Universidad del Valle de Guatemala
Departamento de Matemática
Licenciatura en Matemática Aplicada

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Teoría electromagnética 1 - Catedrático: Eduardo Álvarez
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Tarea

Problema 1.

$$\begin{aligned}\mathbf{A} &= 2\mathbf{a}_x + \mathbf{a}_y - 3\mathbf{a}_z \\ \mathbf{B} &= \mathbf{a}_y - \mathbf{a}_z \\ \mathbf{C} &= 3\mathbf{a}_x + 5\mathbf{a}_y + 7\mathbf{a}_z\end{aligned}$$

Determinar:

1. $\mathrm{A} - 2\ \mathrm{B} + \mathrm{C}$

Solución. Sea

$$\begin{aligned}\mathrm{A} - 2\ \mathrm{B} + \mathrm{C} &= (2, 1, -3) - 2 \cdot (0, 1, -1) + (3, 5, 7) \\ &= (2, 1, -3) - (0, 2, -2) + (3, 5, 7) \\ &= (2, -1, -1) + (3, 5, 7) \\ &= (5, 4, 6)\end{aligned}$$

□

2. $\mathrm{C} - 4(\mathrm{A} + \mathrm{B})$

Solución. Sea

$$\begin{aligned}\mathrm{C} - 4(\mathrm{A} + \mathrm{B}) &= (3, 5, 7) - 4((2, 1, -3) + (0, 1, -1)) \\ &= (3, 5, 7) - 4(2, 2, -4) \\ &= (3, 5, 7) - (8, 8, -16) \\ &= (-5, -3, 23)\end{aligned}$$

□

3. $\frac{2\mathrm{A}-3\mathrm{B}}{|\mathrm{C}|}$

Solución. Sea

$$\begin{aligned}\frac{2\mathrm{A}-3\mathrm{B}}{|\mathrm{C}|} &= \frac{2(2,1,-3)-3(0,1,-1)}{|(3,5,7)|}\\ &= \frac{(4,2,-6)-(0,3,-3)}{\sqrt{3^2+5^2+7^2}}\\ &= \frac{(4,-1,-3)}{\sqrt{9+25+49}}\\ &= \frac{(4,-1,-3)}{\sqrt{83}}\end{aligned}$$

□

4. $\mathrm{A}\cdot\mathrm{C}-|\mathrm{B}|^2$

Solución. Sea

$$\begin{aligned}\mathrm{A}\cdot\mathrm{C}-|\mathrm{B}|^2 &= (2,1,-3)\cdot(3,5,7)-(0,1,-1)\cdot(0,1,-1)\\ &= 2*3+1*5-3*7-(0*0+1*1+(-1)*(-1))\\ &= 6+5-21-(0+1+1)\\ &= -10-2\\ &= -12\end{aligned}$$

□

5. $\frac{1}{2}\mathrm{B}\times\left(\frac{1}{3}\mathrm{A}+\frac{1}{4}\mathrm{C}\right)$

Solución. Sea

$$\begin{aligned}\frac{1}{2}\mathrm{B}\times\left(\frac{1}{3}\mathrm{A}+\frac{1}{4}\mathrm{C}\right) &= \frac{1}{2}\left(0,1,-1\right)\times\left(\frac{1}{3}(2,1,-3)+\frac{1}{4}(3,5,7)\right)\\ &= \frac{1}{2}\left(0,1,-1\right)\times\left(\frac{2}{3}+\frac{3}{4},\frac{1}{3}+\frac{5}{4},\frac{-3}{3}+\frac{7}{4}\right)\\ &= \frac{1}{2}\left(0,1,-1\right)\times\left(\frac{2}{3}+\frac{3}{4},\frac{1}{3}+\frac{5}{4},\frac{-3}{3}+\frac{7}{4}\right)\\ &= \frac{1}{2}\left(0,1,-1\right)\times\left(\frac{15}{12},\frac{19}{12},\frac{-3}{4}\right)\\ &= \frac{1}{2}\left(\frac{-3}{4}+\frac{19}{12},\frac{15}{12},\frac{15}{12}\right)\\ &= \frac{1}{2}\left(\frac{10}{12},\frac{15}{12},\frac{15}{12}\right)\\ &= \left(\frac{10}{24},\frac{15}{24},\frac{15}{24}\right)\end{aligned}$$

□

Problema 2. *Given that*

$$\begin{aligned}\mathbf{P} &= 2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z = (2,-1,-2)\\ \mathbf{Q} &= 4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z = (4,3,2)\\ \mathbf{R} &= -\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z = (-1,1,2)\end{aligned}$$

Find:

1. $|\mathbf{P}+\mathbf{Q}-\mathbf{R}|$

   Solución. Sea

$$\begin{aligned}|\mathbf{P}+\mathbf{Q}-\mathbf{R}| &= |(2,-1,-2)+(4,3,2)-(-1,1,2)|\\ &= |(7,1,-2)|\\ &= \sqrt{7^2+1^2+2^2}\\ &= \sqrt{54}\end{aligned}$$

   □

2. $\mathbf{P}\cdot\mathbf{Q}\times\mathbf{R}$

   Solución. Sea

$$\begin{aligned}\mathbf{P}\cdot\mathbf{Q}\times\mathbf{R} &= (2,-1,-2)\cdot[(4,3,2)\times(-1,1,2)]\\ &= (2,-1,-2)\cdot(4,-10,7)\\ &= 4\end{aligned}$$

   □

3. $\mathbf{Q}\times\mathbf{P}\cdot\mathbf{R}$

   Solución. Sea

$$\begin{aligned}\mathbf{Q}\times\mathbf{P}\cdot\mathbf{R} &= [(4,3,2)\times(2,-1,-2)]\cdot(-1,1,2)\\ &= (-4,12,-10)\cdot(-1,1,2)\\ &= -4\end{aligned}$$

