Universidad del Valle de Guatemala
Departamento de Matemática
Licenciatura en Matemática Aplicada

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Teoría electromagnética 1 - Catedrático: Eduardo Álvarez
5 de febrero de 2023

Tarea

Problema 1.

$$\begin{aligned}\mathbf{A} &= 2\mathbf{a}_x + \mathbf{a}_y - 3\mathbf{a}_z \\ \mathbf{B} &= \mathbf{a}_y - \mathbf{a}_z \\ \mathbf{C} &= 3\mathbf{a}_x + 5\mathbf{a}_y + 7\mathbf{a}_z\end{aligned}$$

Determinar:

1. $\mathrm{A} - 2\ \mathrm{B} + \mathrm{C}$

Solución. Sea

$$\begin{aligned}\mathrm{A} - 2\ \mathrm{B} + \mathrm{C} &= (2, 1, -3) - 2 \cdot (0, 1, -1) + (3, 5, 7) \\ &= (2, 1, -3) - (0, 2, -2) + (3, 5, 7) \\ &= (2, -1, -1) + (3, 5, 7) \\ &= (5, 4, 6)\end{aligned}$$

□

2. $\mathrm{C} - 4(\mathrm{A} + \mathrm{B})$

Solución. Sea

$$\begin{aligned}\mathrm{C} - 4(\mathrm{A} + \mathrm{B}) &= (3, 5, 7) - 4((2, 1, -3) + (0, 1, -1)) \\ &= (3, 5, 7) - 4(2, 2, -4) \\ &= (3, 5, 7) - (8, 8, -16) \\ &= (-5, -3, 23)\end{aligned}$$

□

3. $\frac{2\mathrm{A}-3\mathrm{B}}{|\mathrm{C}|}$

Solución. Sea

$$\begin{aligned}
\frac{2\mathrm{A}-3\mathrm{B}}{|\mathrm{C}|} &= \frac{2(2,1,-3)-3(0,1,-1)}{|(3,5,7)|} \\
&= \frac{(4,2,-6)-(0,3,-3)}{\sqrt{3^2+5^2+7^2}} \\
&= \frac{(4,-1,-3)}{\sqrt{9+25+49}} \\
&= \frac{(4,-1,-3)}{\sqrt{83}}
\end{aligned}$$

□

4. $\mathrm{A}\cdot\mathrm{C}-|\mathrm{B}|^2$

Solución. Sea

$$\begin{aligned}
\mathrm{A}\cdot\mathrm{C}-|\mathrm{B}|^2 &= (2,1,-3)\cdot(3,5,7)-(0,1,-1)\cdot(0,1,-1) \\
&= 2*3+1*5-3*7-(0*0+1*1+(-1)*(-1)) \\
&= 6+5-21-(0+1+1) \\
&= -10-2 \\
&= -12
\end{aligned}$$

□

5. $\frac{1}{2}\mathrm{B}\times\left(\frac{1}{3}\mathrm{A}+\frac{1}{4}\mathrm{C}\right)$

Solución. Sea

$$\begin{aligned}
\frac{1}{2}\mathrm{B}\times\left(\frac{1}{3}\mathrm{A}+\frac{1}{4}\mathrm{C}\right) &= \frac{1}{2}(0,1,-1)\times\left(\frac{1}{3}(2,1,-3)+\frac{1}{4}(3,5,7)\right) \\
&= \frac{1}{2}(0,1,-1)\times\left(\frac{2}{3}+\frac{3}{4},\frac{1}{3}+\frac{5}{4},\frac{-3}{3}+\frac{7}{4}\right) \\
&= \frac{1}{2}(0,1,-1)\times\left(\frac{2}{3}+\frac{3}{4},\frac{1}{3}+\frac{5}{4},\frac{-3}{3}+\frac{7}{4}\right) \\
&= \frac{1}{2}(0,1,-1)\times\left(\frac{15}{12},\frac{19}{12},\frac{-3}{4}\right) \\
&= \frac{1}{2}\left(\frac{-3}{4}+\frac{19}{12},\frac{15}{12},\frac{15}{12}\right) \\
&= \frac{1}{2}\left(\frac{10}{12},\frac{15}{12},\frac{15}{12}\right) \\
&= \left(\frac{10}{24},\frac{15}{24},\frac{15}{24}\right)
\end{aligned}$$

