Universidad del Valle de Guatemala
Departamento de Matemática
Licenciatura en Matemática Aplicada

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Teoría electromagnética 1 - Catedrático: Eduardo Álvarez
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Parcial 1

Problema 1. *Sean*

$$\begin{aligned}
\vec{A} &= 3\vec{a}_x + 1\vec{a}_z = (3, 0, 1)\\
\vec{B} &= -2\vec{a}_x + 5\vec{a}_y - 6\vec{a}_z = (-2, 5, -6)\\
\vec{C} &= -4\vec{a}_x - 3\vec{a}_y - 2\vec{a}_z = (-4, -3, -2)
\end{aligned}$$

1. *Encontrar el ángulo entre los vectores $\vec{B}$ y $\vec{C}$.*

   Solución. Sea

$$\begin{aligned}
&\implies B \cdot C = |B||C| \cos\theta_{AB}\\
&\implies \cos\theta_{AB} = \frac{B \cdot C}{|B||C|}\\
&\implies \theta_{AB} = \arccos\left(\frac{B \cdot C}{|B||C|}\right)
\end{aligned}$$

   Para

$$\begin{aligned}
B \cdot C &= (-2, 5, -6) \cdot (-4, -3, -2)\\
&= (-2)(-4) + (5)(-3) + (-6)(-2)\\
&= 8 - 15 + 12\\
&= 5
\end{aligned}$$

   y

$$\begin{aligned}
|B| &= |(-2, 5, -6)| = \sqrt{4 + 25 + 36} = \sqrt{65}\\
|C| &= |(-4, -3, -2)| = \sqrt{16 + 9 + 4} = \sqrt{29}
\end{aligned}$$

Por lo tanto,

$$\theta_{AB} = \arccos\left(\frac{B \cdot C}{|B||C|}\right) = \arccos\left(\frac{5}{\sqrt{65}\sqrt{29}}\right) = 83{,}39^{\circ}$$

□

2. *Encontrar las componentes de $\vec{A}$ a lo largo de $\vec{B}$.*

Solución.

$$\begin{aligned}
A_B &= A_B a_B = (A \cdot a_B)\, a_B = \\
&= \left((3,0,1) \cdot \frac{(-2,5,-6)}{\sqrt{(-2)^2+(5)^2+(-6)^2}}\right) \frac{(-2,5,-6)}{\sqrt{(-2)^2+(5)^2+(-6)^2}} \\
&= \frac{1}{65}\left((3)(-2)+(0)(5)+(1)(-6)\right)(-2,5,-6) \\
&= \frac{1}{65}(-6+0-6)(-2,5,-6) \\
&= \frac{-12}{65}(-2,5,-6)
\end{aligned}$$

□

3. *Encontrar $\vec{B} \cdot (\vec{C} \times \vec{A})$.*

Solución. Sea

$$\begin{aligned}
\Rightarrow C \times A = &\begin{vmatrix} -4 & -3 & -2 \\ 3 & 0 & 1 \end{vmatrix} \\
=&[(-3)(1)-(-2)(0)]a_x - \\
&-[(-4)(1)-(-2)(3)]a_y + \\
&+[(-4)(0)-(-3)(3)]a_z \\
&= [-3]a_x - [-4+6]a_y + [9]a_z \\
&= -3a_x - 2a_y + 9a_z \\
&= (-3,-2,+9) \\
\Rightarrow B \cdot (C \times A) &= (-2,5,-6) \cdot (-3,-2,9) \\
&= (-2)(-3)+(5)(-2)+(-6)(9) \\
&= +6-10-54 = +6-64 = -58
\end{aligned}$$

□

Problema 2. *Sea $\vec{A} = \rho^3\left(1-z^3\right)\vec{a}_\rho + \rho z^3\cos(5\phi)\vec{a}_\phi + 7\rho^2 z^4 \vec{a}_z$.*
Calcular:

1. *El gradiente de $\vec{A}$.*

Solución. No se puede aplicar, ya que no se cumple la definición de gradiente para un vector. □

2. *El rotor de $\vec{A}$.*

Solución.

$$
\begin{aligned}
\nabla \times \vec{A} &= \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \\
&= \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial\left(\rho A_\phi\right)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi}\right]\mathbf{a}_z \\
&= \left[\frac{1}{\rho}(0) - \left(3\rho z^2\cos(5\phi)\right)\right]\mathbf{a}_\rho + \left[-3z^2\rho^3 - 14\rho z^4\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[2\rho z^3\cos(5\phi) - 0\right]\mathbf{a}_z \\
&= -\left(3\rho z^2\cos(5\phi)\right)\mathbf{a}_\rho - \left[3z^2\rho^3 + 14\rho z^4\right]\mathbf{a}_\phi + \left[2z^3\cos(5\phi)\right]\mathbf{a}_z
\end{aligned}
$$

