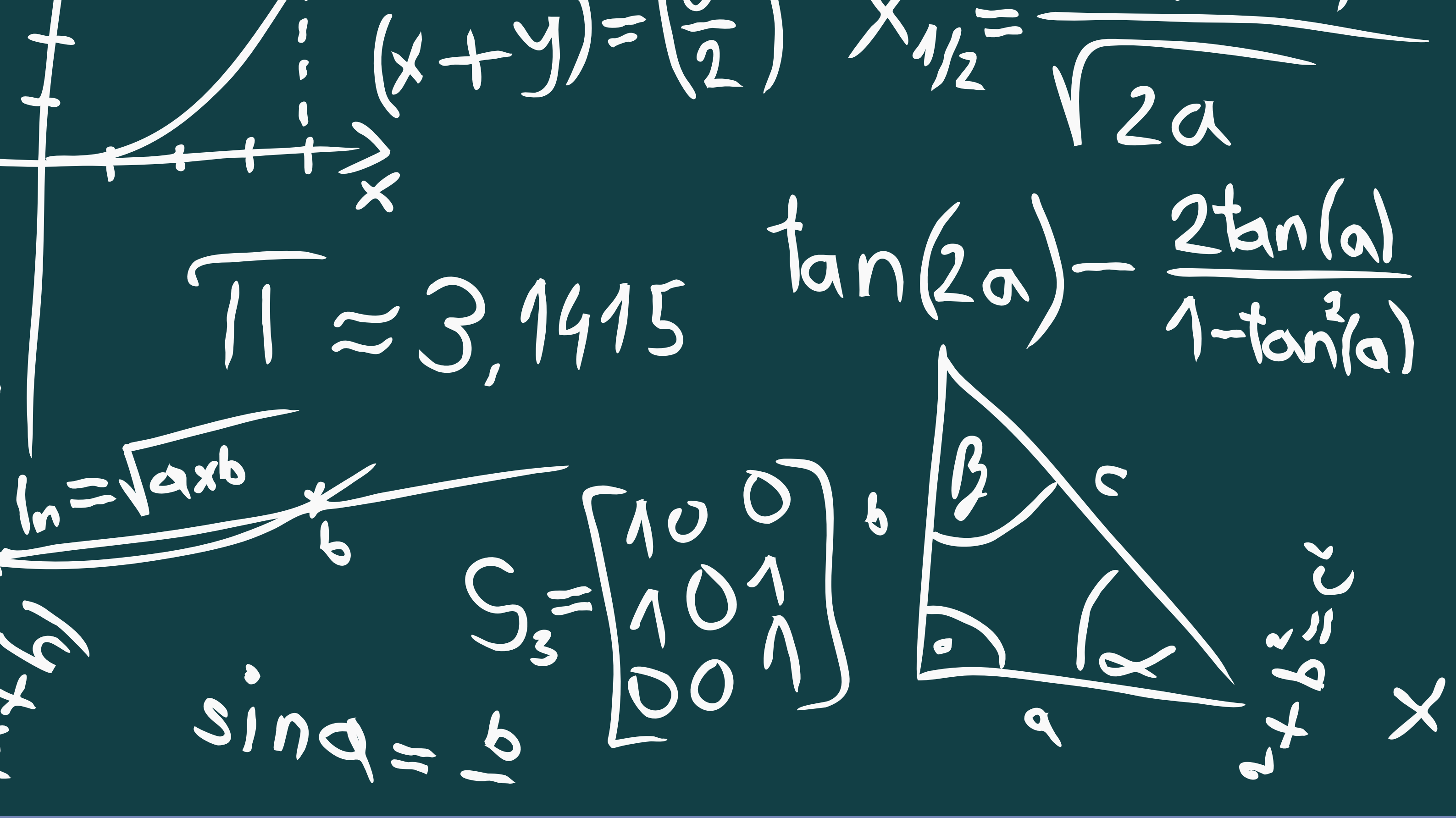
