

$(x+y) = \left(\frac{\pi}{2}\right)$ $x_{1/2} = \sqrt{2a}$
 $\pi \approx 3.1415$ $\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$
 $\ln = \sqrt{a \cdot b}$ $S_3 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\sin \alpha = \frac{b}{a}$

Ecuaciones Diferenciales 1

Notas de clases

Rudik Roberto Rompich

$B \lim_{x \rightarrow 1} \frac{\operatorname{ctgx} - 2}{2\sqrt{11} \times 3}$ Q'' $\int (x \pm a^2)^c$ $e = 2$
 $x^2 + y^2 = z$ $\sum_{n=0}^{+\infty} \frac{x^n}{n!}$ $\phi = \sqrt{\frac{\sum (x-m)^2}{n-1}}$
 $e = \cos x + \operatorname{tg} y$ $\sin \alpha$

Copyright © 2020 Rudik Rompich

PUBLISHED BY RUDIKS

RUDIKS.COM

Licensed under the Creative Commons Attribution-NonCommercial 3.0 Unported License (the “License”). You may not use this file except in compliance with the License. You may obtain a copy of the License at <http://creativecommons.org/licenses/by-nc/3.0>. Unless required by applicable law or agreed to in writing, software distributed under the License is distributed on an “AS IS” BASIS, WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied. See the License for the specific language governing permissions and limitations under the License.

First printing, October 2020

Índice general

I	Transformada de Laplace	
1	Contenido para el parcial 3	7
1.1	14 de octubre de 2020	7
1.1.1	Ejercicio 2	7
1.1.2	Ejercicio para casa	7
1.1.3	Solución de EDO por Series	8
1.2	16 de octubre de 2020	8
1.2.1	Ejercicio 1	9
1.2.2	Ejercicio 2	10



Transformada de Laplace

1	Contenido para el parcial 3	7
1.1	14 de octubre de 2020	
1.2	16 de octubre de 2020	

1. Contenido para el parcial 3

1.1 14 de octubre de 2020

1.1.1 Ejercicio 2

$$y'' + 2y' + y = \delta(t - 1)$$

$$y(0) = 0, y'(0) = 0$$

(1.1)

$$\mathcal{L}[y''] + 2\mathcal{L}[y'] + \mathcal{L}[y] = \mathcal{L}[\delta(t - 1)]$$

(1.2)

$$s^2 \mathcal{L}[y] - sy(0) - y'(0) + 2[s\mathcal{L}[y] - y(0)] + \mathcal{L}[y] = e^{-s}$$

(1.3)

$$(s^2 + 2s + 1)\mathcal{L}[y] = e^{-s}$$

(1.4)

$$\mathcal{L}[y] = e^{-s} \left(\frac{1}{(s+1)^2} \right)$$

(1.5)

Primer teorema de traslación

$$y = \mathcal{L}^{-1} \left[e^{-s} \frac{1}{(s+1)^2} \right]$$

$$\mathcal{L}^{-1} \left[\frac{1}{(s+1)^2} \right] = te^{-t}$$

(1.6)

Segundo teorema de traslación

$$y = (t - 1)e^{-(t-1)} * H(t - 1)$$

(1.7)

1.1.2 Ejercicio para casa

$$y''' + y = e^t + \delta(t - 1)$$

$$y(0) = 1, y'(0) = 1, y''(0) = 2$$

(1.1)

Por ejemplo para orden superior:

$$\mathcal{L}[y'''] = s^3 \mathcal{L}[y] - s^2 y(0) - sy'(0) - y''(0)$$

$$\mathcal{L}[y^{IV}] = s^4 \mathcal{L}[y] - s^3 y(0) - s^2 y'(0) - sy''(0) - y'''(0)$$

1.1.3 Solución de EDO por Series

- Series de potencias
- Series de Frobenius

Hemos resuelto ecuaciones del tipo:

$$y'' + 2y' + y = e^x$$

$$x^2 y'' + 2xy' + y = e^x$$

Pero:

$$(x^2 + 1)y'' - xy' + y = e^x$$

$$xy'' + e^x y = 1$$

Objetivos de los métodos: Resolver ecuaciones diferenciales con coeficientes variables.

Definición 1.1.1 Una serie de potencias es una expresión de la forma siguiente:

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots = \sum_{i=0}^{\infty} a_n x^n \quad (1.2)$$

Nota 1.1. Suponga que se cumplen las condiciones de convergencia que correspondan se tiene:

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \quad (1.1)$$

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots \quad (1.2)$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \quad (1.3)$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} \quad (1.4)$$

Nota 1.2. Suponga que:

$$\sum_{n=1}^{\infty} n a_n x^{n-1} = a_1 + 2a_2 x + 3a_3 x^2 + \dots \quad (1.1)$$

En donde:

$$\sum_{n=0}^{\infty} (n+1) a_{n+1} x^n = a_1 + 2a_2 x + 3a_3 x^2 + \dots \quad (1.1)$$

$$\sum_{n=2}^{\infty} (n-1) a_{n-1} x^{n-2} = a_1 + 2a_2 x + 3a_3 x^2 + \dots \quad (1.2)$$

1.2 16 de octubre de 2020

Nota 1.3.