□

Problema 2. *Given that*

$$\begin{aligned}\mathbf{P} &= 2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z = (2,-1,-2)\\ \mathbf{Q} &= 4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z = (4,3,2)\\ \mathbf{R} &= -\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z = (-1,1,2)\end{aligned}$$

Find:

1. $|\mathbf{P}+\mathbf{Q}-\mathbf{R}|$

   Solución. Sea

$$\begin{aligned}|\mathbf{P}+\mathbf{Q}-\mathbf{R}| &= |(2,-1,-2)+(4,3,2)-(-1,1,2)|\\ &= |(7,1,-2)|\\ &= \sqrt{7^2+1^2+2^2}\\ &= \sqrt{54}\end{aligned}$$

□

2. $\mathbf{P}\cdot\mathbf{Q}\times\mathbf{R}$

   Solución. Sea

$$\begin{aligned}\mathbf{P}\cdot\mathbf{Q}\times\mathbf{R} &= (2,-1,-2)\cdot[(4,3,2)\times(-1,1,2)]\\ &= (2,-1,-2)\cdot(4,-10,7)\\ &= 4\end{aligned}$$

□

3. $\mathbf{Q}\times\mathbf{P}\cdot\mathbf{R}$

   Solución. Sea

$$\begin{aligned}\mathbf{Q}\times\mathbf{P}\cdot\mathbf{R} &= [(4,3,2)\times(2,-1,-2)]\cdot(-1,1,2)\\ &= (-4,12,-10)\cdot(-1,1,2)\\ &= -4\end{aligned}$$

□

4. $(\mathbf{P}\times\mathbf{Q})\cdot(\mathbf{Q}\times\mathbf{R})$

   Solución. Sea

$$\begin{aligned}(\mathbf{P}\times\mathbf{Q})\cdot(\mathbf{Q}\times\mathbf{R}) &= ((2,-1,-2)\times(4,3,2))\cdot((4,3,2)\times(-1,1,2))\\ &= (4,-12,10)\cdot(4,-10,7)\\ &= 206\end{aligned}$$

□

5. $(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R})$

Solución. Sea

$$\begin{aligned}(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R}) &= ((2,-1,-2) \times (4,3,2)) \cdot ((4,3,2) \times (-1,1,2)) \\ &= (4,-12,10) \cdot (4,-10,7) \\ &= 206\end{aligned}$$

□

6. $\cos\theta_{PR}$

Solución. Sea

$$\begin{aligned}\cos\theta_{PR} &= \frac{P \cdot R}{|P||R|} \\ &= \frac{(2,-1,-2) \cdot (-1,1,2)}{|(2,-1,-2)||(-1,1,2)|} \\ &= \frac{-7}{3\sqrt{6}}\end{aligned}$$

□

7. $\sin\theta_{PQ}$

Solución. Sea

$$\begin{aligned}\sin\theta_{PQ} &= \frac{P \times Q}{|P||Q|} \\ &= \frac{(2,-1,-2) \times (4,3,2)}{|(2,-1,-2)||(4,3,2)|} \\ &= \frac{(4,-12,10)}{\sqrt{9}\sqrt{29}} \\ &= \frac{(4,-12,10)}{3\sqrt{29}}\end{aligned}$$

□

Problema 3. *If* $\mathbf{A} = -\mathbf{a}_x + 6\mathbf{a}_y + 5\mathbf{a}_z$ *and* $\mathbf{B} = \mathbf{a}_x + 2\mathbf{a}_. + 3\mathbf{a}_x$*, find:*

1. *the scalar projections of* A *on* B

Solución. Sea

$$\begin{aligned}A_B &= A \cdot \mathbf{a}_B \\ &= (-1,6,5) \cdot \frac{(1,2,3)}{\sqrt{14}} \\ &= \frac{26}{\sqrt{14}}\end{aligned}$$