   □

4. $(\mathbf{P}\times\mathbf{Q})\cdot(\mathbf{Q}\times\mathbf{R})$

   Solución. Sea

$$\begin{aligned}(\mathbf{P}\times\mathbf{Q})\cdot(\mathbf{Q}\times\mathbf{R}) &= ((2,-1,-2)\times(4,3,2))\cdot((4,3,2)\times(-1,1,2))\\ &= (4,-12,10)\cdot(4,-10,7)\\ &= 206\end{aligned}$$

   □

5. $(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R})$

Solución. Sea

$$\begin{aligned}
(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R}) &= ((2,-1,-2) \times (4,3,2)) \cdot ((4,3,2) \times (-1,1,2)) \\
&= (4,-12,10) \cdot (4,-10,7) \\
&= 206
\end{aligned}$$

□

6. $\cos\theta_{PR}$

Solución. Sea

$$\begin{aligned}
\cos\theta_{PR} &= \frac{P \cdot R}{|P||R|} \\
&= \frac{(2,-1,-2) \cdot (-1,1,2)}{|(2,-1,-2)||(-1,1,2)|} \\
&= \frac{-7}{3\sqrt{6}}
\end{aligned}$$

□

7. $\sin\theta_{PQ}$

Solución. Sea

$$\begin{aligned}
\sin\theta_{PQ} &= \frac{P \times Q}{|P||Q|} \\
&= \frac{(2,-1,-2) \times (4,3,2)}{|(2,-1,-2)||(4,3,2)|} \\
&= \frac{(4,-12,10)}{\sqrt{9}\sqrt{29}} \\
&= \frac{(4,-12,10)}{3\sqrt{29}}
\end{aligned}$$

□

Problema 3. *If* $\mathbf{A} = -\mathbf{a}_x + 6\mathbf{a}_y + 5\mathbf{a}_z$ *and* $\mathbf{B} = \mathbf{a}_x + 2\mathbf{a}_. + 3\mathbf{a}_x$*, find:*

1. *the scalar projections of* A *on* B

Solución. Sea

$$\begin{aligned}
A_B &= A \cdot \mathbf{a}_B \\
&= (-1,6,5) \cdot \frac{(1,2,3)}{\sqrt{14}} \\
&= \frac{26}{\sqrt{14}}
\end{aligned}$$

□

2. *the vector projection of B on A.*

Solución. Sea

$$\begin{aligned} B_A &= B_A \mathbf{a}_A \\ &= (B \cdot \mathbf{a}_A)\mathbf{a}_A \\ &= \left((1,2,3) \cdot \frac{(-1,6,5)}{\sqrt{1+36+25}} \right) \frac{(-1,6,5)}{\sqrt{1+36+25}} \\ &= \left((1,2,3) \cdot \frac{(-1,6,5)}{\sqrt{62}} \right) \frac{(-1,6,5)}{\sqrt{62}} \\ &= \left(\frac{26}{\sqrt{62}} \right) \frac{(-1,6,5)}{\sqrt{62}} \\ &= \frac{26}{62}(-1,6,5) \\ &= \frac{13}{31}(-1,6,5) \end{aligned}$$

□

Problema 4. *Let*

1. *If $V = xz - xy + yz$, express V in cylindrical coordinates.*

Solución. Sea

$$\begin{aligned} x &= \rho \cos\phi \\ y &= \rho \sin\phi \\ z &= z \end{aligned}$$

Entonces,

$$\begin{aligned} V &= xz - xy + yz \\ &= (\rho\cos\phi)(z) - (\rho\cos\phi)(\rho\sin\phi) + (\rho\sin\phi)z \\ &= z\rho\cos\phi - \rho^2\cos\phi\sin\phi + z\rho\sin\phi \end{aligned}$$

□

2. *If $U = x^2 + 2y^2 + 3z^2$, express U in spherical coordinates.*

Solución. Sea

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}$$

Entonces,

$$\begin{aligned} U &= x^2 + 2y^2 + 3z^2 \\ &= (r\sin\theta\cos\phi)^2 + 2\,(r\sin\theta\sin\phi)^2 + 3\,(r\cos\theta)^2 \\ &= r^2\left[\sin^2\theta\cos^2\phi + 2\sin^2\theta\sin^2\phi + 3\cos^2\theta\right] \\ &= r^2\left[\sin^2\theta\left(\cos^2\phi + 2\sin^2\phi\right) + 3\cos^2\theta\right] \end{aligned}$$

□

Problema 5. *Express the following vectors in Cartesian coordinates:*

1. $\mathbf{A} = \rho\left(z^2+1\right)\mathbf{a}_\rho - \rho z\cos\phi\mathbf{a}_\phi$

Solución. Sea

$$\begin{aligned}
\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} &= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho(z^2+1) \\ \rho z\cos\phi \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \rho(z^2+1)\cos\phi - \sin\phi\rho z\cos\phi \\ \sin\phi\rho(z^2+1) + \rho z\cos^2\phi \\ 0 \end{bmatrix} = \begin{bmatrix} (z^2+1)x - \sin\phi zx \\ y(z^2+1) + xz\cos\phi \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} (z^2+1)x - \frac{y}{\sqrt{x^2+y^2}}zx \\ y(z^2+1) + xz\frac{x}{\sqrt{x^2+y^2}} \\ 0 \end{bmatrix} = \begin{bmatrix} (z^2+1)x - \frac{yzx}{\sqrt{x^2+y^2}} \\ y(z^2+1) + \frac{x^2z}{\sqrt{x^2+y^2}} \\ 0 \end{bmatrix}
\end{aligned}$$

□

2. $\mathbf{B} = 2r\sin\theta\cos\phi\mathbf{a}_r + r\cos\theta\cos\theta\mathbf{a}_\theta - r\sin\phi\mathbf{a}_\phi$

