Introduction

In a common forest ecosystem, we can examine the interactions between oak tree, rodent, and snake populations. Oak trees reproduce through the release of acorns, which are a significant component of a rodent's diet. Snakes prey on rodents. Oak trees have an interesting mechanism, called "masting", to increase the success of reproduction. They will release an equal amount of acorns for a number of seasons, which will stabilize the rodent population to match the amount of available acorns. They then suddenly release more acorns than normal to "surprise" the rodent population who won't be large enough to compensate for such an unusually large amount of acorns, allowing the oak trees to reproduce more. We will investigate how masting affects the populations of snakes, rodents, and the oak tree.

Assumptions

In order to work with the predator-prey model, we must note any assumptions that our base predator-prey models make will apply to our final product.

1. The prey population will grow exponentially in the predator's presence
2. The predator population will starve and die out in the absence of the prey population rather than changing prey
3. Predators can consume an unlimited amount of prey
4. The environment is homogenous and both predators and prey move randomly through it.

While considering the intricacies of our model, we made additional assumptions.

1. Rodents act as both predators (towards the oak trees' acorns) and as prey to the snake
2. According to trophic level logic, there will be around ten times as many prey as there are predators because only 10% of energy is transferred up the chain

Our Model

Base Predator-Prey Model

$\frac{dR}{dt} = aR - bRS$
$\frac{dS}{dt} = cRS - dS$

Variables
S = population of snakes
R = population of rodents

Constants/Parameters
a is natural birth rate parameter
b and **c** are interaction parameters
d is a natural starvation parameter
This is the base which we will base our model on. The rodent's growth is dependent on their population and their decay is dependent on interactions between both them and the snakes.

Our Model with the Inclusion of Acorns

$\frac{dA}{dt} = gA(1 - hR)$
$\frac{dR}{dt} = aR - bRS - fR$
$\frac{dS}{dt} = cRS - dS$

Variables
S = population of snakes
R = population of rodents
A = population of acorns

We add a second prey acorns and make the rodents be the predators to acorns while being prey to snakes.
Constants/Parameters
a and **h** are interaction parameters between **Acorns** and **Rodents**
b and **c** are interaction parameters between **Rodents** and **Snakes**
d and **f** are natural starvation parameters for **Snakes** and **Rodents** respectively
g is the natural birth rate parameter for **Acorns**

Our Model with Parameters Filled In

$\frac{dA}{dt} = gA(1 + m) - hRA$
$\frac{dR}{dt} = R(aA - bS - f)$

$\frac{dS}{dt} = S(cR - d)$

Variables
S = population of snakes
R = population of rodents
A = population of acorns

Constants/Parameters
a and **h** are interaction parameters between **Acorns** and **Rodents**
b and **c** are interaction parameters between **Rodents** and **Snakes**
d and **f** are natural starvation parameters for **Snakes** and **Rodents** respectively
g is the natural birth rate parameter for **Acorns**
m is the masting bonus parameter for the increase of production of **Acorns**

The relationship of how much prey the predator eats is modeled various times in our equations. With our assumptions that the predator can eat an unlimited number of prey, these parameters help to model how the population increases or decreases with time as a result of interaction from different predator-prey relationships seen here.

Graph of Differential Equation

We used the following values for the parameters.
a = 0.0001
b = 0.001
c = 0.001
d = 0.012
f = 0.008
g = 0.001
h = 0.0001
m = 0.2 OR 0 (depending on if masting or not)

Settings for Euler's Method
We used Euler's Method to approximate results for our graph.
Step size: 0.1
Start to end time: 0 to 1000

(The code for all figures is attached in the Appendix)

Figure 1 (With masting):