□

Problema 3. *Sea $\vec{D} = r^3\cos(5\phi)\vec{a}_r + 28\,\mathrm{sen}^4(4\theta)\vec{a}_\theta + 7r^2\vec{a}_\phi$*

1. *Calcular la divergencia de $\vec{D}$.*

Solución. Sea

$$
\begin{aligned}
\nabla\cdot\mathbf{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial\mathrm{A}_\phi}{\partial\phi} \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^5\cos(5\phi)\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(28\,\mathrm{sen}^4(4\theta)\sin\theta\right) + \frac{1}{r\sin\theta}(0) \\
&= \frac{1}{r^2}\left(5r^4\cos(5\phi)\right) + \frac{28}{r\sin\theta}\left[\sin\theta\left(16\sin^3 4\theta\cos 4\theta\right) + \cos\theta\sin^4(4\theta)\right]
\end{aligned}
$$

□

2. *Calcular el rotor de $\vec{D}$.*

Solución. Sea

$$
\begin{aligned}
\nabla\times\mathbf{A} &= \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & r\sin\theta\mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \\
&= \frac{1}{r\sin\theta}\left[\frac{\partial\left(A_\phi\sin\theta\right)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_r + \\
&+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_\mathrm{r}}{\partial\phi} - \frac{\partial\left(rA_\phi\right)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial\left(rA_\theta\right)}{\partial r} - \frac{\partial A_\mathrm{r}}{\partial\theta}\right]\mathbf{a}_\phi \\
&= \frac{1}{r\sin\theta}\left[\frac{\partial\left(7r^2\sin\theta\right)}{\partial\theta} - 0\right]\mathbf{a}_r + \\
&+ \frac{1}{r}\left[\frac{1}{\sin\theta}\left(-5r^3\sin(5\phi)\right) - \frac{\partial\left(7r^3\right)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial\left(r28\,\mathrm{sen}^4(4\theta)\right)}{\partial r} - 0\right]\mathbf{a}_\phi \\
&= \frac{1}{r\sin\theta}\left[\left(7r^2\cos\theta\right)\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{\left(-5r^3\sin(5\phi)\right)}{\sin\theta} - 21r^2\right]\mathbf{a}_\theta + \frac{1}{r}\left[\left(28\,\mathrm{sen}^4(4\theta)\right)\right]\mathbf{a}_\phi \\
&= \left[(7r\cot\theta)\right]\mathbf{a}_r + \left[\frac{\left(-5r^2\sin(5\phi)\right)}{\sin\theta} - 21r\right]\mathbf{a}_\theta + \frac{1}{r}\left[\left(28\,\mathrm{sen}^4(4\theta)\right)\right]\mathbf{a}_\phi
\end{aligned}
$$

□

Problema 4. *Dado* $\vec{T} = (\alpha xy - \beta z^3)\,\vec{a}_x + (3x^2 + \gamma z)\,\vec{a}_y + (3x^2z^2 - y)\,\vec{a}_z$ *Calcule:*

1. *Si es irrotacional, encuentre* α, β *y* γ.

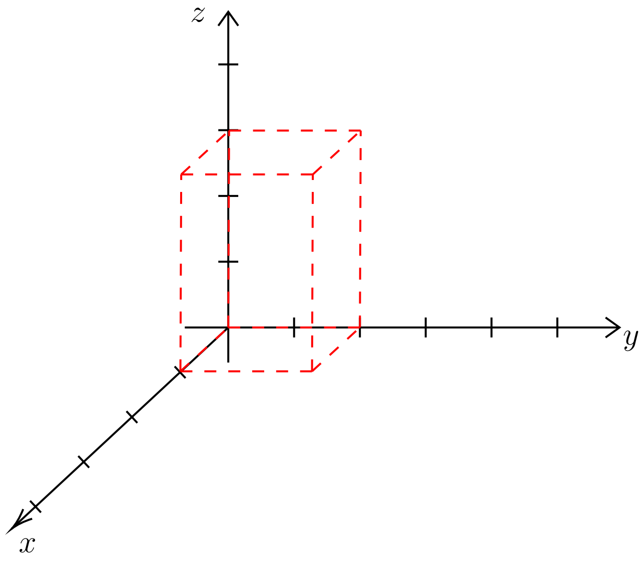
   Solución. Sea
   $$\nabla \times \vec{T} = 0$$
   Entonces,
   $$\begin{aligned}\nabla \times \vec{T} &= \left[\frac{\partial T_z}{\partial y} - \frac{\partial \mathrm{T}_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial T_x}{\partial z} - \frac{\partial \mathrm{T}_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y}\right]\mathbf{a}_z \\ &= [(-1) - \gamma]\,\mathbf{a}_x + \left[-3\beta z^2 - 6xz^2\right]\mathbf{a}_y + [6x - \alpha x]\,\mathbf{a}_z\end{aligned}$$
   Por lo tanto,
   $$\begin{aligned}\gamma &= -1 \\ \beta &= -2x \\ \alpha &= 6\end{aligned}$$
   □