Solución. Sea

$$\begin{aligned}
\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} &= \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 2r\sin\theta\cos\phi \\ r\cos\theta\cos\theta \\ -r\sin\phi \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 2x \\ r\cos^2\theta \\ -r\sin\phi \end{bmatrix} \\
&= \begin{bmatrix} 2x\sin\theta\cos\phi + r\cos^3\theta\cos\phi + r\sin^2\phi \\ 2x\sin\theta\sin\phi + r\cos^3\theta\sin\phi - r\cos\phi\sin\phi \\ 2x\cos\theta - r\cos^2\theta\sin\theta \end{bmatrix} \\
&= \begin{bmatrix} \frac{r}{r}\cdot 2x\sin\theta\cos\phi + \frac{r^2}{r}\cdot r\cos^3\theta\cos\phi + r\sin^2\phi \\ \frac{r}{r}\cdot 2x\sin\theta\sin\phi + \frac{r^2}{r}\cdot r\cos^3\theta\sin\phi - r\cos\phi\sin\phi \\ \frac{r}{r}\cdot 2x\cos\theta - \frac{r}{r}\cdot r\cos^2\theta\sin\theta \end{bmatrix} \\
&= \begin{bmatrix} \frac{2x^2}{r} + \frac{z^3\cos\phi}{r} + r\sin^2\phi \\ \frac{2xy}{r} + \frac{z^3\sin\phi}{r} - r\cos\phi\sin\phi \\ \frac{2xz}{r} - \frac{z^2\sin\theta}{r} \end{bmatrix} = \begin{bmatrix} \frac{2x^2}{r} + \frac{z^3}{r}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + r\left(\frac{y}{x^2+y^2}\right)^2 \\ \frac{2xy}{r} + \frac{z^3}{r}\left(\frac{y}{x^2+y^2}\right) - r\left(\frac{x}{x^2+y^2}\right)\left(\frac{y}{x^2+y^2}\right) \\ \frac{2xz}{r} - \frac{z^2}{r}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\right) \end{bmatrix} \\
&= \begin{bmatrix} \frac{2x^2}{\sqrt{x^2+y^2+z^2}} + \frac{z^3}{\sqrt{x^2+y^2+z^2}}\left(\frac{x}{\sqrt{x^2+y^2}}\right) + \sqrt{x^2+y^2+z^2}\left(\frac{y}{x^2+y^2}\right)^2 \\ \frac{2xy}{\sqrt{x^2+y^2+z^2}} + \frac{z^3}{\sqrt{x^2+y^2+z^2}}\left(\frac{y}{x^2+y^2}\right) - \sqrt{x^2+y^2+z^2}\left(\frac{x}{x^2+y^2}\right)\left(\frac{y}{x^2+y^2}\right) \\ \frac{2xz}{\sqrt{x^2+y^2+z^2}} - \frac{z^2}{\sqrt{x^2+y^2+z^2}}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\right) \end{bmatrix}
\end{aligned}$$

□

Problema 6. *Let*

1. *Express the vector field*

$$\mathbf{H} = xy^2z\mathbf{a}_x + x^2yz\mathbf{a}_y + xyz^2\mathbf{a}_z$$

in cylindrical and spherical coordinates.

Solución. Sea

- Cilíndricas. Sea

$$\begin{aligned}
\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} &= \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} xy^2z \\ x^2yz \\ xyz^2 \end{bmatrix} \\
&= \begin{bmatrix} \cos\phi(xy^2z) + \sin\phi(x^2yz) + 0(xyz^2) \\ -\sin\phi(xy^2z) + \cos\phi(x^2yz) + 0(xyz^2) \\ 0(xy^2z) + 0(x^2yz) + 1(xyz^2) \end{bmatrix} = \begin{bmatrix} \cos\phi(xy^2z) + \sin\phi(x^2yz) \\ -\sin\phi(xy^2z) + \cos\phi(x^2yz) \\ xyz^2 \end{bmatrix} \\
&= \begin{bmatrix} \cos\phi((\rho\cos\phi)(\rho\sin\phi)^2z) + \sin\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ -\sin\phi((\rho\cos\phi)(\rho\sin\phi)^2z) + \cos\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ (\rho\cos\phi)(\rho\sin\phi)z^2 \end{bmatrix}
\end{aligned}$$

- Esféricas. Sea

$$\begin{aligned}
\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} &= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} xy^2z \\ x^2yz \\ xyz^2 \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} (r\sin\theta\cos\phi)(r\sin\theta\sin\phi)^2(r\cos\theta) \\ (r\sin\theta\cos\phi)^2(r\sin\theta\sin\phi)(r\cos\theta) \\ (r\sin\theta\cos\phi)(r\sin\theta\sin\phi)(r\cos\theta)^2 \end{bmatrix} \\
&= \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} r^4(\sin^3\theta\cos\phi\sin^2\phi\cos\theta) \\ r^4(\sin^3\theta\cos^2\phi\sin\phi\cos\theta) \\ r^4(\sin^2\theta\cos\phi\sin\phi\cos^2\theta) \end{bmatrix} \\
&= \begin{bmatrix} r^4\left(\sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^2\theta\cos\phi\sin\phi\cos^3\theta\right) \\ r^4\left(\sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta + \sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta - \sin^3\theta\cos\phi\sin\phi\cos^2\theta\right) \\ r^4\left(-\sin^3\theta\cos\phi\sin^3\phi\cos\theta + \sin^3\theta\cos^3\phi\sin\phi\cos\theta\right) \end{bmatrix}
\end{aligned}$$

□

2. *In both cylindrical and spherical coordinates, determine* H *at* $(3, -4, 5)$.

