Universidad del Valle de Guatemala
Departamento de Matemática
Licenciatura en Matemática Aplicada

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Análisis de Variable Compleja - Catedrático: Dorval Carías
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Exposición - Problema 2

Problema 1. *For each of the following points in $\mathbb{C}$, give the corresponding point of S:*

1. *0*
2. $1+i$
3. $3+2i$.

Solución. Tenemos a S, definido como:

$$S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$$

En donde:

$$x_1 = \frac{2x}{|z|^2+1}, \quad x_2 = \frac{2y}{|z|^2+1}, \quad x_3 = \frac{|z|^2-1}{|z|^2+1}$$

Entonces, tenemos:

1. $z = 0 \implies |0|^2 \implies \sqrt{0} = 0$. Es decir, entonces:

$$\begin{aligned} S &= \left(\frac{2x}{|z|^2+1}, \frac{2y}{|z|^2+1}, \frac{|z|^2-1}{|z|^2+1}\right) \\ &= \left(\frac{0}{0+1}, \frac{0}{0+1}, \frac{0-1}{0+1}\right) \\ &= (0, 0, -1) \end{aligned}$$

2. $z = 1 + i \implies |1 + i|^2 \implies 2$. Es decir, entonces:

$$\begin{aligned} S &= \left(\frac{2x}{|z|^2 + 1}, \frac{2y}{|z|^2 + 1}, \frac{|z|^2 - 1}{|z|^2 + 1} \right) \\ &= \left(\frac{2(1)}{2 + 1}, \frac{2(1)}{2 + 1}, \frac{2 - 1}{2 + 1} \right) \\ &= \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right) \end{aligned}$$

3. $z = 3 + 2i \implies |3 + 2i|^2 \implies 9 + 4 = 13$. Es decir, entonces:

$$\begin{aligned} S &= \left(\frac{2x}{|z|^2 + 1}, \frac{2y}{|z|^2 + 1}, \frac{|z|^2 - 1}{|z|^2 + 1} \right) \\ &= \left(\frac{2(3)}{13 + 1}, \frac{2(2)}{13 + 1}, \frac{13 - 1}{13 + 1} \right) \\ &= \left(\frac{6}{14}, \frac{4}{14}, \frac{12}{14} \right) \\ &= \left(\frac{3}{7}, \frac{2}{7}, \frac{6}{7} \right) \end{aligned}$$

□