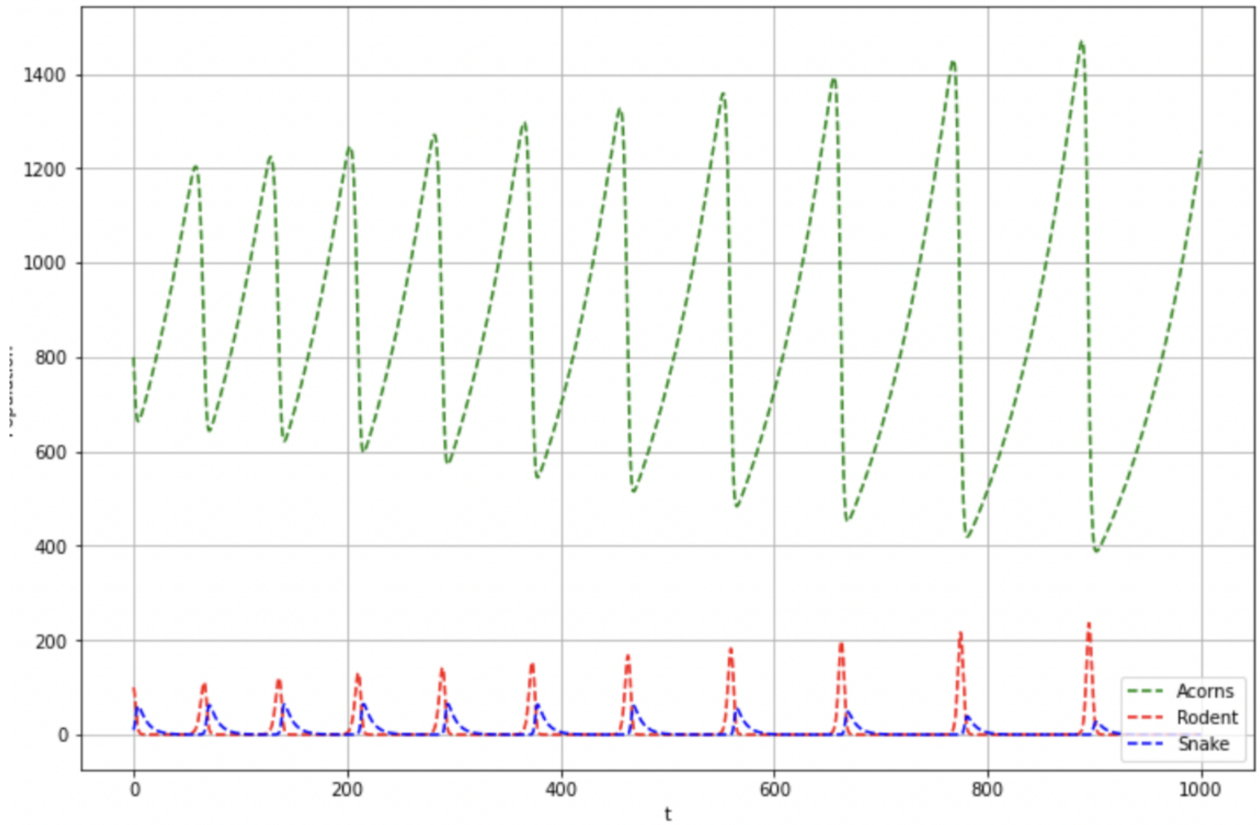
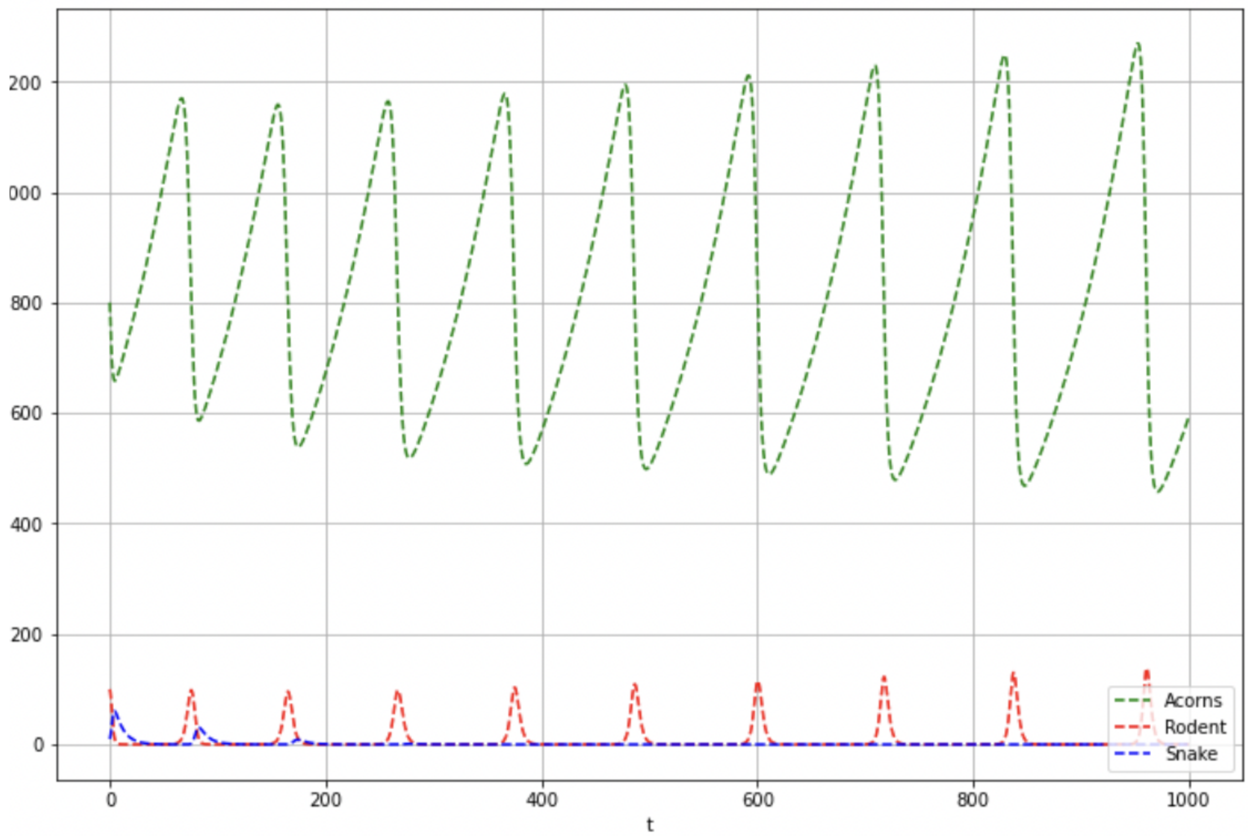