Solución. Sea $x = 3, y = -4, z = 5$,

- Cilíndricas, $H(\rho, \phi, z)$. Tenemos:

- $\rho = \sqrt{3^2+4^2} = \sqrt{25} = 5$
- $\phi = \arctan\left(\frac{-4}{3}\right) = -0{,}927$
- $z = 5$

Con eso, se evalúa en:

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi((\rho\cos\phi)(\rho\sin\phi)^2 z) + \sin\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ -\sin\phi((\rho\cos\phi)(\rho\sin\phi)^2 z) + \cos\phi((\rho\cos\phi)^2(\rho\sin\phi)z) \\ (\rho\cos\phi)(\rho\sin\phi)z^2 \end{bmatrix}$$

- Esféricas, $H(r,\theta,\phi)$. Tenemos:
  - $r = \sqrt{3^2+4^2+5^2} = \sqrt{9+16+25} = \sqrt{50}$
  - $\theta = \arctan\left(\frac{\sqrt{3^2+4^2}}{5}\right) = \arctan\left(\frac{\sqrt{25}}{5}\right) = \arctan(1) = \pi/4$
  - $\phi = \arctan\left(\frac{-4}{3}\right) = -0{,}927$

  Con eso, se evalúa en:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} r^4\left(\sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^4\theta\cos^2\phi\sin^2\phi\cos\theta + \sin^2\theta\cos\phi\sin\phi\cos^3\theta\right) \\ r^4\left(\sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta + \sin^3\theta\cos^2\phi\sin^2\phi\cos^2\theta - \sin^3\theta\cos\phi\sin\phi\cos^2\theta\right) \\ r^4\left(-\sin^3\theta\cos\phi\sin^3\phi\cos\theta + \sin^3\theta\cos^3\phi\sin\phi\cos\theta\right) \end{bmatrix}$$

□

Problema 7. *Given vectors* $\mathbf{A} = 2\mathbf{a}_x + 4\mathbf{a}_y + 10\mathbf{a}_z$ *and* $\mathbf{B} = -5\mathbf{a}_\rho + \mathbf{a}_\phi - 3\mathbf{a}_z$*, find*

1. $\mathbf{A}+\mathbf{B}$ *at* $P(0,2,-5)$

Solución. Sea

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -5 \\ 1 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -5\cos\phi - \sin\phi \\ -5\sin\phi + \cos\phi \\ -3 \end{bmatrix} = \begin{bmatrix} -5\left(\frac{x}{\sqrt{x^2+y^2}}\right) - \left(\frac{y}{\sqrt{x^2+y^2}}\right) \\ -5\left(\frac{y}{\sqrt{x^2+y^2}}\right) + \left(\frac{x}{\sqrt{x^2+y^2}}\right) \\ -3 \end{bmatrix}$$

Entonces $A+B$ en $P(0,2,-5)$

$$A+B = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} 2-5\left(\frac{x}{\sqrt{x^2+y^2}}\right) - \left(\frac{y}{\sqrt{x^2+y^2}}\right) \\ 4-5\left(\frac{y}{\sqrt{x^2+y^2}}\right) + \left(\frac{x}{\sqrt{x^2+y^2}}\right) \\ 10-3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\left(\frac{2}{\sqrt{4}}\right) \\ 4-5\left(\frac{2}{\sqrt{4}}\right) \\ 10-3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix}$$

□

2. *The angle between* **A** *and* **B** *at* P

   Solución. Por la propiedad:

$$\mathbf{A}\cdot\mathbf{B} = |A||B|\cos\theta_{AB}$$
$$\arccos\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|A||B|}\right) = \theta_{AB}$$

   Considerando, $B = (-1,-5,-3)$ tenemos:

$$\begin{aligned}\theta_{AB} &= \arccos\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|A||B|}\right)\\ &= \arccos\left(\frac{(2,4,10)\cdot(-1,-5,-3)}{|(2,4,10)||(-1,-5,-3)|}\right)\\ &= \arccos\left(\frac{-2-20-30}{\sqrt{2^2+4^2+10^2}\sqrt{1^2+5^2+3^2}}\right)\\ &= \arccos\left(\frac{-52}{\sqrt{120}\sqrt{35}}\right)\\ &= 143{,}4^\circ\end{aligned}$$

□

3. *The scalar component of* **A** *along* **B** *at* P

   Solución. Sea

$$\begin{aligned}A_B &= A\cdot\mathbf{a}_B\\ &= (2,4,10)\cdot\frac{(-1,-5,-3)}{|(-1,-5,-3)|}\\ &= (2,4,10)\cdot\frac{(-1,-5,-3)}{\sqrt{35}}\\ &= \frac{-2-20-30}{\sqrt{35}}\\ &= \frac{-52}{\sqrt{35}}\end{aligned}$$

□

Problema 8. *Using the differential length dl, find the length of each of the following curves:*

1. $\rho = 3, \pi/4 < \phi < \pi/2, z =$ *constant*

   Solución. Sea

$$dl = \rho d\phi$$
$$l = 3\int_{\pi/4}^{\pi/2} d\phi = 3\left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \frac{3\pi}{4}$$

□

2. $r = 1, \theta = 30°, 0 < \phi < 60°$

   Solución. Sea

$$\begin{aligned} dl &= r\sin\theta d\phi \\ l &= r\sin\theta \int_0^{60°} d\phi = r\sin\theta(60° - 0) = 1\sin 30°(60°) = \frac{1}{2}\left(\frac{\pi}{3}\right) = \frac{\pi}{6} \end{aligned}$$

$\square$

3. $r = 4, 30° < \theta < 90°, \phi =$ *constant*

   Solución. Sea

$$\begin{aligned} dl &= r d\theta \\ l &= 4\int_{30°}^{90°} d\theta = 4(90° - 30°) = 4(60°) = \frac{4\pi}{3} \end{aligned}$$

$\square$

Problema 9. *Calculate the areas of the following surfaces using the differential surface area dS :*

1. $\rho = 2, 0 < z < 5, \pi/3 < \phi < \pi/2$

   Solución. Sea

$$\begin{aligned} dS &= \rho d\phi dz \\ S &= 2\int_{\pi/3}^{\pi/2} d\phi \int_0^5 dz = 2\left(\pi/2 - \pi/3\right)(5 - 0) = \frac{10\pi}{6} \end{aligned}$$