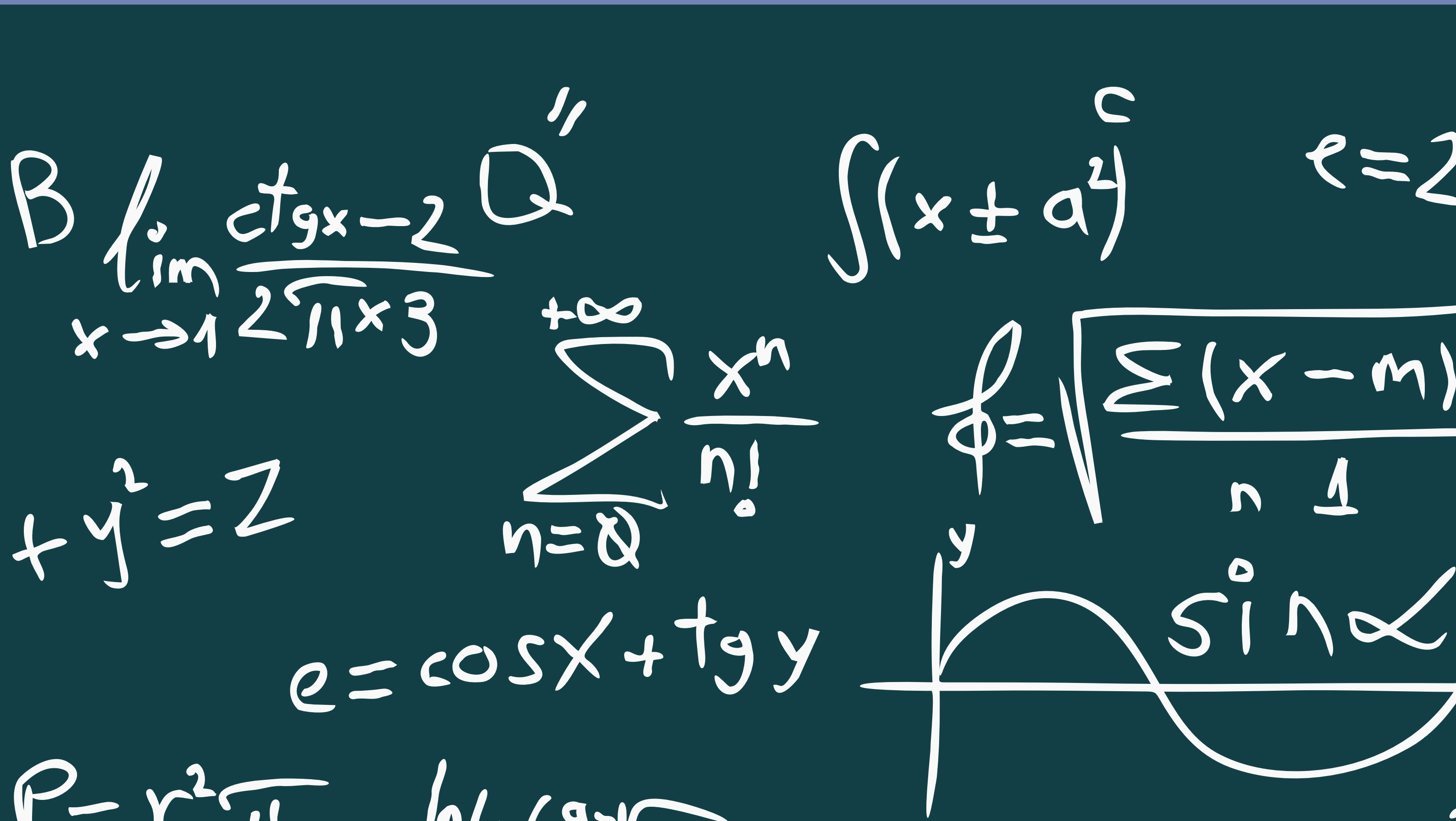