$$\sum_{n=1}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots = 0 \quad (1.1)$$

$$\implies a_n = 0 \quad (1.2)$$

1.2.1 Ejercicio 1

Resuelva:

$$y' - y = 0 \qquad y' = y \qquad (1.1)$$

Suponemos:

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots \qquad (1.2)$$

$$\implies y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \qquad (1.3)$$

Sustituyendo en la ecuación diferencial:

$$\implies \sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 0 \qquad (1.4)$$

$$\implies \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - \sum_{n=0}^{\infty} a_n x^n = 0 \qquad (1.5)$$

$$\implies \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - \sum_{n=0}^{\infty} a_n x^n = 0 \qquad (1.6)$$

$$\implies \sum_{n=0}^{\infty} [(n+1) a_{n+1} - a_n] x^n = 0 + 0x + 0x^2 + 0x^3 = 0 \qquad (1.7)$$

$$\implies [(n+1) a_{n+1} - a_n] = 0 \qquad n = 0, 1, 2, \dots \qquad (1.8)$$

Ecuación indicial:

$$\implies a_{n+1} = \frac{a_n}{n+1} \qquad n = 0, 1, 2, \dots \qquad (1.9)$$

n=0:

$$a_1 = \frac{a_0}{1} \qquad (1.10)$$

n=1:

$$a_2 = \frac{a_1}{2} = \frac{a_0}{2 * 1} \qquad (1.11)$$

n=2

$$: a_3 = \frac{a_2}{3} = \frac{a_0}{3 * 2 * 1} \qquad (1.12)$$

$n=3$:

$$a_4 = \frac{a_3}{4} = \frac{a_0}{4 * 3 * 2 * 1} \quad (1.13)$$

$n=n$:

$$a_n = \frac{a_0}{n!} \quad (1.14)$$

$$\Rightarrow y(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \quad (1.15)$$

$$\Rightarrow y(x) = a_0 + \frac{a_0}{1!}x + \frac{a_0}{2!}x^2 + \frac{a_0}{3!}x^3 + \frac{a_0}{4!}x^4 + \dots \quad (1.16)$$

Un supuesto:

$$\Rightarrow y(x) = a_0 \left[1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right] \frac{x^0}{0!} = 1 \quad (1.17)$$

$$\Rightarrow y = a_0 \left[1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right] \quad (1.18)$$

Serie de McLaurin (Taylor) para $f(x) = e^x$:

$$\Rightarrow y = a_0 \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad (1.19)$$

$$\Rightarrow y = a_0 e^x \quad (1.20)$$

1.2.2 Ejercicio 2

Resuelva:

$$y'' + y = 0 \quad (1.1)$$

$$\text{ya se sabe } y = c_1 \cos x + c_2 \sin x \quad (1.2)$$

Suponemos que:

$$y = \sum_{n=0}^{\infty} a_n x^n \quad (1.3)$$

$$\Rightarrow y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \quad (1.4)$$

$$\Rightarrow y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} \quad (1.5)$$

Sustituyendo en la ecuación diferencial

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=1}^{\infty} n a_n x^{n-1} = 0 \quad (1.6)$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n + \sum_{n=1}^{\infty} n a_n x^{n-1} = 0 \quad (1.7)$$

$$\Rightarrow \sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} + a_n] x^n \quad (1.8)$$

$$(n+2)(n+1)a_{n+2} + a_n \quad n = 0, 1, 2, \dots \quad (1.9)$$

$$+ \quad (1.10)$$

Ecuación indicial:

$$\Rightarrow a_{n+2} = \frac{-a_n}{(n+2)(n+1)} \quad n = 0, 1, 2, \dots \quad (1.11)$$

n=0:

$$a_2 = \frac{-a_0}{2 * 1} \quad (1.12)$$

n=1:

$$a_3 = \frac{-a_1}{3 * 2} \quad (1.13)$$

n=2:

$$a_4 = \frac{-a_2}{4 * 3} = \frac{\frac{-a_0}{2 * 1}}{4 * 3} = a_4 = \frac{a_0}{4 * 3 * 2 * 1} \quad (1.14)$$

n=3:

$$a_5 = \frac{-a_3}{5 * 4} = \frac{\frac{-a_1}{3 * 2}}{5 * 4 * 3 * 2} \quad (1.15)$$

n=4:

$$a_6 = \frac{-a_4}{6 * 5} = \frac{-a_0}{6 * 5 * 4 * 3 * 2} \quad (1.16)$$

n=5

$$a_7 = \frac{-a_5}{7 * 6} = \frac{-a_1}{7 * 6 * 5 * 4 * 3 * 2} \quad (1.17)$$

Solución de la ecuación diferencial

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \quad (1.18)$$

$$\Rightarrow y = a_0 + a_1 x + \frac{a_0}{2!} x^2 - \frac{a_1}{3!} x^3 + \frac{a_0}{4!} x^4 + \frac{a_1}{5!} x^5 \quad (1.19)$$

$$\Rightarrow y = a_0 \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right) + a_1 \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) \quad (1.20)$$

$$\Rightarrow y = a_0 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} + a_1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad (1.21)$$

$$= a_0 \cos x + a_1 \sin x \quad (1.22)$$