□

2. *the vector projection of B on A.*

Solución. Sea

$$\begin{aligned}
B_A &= B_A\mathbf{a}_A \\
&= (B\cdot\mathbf{a}_A)\mathbf{a}_A \\
&= \left((1,2,3)\cdot\frac{(-1,6,5)}{\sqrt{1+36+25}}\right)\frac{(-1,6,5)}{\sqrt{1+36+25}} \\
&= \left((1,2,3)\cdot\frac{(-1,6,5)}{\sqrt{62}}\right)\frac{(-1,6,5)}{\sqrt{62}} \\
&= \left(\frac{26}{\sqrt{62}}\right)\frac{(-1,6,5)}{\sqrt{62}} \\
&= \frac{26}{62}(-1,6,5) \\
&= \frac{13}{31}(-1,6,5)
\end{aligned}$$

□

Problema 4. *Let*

1. *If $V = xz - xy + yz$, express V in cylindrical coordinates.*

Solución. Sea

$$\begin{aligned}
x &= \rho\cos\phi \\
y &= \rho\sin\phi \\
z &= z
\end{aligned}$$

Entonces,

$$\begin{aligned}
V &= xz - xy + yz \\
&= (\rho\cos\phi)(z) - (\rho\cos\phi)(\rho\sin\phi) + (\rho\sin\phi)z \\
&= z\rho\cos\phi - \rho^2\cos\phi\sin\phi + z\rho\sin\phi
\end{aligned}$$

□

2. *If $U = x^2 + 2y^2 + 3z^2$, express U in spherical coordinates.*

Solución. Sea

$$\begin{aligned}
x &= r\sin\theta\cos\phi \\
y &= r\sin\theta\sin\phi \\
z &= r\cos\theta
\end{aligned}$$

Entonces,

$$\begin{aligned}
U &= x^2 + 2y^2 + 3z^2 \\
&= (r\sin\theta\cos\phi)^2 + 2\,(r\sin\theta\sin\phi)^2 + 3\,(r\cos\theta)^2 \\
&= r^2\left[\sin^2\theta\cos^2\phi + 2\sin^2\theta\sin^2\phi + 3\cos^2\theta\right] \\
&= r^2\left[\sin^2\theta\left(\cos^2\phi + 2\sin^2\phi\right) + 3\cos^2\theta\right]
\end{aligned}$$

□

Problema 5. *Express the following vectors in Cartesian coordinates:*

1. $\mathbf{A} = \rho\left(z^2+1\right)\mathbf{a}_\rho - \rho z\cos\phi\mathbf{a}_\phi$

Solución. Sea

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \rho(z^2+1) \\ \rho z\cos\phi \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} \rho(z^2+1)\cos\phi - \sin\phi\rho z\cos\phi \\ \sin\phi\rho(z^2+1) + \rho z\cos^2\phi \\ 0 \end{bmatrix} = \begin{bmatrix} (z^2+1)x - \sin\phi zx \\ y(z^2+1) + xz\cos\phi \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} (z^2+1)x - \frac{y}{\sqrt{x^2+y^2}}zx \\ y(z^2+1) + xz\frac{x}{\sqrt{x^2+y^2}} \\ 0 \end{bmatrix} = \begin{bmatrix} (z^2+1)x - \frac{yzx}{\sqrt{x^2+y^2}} \\ y(z^2+1) + \frac{x^2z}{\sqrt{x^2+y^2}} \\ 0 \end{bmatrix}$$

□

2. $\mathbf{B} = 2r\sin\theta\cos\phi\mathbf{a}_r + r\cos\theta\cos\theta\mathbf{a}_\theta - r\sin\phi\mathbf{a}_\phi$