$\square$

2. $z = 1, 1 < \rho < 3, 0 < \phi < \pi/4$

   Solución. Sea

$$\begin{aligned} dS &= \rho d\phi d\rho \\ S &= \int_1^3 \rho d\rho \int_0^{\pi/4} d\phi = \frac{\rho^2}{2}\Big|_1^3 (\pi/4) = \left(\frac{9}{2} - \frac{1}{2}\right)\frac{\pi}{4} = \pi \end{aligned}$$

$\square$

3. $r = 10, \pi/4 < \theta < 2\pi/3, 0 < \phi < 2\pi$

Solución. Sea

$$\begin{aligned} dS &= r^2 \sin\theta d\theta d\phi \\ &= r^2 \int_{\pi/4}^{2\pi/3} \sin\theta d\theta \int_0^{2\pi} d\phi \\ &= -100(\cos(2\pi/3) - \cos(\pi/4))(2\pi - 0) \\ &= -200\pi\left(-\frac{1}{2}(1+\sqrt{2})\right) \\ &= 100\pi(1+\sqrt{2}) \end{aligned}$$

□

4. $0 < r < 4, 60^\circ < \theta < 90^\circ$, $\phi =$ *constant*

Solución. Sea

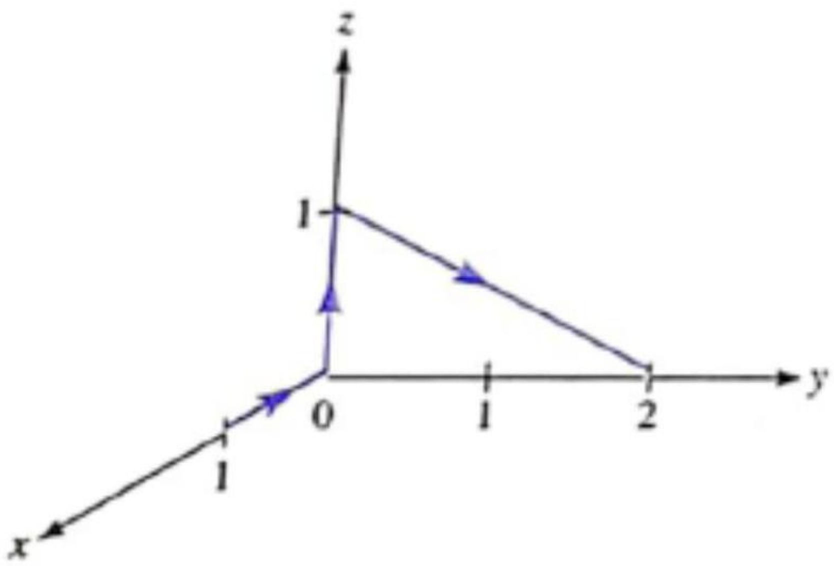
$$\begin{aligned} dS &= r dr d\theta \\ &= \int_0^4 r dr \int_{60^\circ}^{90^\circ} d\theta \\ &= \frac{1}{2}(4^2 - 0^2)(30^\circ) \\ &= 8\left(\frac{\pi}{6}\right) \\ &= \frac{4\pi}{3} \end{aligned}$$

□

Problema 10. *If*

$$\mathbf{H} = (x-y)\mathbf{a}_x + \left(x^2 + zy\right)\mathrm{a}_y + 5yz\mathrm{a}_z$$

evaluate $\int H \cdot dl$ *along the contour of Figure* 3,28.



Solución. Sea

$$\begin{aligned}
\int \mathbf{H} \cdot dl &= \left(\int_1 + \int_2 + \int_3\right)(x-y, x^2+zy, 5yz) \cdot dl \\
&= \int_1 (x-y, x^2+zy, 5yz) \cdot (dx, 0, 0) + \int_2 (x-y, x^2+zy, 5yz) \cdot (0,0,dz) + \\
&+ \int_3 (x-y, x^2+zy, 5yz) \cdot (0, dy, dz) \\
&= \int_1 (x-y)dx + \int_2 (5yz)dz + \int_3 ((x^2+zy)dy + 5yzdz) \\
&= \int_1^0 (x-y)dx + 5y\int_0^1 zdz + \int_0^2 ((x^2+zy)dy + 5yzdz) \\
&= \int_1^0 (x-0)dx + 5*0\int_0^1 zdz + \int_0^2 ((0^2+zy)dy + 5yzdz) \\
&= \int_1^0 xdx + \int_0^2 \left(-\frac{y}{2}+1\right) ydy + 5\int_0^2 (y)\left(-\frac{y}{2}+1\right)\left(-\frac{dy}{2}\right) \\
&= \int_1^0 xdx + \int_0^2 \left(-\frac{y^2}{2}+y\right) dy - \frac{5}{2}\int \left(-\frac{y^2}{2}+y\right) dy \\
&= 1/2 - 2/3 - 5/2(-2/3) \\
&= 3/2
\end{aligned}$$

□

Problema 11. *Find the gradient of the these scalar fields:*

1. $U = 4xz^2 + 3yz$

   Solución. Sea

$$\begin{aligned}
\nabla &= \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) \\
&= \left(4z^2, 3z, 8xz + 3y\right)
\end{aligned}$$

□

2. $W = 2\rho\,(z^2+1)\cos\phi$

   Solución. Sea

$$\begin{aligned}
\nabla &= \left(\frac{\partial W}{\partial \rho}, \frac{1}{\rho}\frac{\partial W}{\partial \phi}, \frac{\partial W}{\partial z}\right) \\
&= \left(2\left(z^2+1\right)\cos\phi, -2\left(z^2+1\right)\sin\phi, z\right)
\end{aligned}$$

□

3. $H = r^2\cos\theta\cos\phi$

Solución. Sea

$$\begin{aligned}\nabla &= \left(\frac{\partial H}{\partial r}, \frac{1}{r}\frac{\partial H}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial H}{\partial \phi}\right)\\ &= (2r\cos\theta\cos\phi, -r\sin\theta\cos\phi, -r\cot\theta\sin\phi)\end{aligned}$$