Ecuaciones Diferenciales 1

Notas de clases

Rudik Roberto Rompich



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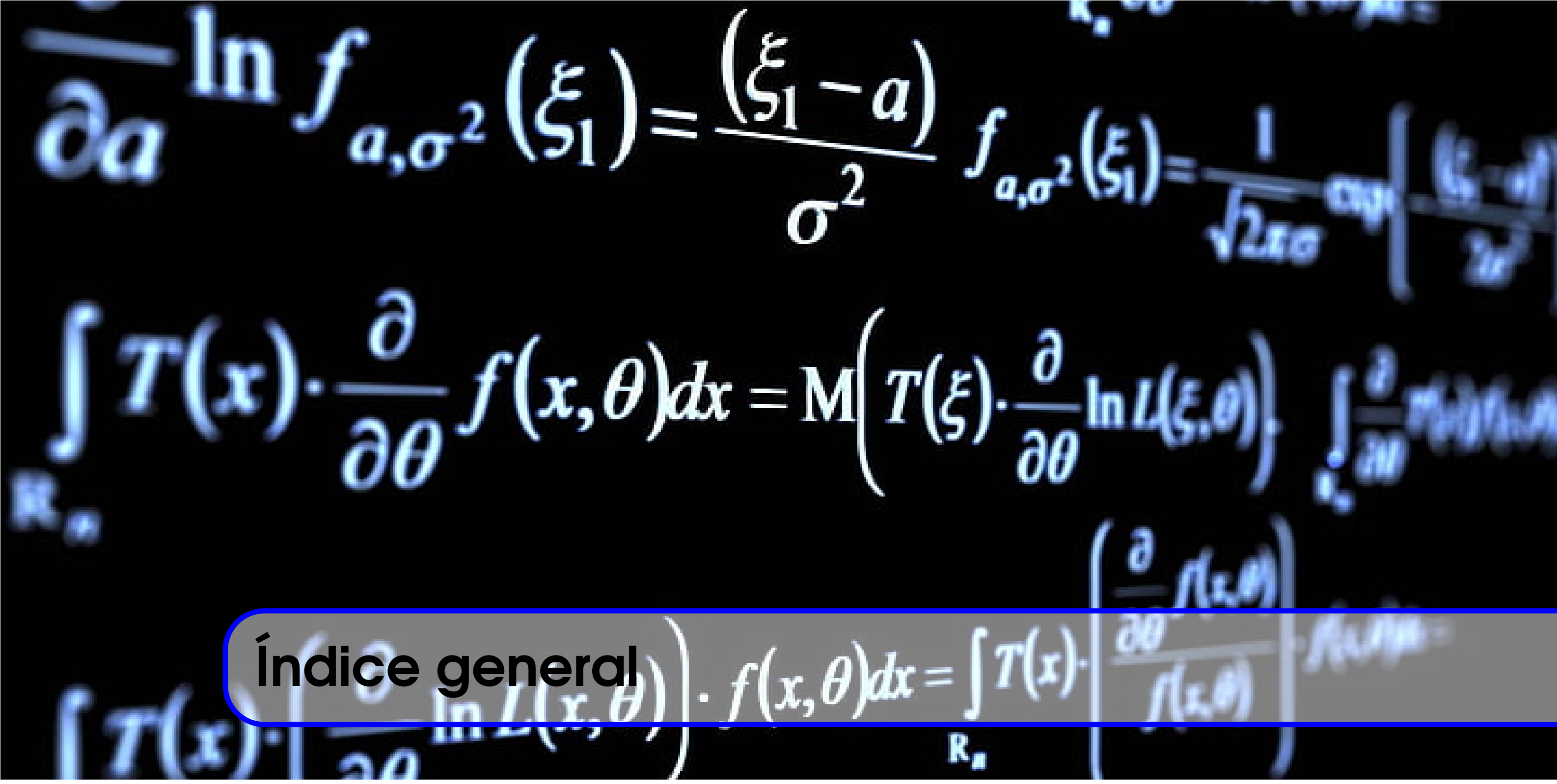
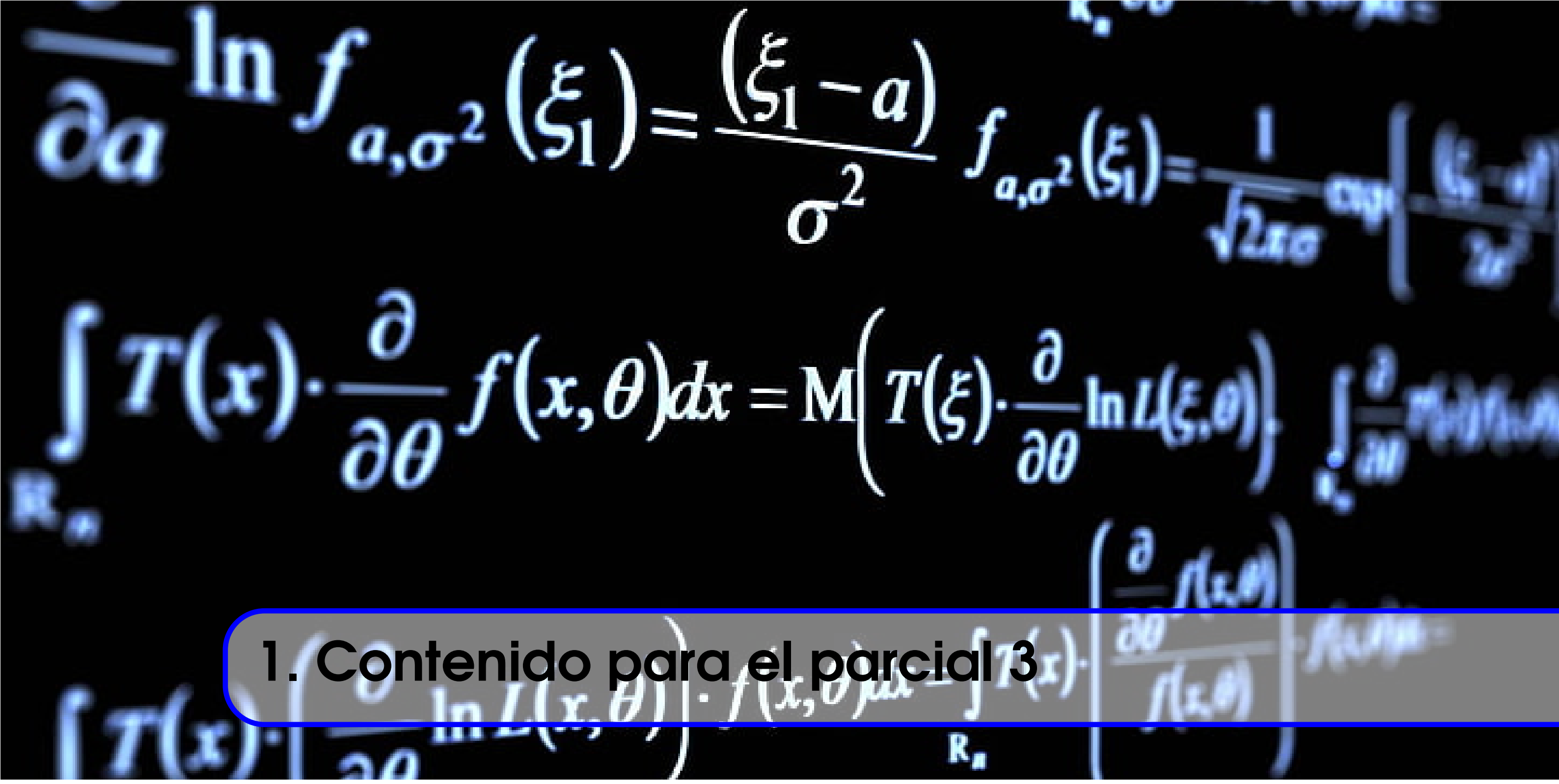