Solución. Sea

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$
$$= \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}\begin{bmatrix} 2r\sin\theta\cos\phi \\ r\cos\theta\cos\theta \\ -r\sin\phi \end{bmatrix}$$
$$= \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}\begin{bmatrix} 2x \\ r\cos^2\theta \\ -r\sin\phi \end{bmatrix}$$
$$= \begin{bmatrix} 2x\sin\theta\cos\phi + r\cos^3\theta\cos\phi + r\sin^2\phi \\ 2x\sin\theta\sin\phi + r\cos^3\theta\sin\phi - r\cos\phi\sin\phi \\ 2x\cos\theta - r\cos^2\theta\sin\theta \end{bmatrix}$$
$$= \begin{bmatrix} \frac{r}{r}\cdot 2x\sin\theta\cos\phi + \frac{r^2}{r}\cdot r\cos^3\theta\cos\phi + r\sin^2\phi \\ \frac{r}{r}\cdot 2x\sin\theta\sin\phi + \frac{r^2}{r}\cdot r\cos^3\theta\sin\phi - r\cos\phi\sin\phi \\ \frac{r}{r}\cdot 2x\cos\theta - \frac{r}{r}\cdot r\cos^2\theta\sin\theta \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2x^2}{r} + \frac{z^3\cos\phi}{r} + r\sin^2\phi \\ \frac{2xy}{r} + \frac{z^3\sin\phi}{r} - r\cos\phi\sin\phi \\ \frac{2xz}{r} - \frac{z^2\sin\theta}{r} \end{bmatrix} = \begin{bmatrix} \frac{2x^2}{r} + \frac{z^3}{r}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + r\left(\frac{y}{x^2+y^2}\right)^2 \\ \frac{2xy}{r} + \frac{z^3}{r}\left(\frac{y}{x^2+y^2}\right) - r\left(\frac{x}{x^2+y^2}\right)\left(\frac{y}{x^2+y^2}\right) \\ \frac{2xz}{r} - \frac{z^2}{r}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\right) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2x^2}{\sqrt{x^2+y^2+z^2}} + \frac{z^3}{\sqrt{x^2+y^2+z^2}}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + \sqrt{x^2+y^2+z^2}\left(\frac{y}{x^2+y^2}\right)^2 \\ \frac{2xy}{\sqrt{x^2+y^2+z^2}} + \frac{z^3}{\sqrt{x^2+y^2+z^2}}\left(\frac{y}{x^2+y^2}\right) - \sqrt{x^2+y^2+z^2}\left(\frac{x}{x^2+y^2}\right)\left(\frac{y}{x^2+y^2}\right) \\ \frac{2xz}{\sqrt{x^2+y^2+z^2}} - \frac{z^2}{\sqrt{x^2+y^2+z^2}}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\right) \end{bmatrix}$$

□

Problema 6. *Let*

1. *Express the vector field*

$$\mathbf{H} = xy^2z\mathbf{a}_x + x^2yz\mathbf{a}_y + xyz^2\mathbf{a}_z$$

in cylindrical and spherical coordinates.

Solución. Sea

- Cilíndricas. Sea

$$\begin{aligned}
\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} &= \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} xy^2z \\ x^2yz \\ xyz^2 \end{bmatrix} \\
&= \begin{bmatrix} \cos\phi(xy^2z) + \sin\phi(x^2yz) + 0(xyz^2) \\ -\sin\phi(xy^2z) + \cos\phi(x^2yz) + 0(xyz^2) \\ 0(xy^2z) + 0(x^2yz) + 1(xyz^2) \end{bmatrix} = \begin{bmatrix} \cos\phi(xy^2z) + \sin\phi(x^2yz) \\ -\sin\phi(xy^2z) + \cos\phi(x^2yz) \\ xyz^2 \end{bmatrix} \\
&= \begin{bmatrix} \cos\phi((\rho\cos\phi)(\rho\sin\phi)^2z) + \sin\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ -\sin\phi((\rho\cos\phi)(\rho\sin\phi)^2z) + \cos\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ (\rho\cos\phi)(\rho\sin\phi)z^2 \end{bmatrix}
\end{aligned}$$

- Esféricas. Sea

$$\begin{aligned}
\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} &= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} xy^2z \\ x^2yz \\ xyz^2 \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} (r\sin\theta\cos\phi)(r\sin\theta\sin\phi)^2(r\cos\theta) \\ (r\sin\theta\cos\phi)^2(r\sin\theta\sin\phi)(r\cos\theta) \\ (r\sin\theta\cos\phi)(r\sin\theta\sin\phi)(r\cos\theta)^2 \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} r^4(\sin^3\theta\cos\phi\sin^2\phi\cos\theta) \\ r^4(\sin^3\theta\cos^2\phi\sin\phi\cos\theta) \\ r^4(\sin^2\theta\cos\phi\sin\phi\cos^2\theta) \end{bmatrix} \\
&= \begin{bmatrix} r^4\left(\sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^2\theta\cos\phi\sin\phi\cos^3\theta\right) \\ r^4\left(\sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta + \sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta - \sin^3\theta\cos\phi\sin\phi\cos^2\theta\right) \\ r^4\left(-\sin^3\theta\cos\phi\sin^3\phi\cos\theta + \sin^3\theta\cos^3\phi\sin\phi\cos\theta\right) \end{bmatrix}
\end{aligned}$$

□

2. *In both cylindrical and spherical coordinates, determine* H *at* $(3, -4, 5)$.