□

Problema 12. *The temperature in an auditorium is given by* $T = x^2 + y^2 - z$. *A mosquito located at* $(1, 1, 2)$ *in the auditorium desires to fly in such a direction that it will get warm as soon as possible. In what direction must it fly?*

Solución. Sea

$$\begin{aligned}\nabla &= \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right)\\ &= (2x, 2y, -1)\end{aligned}$$

Entonces, en el punto $(1, 1, 2)$, debe seguir el vector:

$$(2, 2, -1)$$

□

Problema 13. *Find the divergence and curl of the following vectors:*

1. $\mathbf{A} = e^{xy}\mathbf{a}_x + \sin xy\mathbf{a}_y + \cos^2 xz\mathbf{a}_z$

Solución. Sea 5

- Divergencia

$$\begin{aligned}\nabla \cdot A &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\\ &= ye^{xy} + x\cos xy - 2x\cos zx\sin zxs\end{aligned}$$

- Rotor

$$\begin{aligned}\nabla \times A &= \left[\frac{\partial A_z}{\partial y} - \frac{\partial \mathrm{A}_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial \mathrm{A}_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial \mathrm{A}_x}{\partial y}\right]\mathbf{a}_z\\ &= (0, z2\sin xz, y\cos xy - xe^{xy})\end{aligned}$$

□

2. $\mathbf{B} = \rho z^2\cos\phi\mathbf{a}_p + z\sin^2\phi\mathbf{a}_z$

Solución. Sea

- Divergencia

$$\begin{aligned}\nabla \cdot B &= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}\\ &= 2z^2\cos\phi + \sin^2\phi\end{aligned}$$

- Rotor

$$\begin{aligned}\nabla \times B &= \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial\left(\rho A_\phi\right)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\mathbf{a}_z \\ &= \left(\frac{z\sin 2\phi}{\rho}, 2\rho z\cos\phi, z^2\sin\phi\right)\end{aligned}$$

□

3. $\mathbf{C} = r\cos\theta\mathbf{a}_r - \frac{1}{r}\sin\theta\mathbf{a}_\theta + 2r^2\sin\theta\mathbf{a}_\phi$

Solución. Sea

- Divergencia

$$\begin{aligned}\nabla \cdot C &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(A_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial \mathrm{A}_\phi}{\partial \phi} \\ &= 3\cos\theta - \frac{2\cos\theta}{r^2}\end{aligned}$$

- Rotor

$$\begin{aligned}\nabla \times C &= \frac{1}{r\sin\theta}\left[\frac{\partial\left(A_\phi\sin\theta\right)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_r \\ &+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial\left(rA_\phi\right)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial\left(rA_\theta\right)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\mathbf{a}_\phi \\ &= \left(r\cos\theta, -6r\sin\theta, \sin\theta\right)\end{aligned}$$

□

Problema 14. *Verify the divergence theorem*

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{A}dv$$

for each of the following cases:

1. $\mathbf{A} = xy^2\mathbf{a}_x + y^3\mathbf{a}_y + y^2z\mathbf{a}_z$ *and S is the surface of the cuboid defined by* $0 < x < 1$, $0 < y < 1, 0 < z < 1$

Solución. Debemos comprobar los dos lados de la igualdad

- Sea

$$\begin{aligned}
\oint_S \mathbf{A} \cdot d\mathbf{S} &= \oint_S \left(xy^2, y^3, y^2z\right) \cdot (dydz, dxdz, dxdy) \\
&= \oint_S \left(xy^2dydz + y^3dxdz + y^2zdxdy\right) \\
&= \left(\iint_{x=0} + \iint_{x=1} + \iint_{y=0} + \iint_{y=1} + \iint_{z=0} + \iint_{z=1}\right) \\
&\quad \left(xy^2dydz + y^3dxdz + y^2zdxdy\right) \\
&= x\int_0^1 y^2dy \int_0^1 dz + y^3\int_0^1 dx \int_0^1 dz + z\int_0^1 y^2dy \int_0^1 dx \\
&= (1)\left(\frac{(1)^3}{3}\right) + (1)^3 + (1)\left(\frac{(1)^3}{3}\right) \\
&= \frac{1}{3} + 1^3 + \frac{1}{3} = \frac{5}{3}
\end{aligned}$$

- Sea

$$\begin{aligned}
\int_v \nabla \cdot \mathbf{A} dv &= \int_v \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) dv \\
&= \int_v \left(y^2 + 3y^2 + y^2\right) dxdydz \\
&= \int_v \left(5y^2\right) dxdydz \\
&= \int_0^1 \int_0^1 \int_0^1 \left(5y^2\right) dxdydz \\
&= \frac{5}{3}(1)^3 = \frac{5}{3}
\end{aligned}$$

Por lo tanto, se cumple el teorema de la divergencia. □

2. $\mathbf{A} = 2\rho z\mathbf{a}_\rho + 3z\sin\phi\mathbf{a}_\phi - 4\rho\cos\phi\mathbf{a}_z$ *and* S *is the surface of the wedge* $0 < \rho < 2$, $0 < \phi < 45° = \pi/4, 0 < z < 5$