Índice general

I Transformada de Laplace



1. Contenido para el parcial 3

1.1 14 de octubre de 2020

1.1.1 Ejercicio 2

$$y'' + 2y' + y = \delta(t-1) \qquad y(0) = 0, y'(0) = 0 \tag{1.1}$$

$$\mathscr{L}[y''] + 2\mathscr{L}[y'] + \mathscr{L}[y] = \mathscr{L}[\delta(t-1)] \tag{1.2}$$

$$s^2\mathscr{L}[y] - sy(0) - y'(0) + 2[s\mathscr{L}[y] - y(0)] + \mathscr{L}[y] = e^{-s} \tag{1.3}$$

$$(s^2 + 2s + 1)\mathscr{L}[y] = e^{-s} \tag{1.4}$$

$$\mathscr{L}[y] = e^{-s}(\frac{1}{(s+1)^2}) \tag{1.5}$$

Primer teorema de traslación

$$y = \mathscr{L}^{-1}[e^{-s}\frac{1}{(s+1)^2}] \qquad \mathscr{L}^{-1}[\frac{1}{(s+1)^2}] = te^{-t} \tag{1.6}$$

Segundo teorema de traslación

$$y = (t-1)e^{-(t-1)} * H(t-1) \tag{1.7}$$

1.1.2 Ejercicio para casa

$$y''' + y = e^t + \delta(t-1) \qquad y(0) = 1, y'(0) = 1, y''(0) = 2 \tag{1.1}$$

Por ejemplo para orden superior:

$$\mathscr{L}[y'''] = s^3\mathscr{L}[y] - s^2y(0) - sy'(0) - y''(0)$$
$$\mathscr{L}[y^{IV}] = s^4\mathscr{L}[y] - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0)$$

1.1.3 Solución de EDO por Series

- Series de potencias
- Series de Frobenius

Hemos resuelto ecuaciones del tipo:

$$y'' + 2y' + y = e^x$$
$$x^2y'' + 2xy' + y = e^x$$

Pero:

$$(x^2+1)y'' - xy' + y = e^x$$
$$xy'' + e^x y = 1$$

Objetivos de los métodos: Resolver ecuaciones diferenciales con coeficientes variables.

Definición 1.1.1 Una serie de potencias es una expresión de la forma siguiente:

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots = \sum_{i=0}^{\infty} a_nx^n \tag{1.2}$$

Nota 1.1. *Suponga que se cumplen las condiciones de convergencia que correspondan se tiene:*

$$y = \sum n = 0^{\infty} a_nx^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots \tag{1.1}$$
$$y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \ldots \tag{1.2}$$
$$y' = \sum_{n=1}^{\infty} na_nx^{n-1} \tag{1.3}$$
$$y'' = \sum_{n=2}^{\infty} n(n-1)a_nx^{n-2} \tag{1.4}$$

Nota 1.2. *Suponga que:*

$$\sum_{n=1}^{\infty} na_nx^{n-1} = a_1 + 2a_2x + 3a_3x^2 + \ldots \tag{1.1}$$

En donde:

$$\sum_{n=0}^{\infty} (n+1)a_{n+1}x^n = a_1 + 2a_2x + 3a_3x^2 + \ldots \tag{1.1}$$
$$\sum_{n=2}^{\infty} (n-1)a_{n-1}x^{n-2} = a_1 + 2a_2x + 3a_3x^2 + \ldots \tag{1.2}$$

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Nota 1.3.

$$\sum_{n=1}^{\infty} a_nx^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots = 0 \tag{1.1}$$
$$\implies a_n = 0 \tag{1.2}$$

1.2.1 Ejercicio 1

Resuelva:

$$y' - y = 0 \qquad y' = y \tag{1.1}$$

Suponemos:

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots \tag{1.2}$$

$$\implies y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \tag{1.3}$$

Sustituyendo en la ecuación diferencial:

$$\implies \sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 0 \tag{1.4}$$

$$\implies \sum_{n=0}^{\infty} (n+1) a_{n+1} x^x n + 1 - 1 - \sum_{n=0}^{\infty} a_n x^n = 0 \tag{1.5}$$

$$\implies \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - \sum_{n=0}^{\infty} a_n x^n = 0 \tag{1.6}$$

$$\implies \sum_{n=0}^{\infty} [(n+1) a_{n+1} - a_n] x^n = 00 + 0x + 0x^2 + 0x^3 = 0 \tag{1.7}$$

$$\implies [(n+1) a_{n+1} - a_n] = 0 \qquad n =, 0, 1, 2 \dots \tag{1.8}$$

Ecuación indicial:

$$\implies a_{n+1} = \frac{a_n}{n+1} \qquad n = 0, 1, 2, \dots \tag{1.9}$$

n=0:

$$a_1 = \frac{a_0}{1} \tag{1.10}$$

n=1:

$$a_2 = \frac{a_1}{2} = \frac{a_0}{2 * 1} \tag{1.11}$$

n=2

$$: a_3 = \frac{a_2}{3} = \frac{a_0}{3 * 2 * 1} \tag{1.12}$$

n=3:

$$a_4 = \frac{a_3}{4} = \frac{a_0}{4*3*2*1} \tag{1.13}$$

n=n:

$$a_n = \frac{a_0}{n!} \tag{1.14}$$

$$\implies y(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ... \tag{1.15}$$

$$\implies y(x) = a_0 + \frac{a_0}{1!}x + \frac{a_0}{2!}x^2 + \frac{a_0}{3!}x^3 + \frac{a_0}{4!}x^4 + ... \tag{1.16}$$

Un supuesto:

$$\implies y(x) = a_0[1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...]\frac{x^0}{0!} = 1 \tag{1.17}$$

$$\implies y = a_0[1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...] \tag{1.18}$$

Serie de McLaurin (Taylor) para $f(x) = e^x$:

$$\implies y = a_0 \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{1.19}$$

$$\implies y = a_0 e^x \tag{1.20}$$

1.2.2 Ejercicio 2

Resuelva:

$$y'' + y = 0 \tag{1.1}$$

$$\text{ya se sabe} y = c_1 \cos x + c_2 \sin x \tag{1.2}$$

Suponemos que:

$$y = \sum_{n=0}^{\infty} a_n x^n \tag{1.3}$$

$$\implies y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \tag{1.4}$$

$$\implies y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} \tag{1.5}$$

Sustituyendo en la ecuación diferencial

$$\Longrightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=1}^{\infty} n a_n x^{n-1} = 0 \tag{1.6}$$

$$\Longrightarrow \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n + \sum_{n=1}^{\infty} n a_n x^{n-1} = 0 \tag{1.7}$$

$$\Longrightarrow \sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} + a_n]x^n \tag{1.8}$$

$$(n+2)(n+1)a_{n+2} + a_n \qquad n=,0,1,2... \tag{1.9}$$

$$+ \tag{1.10}$$

Ecuación indicial:

$$\Longrightarrow a_{n+2} = \frac{-a_n}{(n+2)(n+1)} \qquad n=0,1,2,... \tag{1.11}$$

n=0:

$$a_2 = \frac{-a_0}{2*1} \tag{1.12}$$

n=1:

$$a_3 = \frac{-a_1}{3*2} \tag{1.13}$$

n=2:

$$a_4 = \frac{-a_2}{4*3} = \frac{\frac{-a_0}{2*1}}{4*3} = a_4 = \frac{a_0}{4*3*2*1} \tag{1.14}$$

n=3:

$$a_5 = \frac{-a_3}{5*4} = \frac{\frac{-a_1}{3*2}}{5*4*3*2} \tag{1.15}$$

n=4:

$$a_6 = \frac{-a_4}{6*5} = \frac{-a_0}{6*5*4*3*2} \tag{1.16}$$

n=5

$$a_7 = \frac{-a_5}{7*6} = \frac{-a_1}{7*6*5*4*3*2} \tag{1.17}$$

Solución de la ecuación diferencial

$$y=\sum_{n=0}^{\infty}a_nx^n=a_0*a_1x+a_2x^2+a_3x^3+... \tag{1.18}$$

$$\implies y=a_0+a_1x+\frac{a_0}{2!}x^2-\frac{a_1}{3!}x^3+\frac{a_0}{4!}x^4+\frac{a_1}{5!}x^5 \tag{1.19}$$

$$\implies y=a_0(1-\frac{x^2}{2!}+\frac{x^4}{4!}+...)+a_1(x-\frac{x^3}{3!}-\frac{x^5}{5!}+...) \tag{1.20}$$

$$\implies y=a_0\sum_{n=0}^{\infty}\frac{(-1)^nx^{2n}}{(2n)!}+a_1\sum_{n=0}^{\infty}\frac{(-1)^nx^{2n+1}}{(2n+1)!} \tag{1.21}$$

$$=a_0\cos x+a_1\sin x \tag{1.22}$$