Solución. Sea $x = 3, y = -4, z = 5$,

- Cilíndricas, $H(\rho, \phi, z)$. Tenemos:

- $\rho = \sqrt{3^2+4^2} = \sqrt{25} = 5$
- $\phi = \arctan\left(\frac{-4}{3}\right) = -0{,}927$
- $z = 5$

Con eso, se evalúa en:

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi((\rho\cos\phi)(\rho\sin\phi)^2 z) + \sin\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ -\sin\phi((\rho\cos\phi)(\rho\sin\phi)^2 z) + \cos\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ (\rho\cos\phi)(\rho\sin\phi)z^2 \end{bmatrix}$$

- Esféricas, $H(r,\theta,\phi)$. Tenemos:
  - $r = \sqrt{3^2+4^2+5^2} = \sqrt{9+16+25} = \sqrt{50}$
  - $\theta = \arctan\left(\frac{\sqrt{3^2+4^2}}{5}\right) = \arctan\left(\frac{\sqrt{25}}{5}\right) = \arctan(1) = \pi/4$
  - $\phi = \arctan\left(\frac{-4}{3}\right) = -0{,}927$

  Con eso, se evalúa en:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} r^4\left(\sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^2\theta\cos\phi\sin\phi\cos^3\theta\right) \\ r^4\left(\sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta + \sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta - \sin^3\theta\cos\phi\sin\phi\cos^2\theta\right) \\ r^4\left(-\sin^3\theta\cos\phi\sin^3\phi\cos\theta + \sin^3\theta\cos^3\phi\sin\phi\cos\theta\right) \end{bmatrix}$$

□

Problema 7. *Given vectors* $\mathbf{A} = 2\mathbf{a}_x + 4\mathbf{a}_y + 10\mathbf{a}_z$ *and* $\mathbf{B} = -5\mathbf{a}_\rho + \mathbf{a}_\phi - 3\mathbf{a}_z$*, find*

1. $\mathbf{A}+\mathbf{B}$ *at* $P(0,2,-5)$

   Solución. Sea

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -5\cos\phi - \sin\phi \\ -5\sin\phi + \cos\phi \\ -3 \end{bmatrix} = \begin{bmatrix} -5\left(\frac{x}{\sqrt{x^2+y^2}}\right) - \left(\frac{y}{\sqrt{x^2+y^2}}\right) \\ -5\left(\frac{y}{\sqrt{x^2+y^2}}\right) + \left(\frac{x}{\sqrt{x^2+y^2}}\right) \\ -3 \end{bmatrix}$$

   Entonces $A+B$ en $P(0,2,-5)$

$$A+B = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} 2-5\left(\frac{x}{\sqrt{x^2+y^2}}\right) - \left(\frac{y}{\sqrt{x^2+y^2}}\right) \\ 4-5\left(\frac{y}{\sqrt{x^2+y^2}}\right) + \left(\frac{x}{\sqrt{x^2+y^2}}\right) \\ 10-3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\left(\frac{2}{\sqrt{4}}\right) \\ 4-5\left(\frac{2}{\sqrt{4}}\right) \\ 10-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}$$

□

2. *The angle between* $\mathbf{A}$ *and* $\mathbf{B}$ *at* P

Solución. Por la propiedad:

$$\mathbf{A}\cdot\mathbf{B} = |A||B|\cos\theta_{AB}$$
$$\arccos\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|A||B|}\right) = \theta_{AB}$$

Considerando, $B = (-1,-5,-3)$ tenemos:

$$\begin{aligned}\theta_{AB} &= \arccos\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|A||B|}\right)\\ &= \arccos\left(\frac{(2,4,10)\cdot(-1,-5,-3)}{|(2,4,10)||(-1,-5,-3)|}\right)\\ &= \arccos\left(\frac{-2-20-30}{\sqrt{2^2+4^2+10^2}\sqrt{1^2+5^2+3^2}}\right)\\ &= \arccos\left(\frac{-52}{\sqrt{120}\sqrt{35}}\right)\\ &= 143{,}4^\circ\end{aligned}$$