Solución. Debemos comprobar los dos lados de la igualdad

- Sea

$$
\begin{aligned}
\oint_S \mathbf{A}\cdot d\mathbf{S} &= \oint_S (2\rho z, +3z\sin\phi, -4\rho\cos\phi)\cdot(\rho d\phi dz, d\rho dz, \rho d\phi d\rho) \\
&= \oint_S (2\rho z\rho d\phi dz + 3z\sin\phi d\rho dz - 4\rho\cos\phi\rho d\phi d\rho) \\
&= \left(\iint_{\rho=0} + \iint_{\rho=2} + \iint_{\phi=0} + \iint_{\phi=\pi/4} + \iint_{z=0} + \iint_{z=5}\right) \\
&\quad \left(2\rho^2 z d\phi dz + 3z\sin\phi d\rho dz - 4\rho^2\cos\phi d\phi d\rho\right) \\
&= \iint_{\rho=2} 2\rho^2 z d\phi dz + \iint_{\phi=\pi/4} 3z\sin\phi d\rho dz + \\
&\quad \underbrace{-\iint_{z=0} -4\rho^2\cos\phi d\phi d\rho + \iint_{z=5} -4\rho^2\cos\phi d\phi d\rho}_{\text{se cancelan}} \\
&= 8\int_0^{\pi/4} d\phi \int_0^5 zdz + \frac{3}{\sqrt{2}}\int_0^5 zdz\int_0^2 d\rho \\
&= 8\left(\frac{\pi}{4}\right)\left(\frac{5^2}{2}\right) + \frac{3}{\sqrt{2}}\left(\frac{5^2}{2}\right)(2) \\
&= 25\pi + \frac{75}{\sqrt{2}}
\end{aligned}
$$

- Sea

$$
\begin{aligned}
\int_v \nabla\cdot\mathbf{A}dv &= \int_v \left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right)\rho d\rho d\phi dz \\
&= \int_v \left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(2\rho^2 z\right) + \frac{1}{\rho}\frac{\partial(3z\sin\phi)}{\partial\phi} + \frac{\partial(-4\rho\cos\phi)}{\partial z}\right)\rho d\rho d\phi dz \\
&= \int_v \left(4z + \frac{3z\cos\phi}{\rho} + 0\right)\rho d\rho d\phi dz \\
&= \int_v (4\rho z + 3z\cos\phi)\, d\rho d\phi dz \\
&= 4\int_0^2 \rho d\rho \int_0^{\pi/4} d\phi \int_0^5 zdz + 3\int_0^2 d\rho\int_0^{\pi/4}\cos\phi d\phi\int_0^5 zdz \\
&= 4\left(\frac{2^2}{2}\right)\left(\frac{\pi}{4}\right)\left(\frac{5^2}{2}\right) + 3(2)\left(\sin\left(\frac{\pi}{4}\right)\right)\left(\frac{5^2}{2}\right) \\
&= 25\pi + \frac{150}{2\sqrt{2}}
\end{aligned}
$$

$\square$

3. *$\mathbf{A} = r^2\mathbf{a}_r + r\sin\theta\cos\phi\mathbf{a}_\theta$ and S is the surface of a quarter of a sphere defined by* $0 < r < 3, 0 < \phi < \pi/2, 0 < \theta < \pi/2$

Solución. Debemos comprobar los dos lados de la igualdad

- Sea

$$\oint_S \mathbf{A}\cdot d\mathbf{S} = \left(\iint_{r=0} + \iint_{r=3} + \iint_{\phi=0} + \iint_{\phi=\pi/2} + \iint_{\theta=0} + \iint_{\theta=\pi/2}\right)$$
$$\left(r^2, r\sin\theta\cos\phi, 0\right)\cdot\left(r^2\sin\theta d\theta d\phi, r\sin\theta dr d\phi, r dr d\theta\right)$$
$$= \iint_{r=3} r^4\sin\theta d\theta d\phi + \iint_{\theta=\pi/2} r^2\sin^2\theta\cos\phi dr d\phi$$
$$= 3^4\int_0^{\pi/2}\sin\theta d\theta\int_0^{\pi/2}d\phi + (1)^2\int_0^3 r^2 dr\int_0^{\pi/2}\cos\phi d\phi$$
$$= 3^4(-\cos\frac{\pi}{2}+\cos 0)\left(\frac{\pi}{2}\right) + \frac{3^3}{3}\left(\sin\frac{\pi}{2}-\sin 0\right)$$
$$= \frac{81\pi}{2}+9$$

- Sea

$$\int_v \nabla\cdot\mathbf{A}dv = \int_v\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\phi}\right)dv$$
$$= \int_v\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^4) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial(0)}{\partial\phi}\right)dv$$
$$= \int_v\left(4r + \frac{r\cos\phi 2\sin\theta\cos\theta}{r\sin\theta}\right)dv$$
$$= \int_v(4r+2\cos\phi\cos\theta)\, r^2\sin\theta dr d\theta d\phi$$
$$= \int_0^3\int_0^{\pi/2}\int_0^{\pi/2}\left(4r^3\sin\theta + 2r^2\cos\phi\cos\theta\sin\theta\right)dr d\theta d\phi$$
$$= 4\int_0^3 r^3 dr\int_0^{\pi/2}\sin\theta d\theta\int_0^{\pi/2}d\phi + 2\int_0^3 r^2 dr\int_0^{\pi/2}\cos\theta\sin\theta d\theta\int_0^{\pi/2}\cos\phi d\phi$$
$$= 4\left(\frac{3^4}{4}\right)\left(-\cos\frac{\pi}{2}+\cos 0\right)\left(\frac{\pi}{2}\right) + 2\left(\frac{3^3}{3}\right)\left(\frac{1}{2}\right)\left(\sin\frac{\pi}{2}-\sin 0\right)$$
$$= 81\,(1)\left(\frac{\pi}{2}\right)+9$$
$$= \frac{81\pi}{2}+9$$

□

Problema 15. *Given that* $\mathbf{F} = x^2y\mathbf{a}_x - y\mathbf{a}_y$, *find*

1. $\oint_L \mathrm{F}\cdot d\mathrm{l}$ *where* L *is shown in Figure* 3,29.