2. *Calcular la divergencia de* $\vec{T}$ *valuada en* $(2, -1, -3)$

   Solución. Tenemos:
   $$\vec{T} = \left(6xy + 2xz^3\right)\vec{a}_x + \left(3x^2 - z\right)\vec{a}_y + \left(3x^2z^2 - y\right)\vec{a}_z$$
   Y la divergencia:
   $$\begin{aligned}\nabla \cdot \mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\ &= (6y + 2z^3) + (0) + (6x^2 z) \\ &= 6y + 2z^3 + 6x^2 z\end{aligned}$$
   Evaluando:
   $$\nabla \cdot \mathbf{A}(2, -1, -3) = 6(-1) + 2(-3)^3 + 6(2)^2(-3) = -132$$
   □

Problema 5. *Sea* $\vec{D} = 2xy\vec{a}_y + x^2 z\vec{a}_z$ *y el paralelepípedo rectangular formado por* $\mathrm{x} = 0$ *y* $\mathrm{x} = 1, \mathrm{y} = 0$ *y* $\mathrm{y} = 2, z = 0$ *y z=3. Mostrar si se cumple el teorema de la divergencia.*

$$\oint_S \vec{D} \cdot d\vec{S} = \int_{Vol} \nabla \cdot \vec{D} dVol$$



Solución. Sea

- Sea

$$
\begin{aligned}
\oint_s \vec{D}\cdot d\vec{S} &= \oint_s \left(0,2xy,x^2z\right)\cdot d\vec{S} \\
&= \left(\int\Big|_{y=0} + \int\Big|_{x=1} + \int\Big|_{y=2} + \int\Big|_{x=0} + \int\Big|_{z=3} + \int\Big|_{z=0} \right) \left(0,2xy,x^2z\right)\cdot d\vec{S} \\
&= \int\Big|_{y=0} \left(0,2xy,x^2z\right)\cdot d\vec{S} + \int\Big|_{x=1} \left(0,2xy,x^2z\right)\cdot d\vec{S} \\
&+ \int\Big|_{y=2} \left(0,2xy,x^2z\right)\cdot d\vec{S} + \int\Big|_{x=0} \left(0,2xy,x^2z\right)\cdot d\vec{S} \\
&+ \int\Big|_{z=3} \left(0,2xy,x^2z\right)\cdot d\vec{S} + \int\Big|_{z=0} \left(0,2xy,x^2z\right)\cdot d\vec{S} \\
&= \int\Big|_{y=0} \left(0,0,x^2z\right)\cdot (0,dxdz,0) + \int\Big|_{x=1} (0,2y,z)\cdot (dydz,0,0) \\
&+ \int\Big|_{y=2} \left(0,4x,x^2z\right)\cdot (0,dxdz,0) + \int\Big|_{x=0} (0,0,0)\cdot (dydz,0,0) \\
&+ \int\Big|_{z=3} \left(0,2xy,3x^2\right)\cdot (0,0,dxdy) + \int\Big|_{z=0} (0,2xy,0)\cdot (0,0,dxdy) \\
&= \int\int 4x dxdz + \int\int 3x^2 dxdy \\
&= 4\int_0^1 xdx \int_0^3 dz + 3\int_0^1 x^2dx \int_0^2 dy \\
&= (2)(3) + (1)(2) \\
&= 8
\end{aligned}
$$

- Sea

$$
\begin{aligned}
\int_v \nabla \cdot D dv &= \int_V \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) (dxdydz) \\
&= \int_V \left(0 + 2x + x^2\right) dxdydz \\
&= \int_0^1 \left(2x + x^2\right) dx \int_0^2 dy \int_0^3 dz \\
&= \left[\frac{2}{2} x^2 + \frac{x^3}{3} \right] \Bigg|_0^1 (2)(3) \\
&= \left[(1) + \frac{1}{3} \right] (2)(3) = \frac{4}{3}(2)(3) = 8
\end{aligned}
$$

$\square$