□

3. *The scalar component of* $\mathbf{A}$ *along* $\mathbf{B}$ *at* P

Solución. Sea

$$\begin{aligned}A_B &= A\cdot\mathbf{a}_B\\ &= (2,4,10)\cdot\frac{(-1,-5,-3)}{|(-1,-5,-3)|}\\ &= (2,4,10)\cdot\frac{(-1,-5,-3)}{\sqrt{35}}\\ &= \frac{-2-20-30}{\sqrt{35}}\\ &= \frac{-52}{\sqrt{35}}\end{aligned}$$

□

Problema 8. *Using the differential length* dl*, find the length of each of the following curves:*

1. *(a)* $\rho = 3, \pi/4 < \phi < \pi/2, z =$ *constant*
2. *(b)* $r = 1, \theta = 30^\circ, 0 < \phi < 60^\circ$
3. *(c)* $r = 4, 30^\circ < \theta < 90^\circ, \phi =$ *constant*

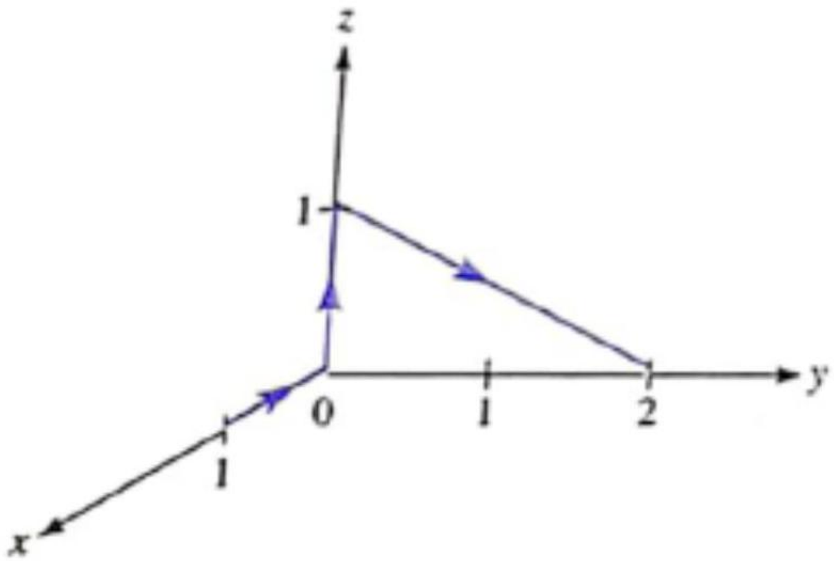
Problema 9. *Calculate the areas of the following surfaces using the differential surface area* dS *:*

1. *(a)* $\rho = 2, 0 < z < 5, \pi/3 < \phi < \pi/2$
2. *(b)* $z = 1, 1 < \rho < 3, 0 < \phi < \pi/4$
3. *(c)* $r = 10, \pi/4 < \theta < 2\pi/3, 0 < \phi < 2\pi$
4. *(d)* $0 < r < 4, 60^\circ < \theta < 90^\circ$, $\phi =$ *constant*

Problema 10. *If*

$$\mathbf{H} = (x-y)\mathbf{a}_x + \left(x^2 + zy\right)\mathrm{a}_y + 5yz\mathrm{a}_z$$

evaluate $\int H \cdot d\ I$ *along the contour of Figure* 3,28.



Problema 11. *Find the gradient of the these scalar fields:*

1. *(a)* $U = 4xz^2 + 3yz$
2. *(b)* $W = 2\rho\left(z^2+1\right)\cos\phi$
3. *(c)* $H = r^2\cos\theta\cos\phi$

Problema 12. *The temperature in an auditorium is given by* $T = x^2 + y^2 - z$. *A mosquito located at* $(1, 1, 2)$ *in the auditorium desires to lly in such a direction that it will get warm as soon as possible. In what direction must it fly?*