Solución. Sea

> Considerando:
>
> - Para (1): $y = x \implies dy = dx.$
> - Para (2): $y = -x+2 \implies dy = -dx$
> - Para (3): $y = 0 \implies dy = 0$

$$\begin{aligned}
\oint_L \mathrm{F}\cdot d\mathrm{l} &= \oint_L \left(x^2y,-y,0\right)\cdot(dx,dy,dz)\\
&= \left(\int_1+\int_2+\int_3\right)\left(x^2ydx-ydy\right)\\
&= \int_1(x^2ydx-ydy)+\int_2(x^2ydx-ydy)+\int_3(x^2ydx-ydy)\\
&= \int_1(x^2(x)dx-xdx)+\int_2(x^2(-x+2)dx-(-x+2)(-dx))+0\\
&= \int_0^1(x^3-x)dx+\int_1^2\left(-x^3+2x^2-x+2\right)dx\\
&= \frac{(1)^4}{4}-\frac{1^2}{2}+\left(\frac{17}{12}\right)\\
&= \frac{7}{6}
\end{aligned}$$

□

2. *$\int_S(\nabla\times\mathbf{F})\cdot dS$ where S is the area bounded by L.*

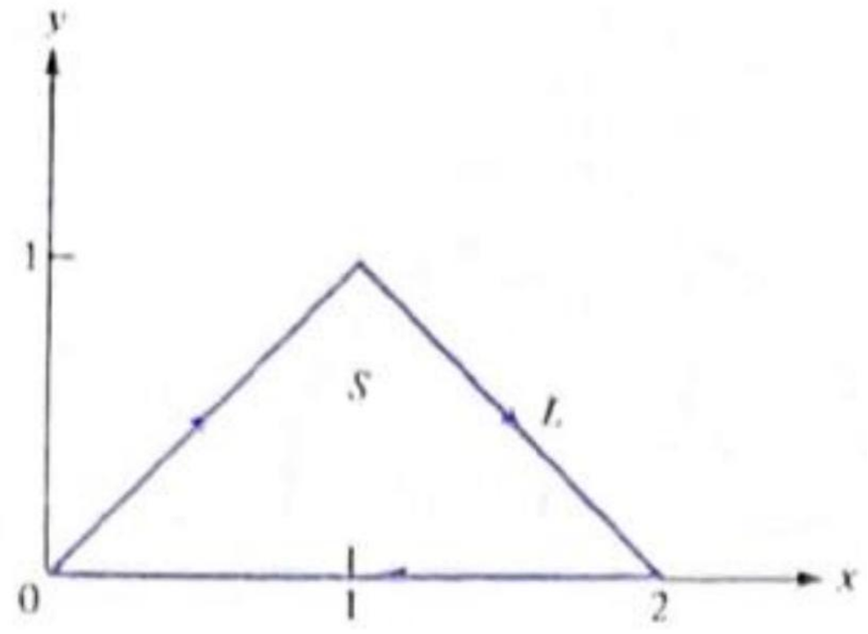
Solución. Sea

$$\begin{aligned}
\int_S(\nabla\times\mathbf{F})\cdot dS &= \left(\iint_1+\iint_2+\iint_3\right)\\
&\quad\left(\frac{\partial A_z}{\partial y}-\frac{\partial \mathrm{A}_y}{\partial z},\frac{\partial A_x}{\partial z}-\frac{\partial \mathrm{A}_z}{\partial x},\frac{\partial A_y}{\partial x}-\frac{\partial \mathrm{A}_x}{\partial y}\right)\cdot(dydx,dxdz,-dxdy)\\
&= \left(\iint_1+\iint_2+\iint_3\right)\left(0-0,0-0,0-x^2\right)\cdot(dydx,dxdz,-dxdy)\\
&= \left(\iint_1+\iint_2+\iint_3\right)\left(x^2dxdy\right)\\
&= \iint_1\left(x^2dxdy\right)+\iint_2\left(x^2dxdy\right)+\iint_3\left(x^2dxdy\right)\\
&= \int_0^1 x^2dx\int_0^x dy+\int_1^2 x^2dx\int_0^{-x+2}dy+0\\
&= \int_0^1 x^2(x)dx+\int_1^2 x^2(-x+2)dx\\
&= \frac{7}{6}
\end{aligned}$$

□

3. *Is Stokes's theorem satisfied?*

Solución. Sí, se cumple la igualdad. □



Problema 16. *Given the vector field*

$$\mathbf{G} = (16xy - z)\mathrm{a}_x + 8x^2\mathrm{a}_y - x\mathrm{a}_z$$

Assume anticlockwise direction.

1. *Is* **G** *irrotational (or conservative)?*

   Solución. Es necesario determinar si se cumple o no: $\nabla \times G = 0$.

$$\begin{aligned}\nabla \times G &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial \mathrm{A}_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial \mathrm{A}_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial \mathrm{A}_x}{\partial y}\right)\\ &= (0,0,0)\end{aligned}$$

   Entonces es irrotacional. □

2. *Find the net flux of* **G** *over the cube* $0 < x, y, z < 1$.

   Solución. Sea

$$\begin{aligned}\oint G \cdot dS &= \int \nabla \cdot G dv\\ &= \iiint 16y dx dy dz = 16 \int_0^1 dx \int_0^1 dz \int_0^1 y dy\\ &= 8\end{aligned}$$

□

3. *Determine the circulation of* **G** *around the edge of the square* $z = 0, 0 < x, y < 1$.

Solución. Sea

$$\begin{aligned}
\oint G \cdot dl &= \left(\int_1 + \int_2 + \int_3 + \int_4\right)(16xy - z, 8x^2, -x)\cdot(dx, dy, -dz)\\
&= \left(\int_{x=0,z=0} + \int_{y=1,z=0} + \int_{x=1,z=0} + \int_{y=0,z=0}\right)((16xy - z)dx + (8x^2)dy + xdz)\\
&= \int_{y=1,z=0}((16xy - z)dx + (8x^2)dy + xdz)\\
&\quad + \int_{x=1,z=0}((16xy - z)dx + (8x^2)dy + xdz)\\
&= 16\int_0^1 xdx + 8\int_1^0 dy\\
&= 16\left(\frac{1}{2}\right) - 8\\
&= 0
\end{aligned}$$

□