Problema 13. *Find the divergence and curl of the following vectors:*

1. *(a)* $\mathbf{A} = e^{vy}\mathbf{a}_x + \sin xy\mathbf{a}_y + \cos^2 xz\mathbf{a}_z$
2. *(b)* $\mathbf{B} = \rho z^2\cos\phi\mathbf{a}_p + z\sin^2\phi\mathbf{a}_z$
3. *(c)* $\mathbf{C} = r\cos\theta\mathrm{a}_r - \frac{1}{r}\sin\theta\mathrm{a}_\theta + 2r^2\sin\theta\mathrm{a}_\phi$

Problema 14. *Verify the divergence theorem*

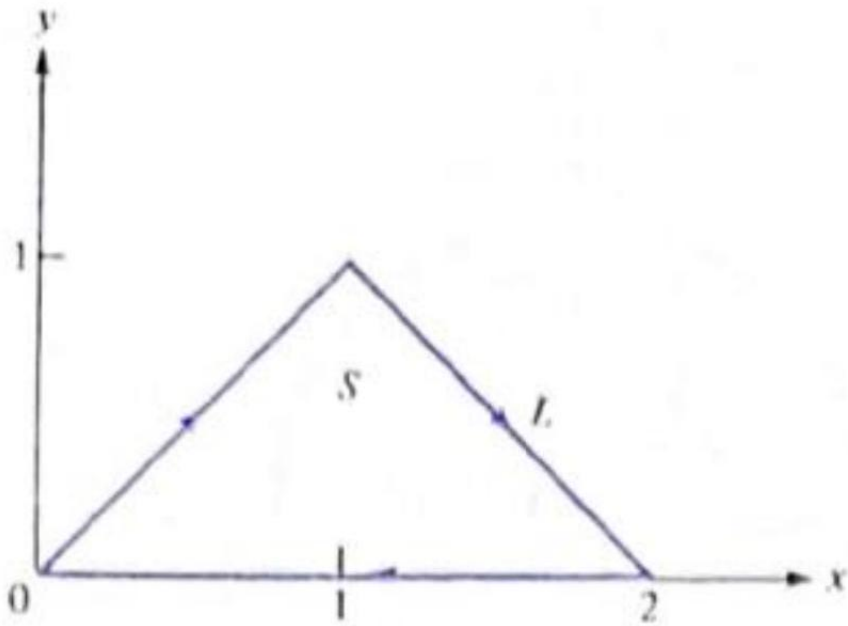
$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}dv$$

for each of the following cases:

1. *(a)* $\mathbf{A} = xy^2\mathbf{a}_x + y^3\mathbf{a}_y + y^2z\mathbf{a}_z$ *and* S *is the surface of the cuboid defined by* $0 < x < 1$, $0 < y < 1, 0 < z < 1$
2. *(b)* $\mathbf{A} = 2\rho z\mathbf{a}_\rho + 3z\sin\phi\mathbf{a}_\phi - 4\rho\cos\phi\mathbf{a}_z$ *and* S *is the surface of the wedge* $0 < \rho < 2$, $0 < \phi < 45°, 0 < z < 5$
3. *(c)* $\mathbf{A} = r^2\mathbf{a}_r + r\sin\theta\cos\phi\mathbf{a}_\theta$ *and* S *is the surface of a quarter of a sphere defined by* $0 < r < 3, 0 < \phi < \pi/2, 0 < \theta < \pi/2$

Problema 15. *Given that* $\mathbf{F} = x^2y\mathbf{a}_x - y\mathbf{a}_y$*, find*

1. *(a)* $\oint_L \mathrm{F} \cdot \mathrm{dl}$ *where* L *is shown in Figure* 3,29.
2. *(b)* $\int_S(\nabla \times \mathbf{I}) \cdot dS$ *where* S *is the area bounded by* L.
3. *(c) Is Stokes's theorem satisfied?*



Problema 16. *Given the vector field*

$$\mathbf{G} = (16xy - z)\mathrm{a}_x + 8x^2\mathrm{a}_y - x\mathrm{a}_z$$

Assume anticlockwise direction.

1. *(a) Is* $\mathbf{G}$ *irrotational (or conservative)?*
2. *(b) Find the net flux of* $\mathbf{G}$ *over the cube* $0 < x, y, z < 1$.
3. *(c) Determine the circulation of* $\mathbf{G}$ *around the edge of the square* $z = 0, 0 < x, y < 1$.