Figure 2 (Without Masting):



Analysis

(Population is graphed on Y axis and Time is graphed on x axis.)Masting and not masting are similar except with masting it accelerates the cycle etc.

Our graphs feature the food chain between the acorn, rodent, and snake populations that split between the effects with and without masting. Both graphs maintain a similar shape however the intensity of the graph changes. When masting is present, the population of acorns significantly increases. There is also small increases in the rodent and snake populations although not as significant. From the model, we can see the continual cycle of increasing and decreasing populations of each animal population. When the population of acorns decreases, the rodent population increases and vice versa. This cycle continues with an increasing peak each time for each prey population. The interesting part is the predator population's peak decreases after every cycle (contrary to the other animal populations). However, the acorn population peak increases each time. We also notice that the snakes can't handle such unpredictability and eventually die out.

Conclusion

To conclude, our project encompasses the gradual rise and decline of an ecosystem of three different life systems. The introduction of masting brings a unique twist to the problem where the increased prey population also leads to increased predator populations. With our assumption that predators can eat unlimited prey and that prey grow without a set carrying capacity, this is a limited understanding of the growing diverse ecosystem that we live in today. Although this model can be applicable in this instance, it becomes increasingly difficult to model what can occur in our everyday world. We realize that the reason trees mast is because the delayed effect of predation (since the rats don't catch up quickly) lets them achieve high peaks faster. As Acorns grow so do rodents and as rodents grow acorns fall (and same for rodents and snakes) This scales relatively to a realistic predator prey situation that we learned in K-12 education. Our approach to this problem was based on the base predator that we learned in class and we adapted it to a three layer model with an additional masting effect. We could have modeled the trees growth as logarithmic rather than exponential however, we decided not to because we stuck to the base assumptions of predator prey model. We also were limited by the base limitations of the predator prey model. The snakes dying out however, does not align with reality so more tweaking of our model needs to be done to improve it.

References:

"The Euler Method — Python Numerical Methods." *Python Numeral Methods*, UC Berkeley, https://pythonnumericalmethods.berkeley.edu/notebooks/chapter22.03-The-Euler-Method.html.

Judson, T. W. (2022, July 20). *The ordinary differential equations project*. ODE-Project Modeling with Differential Equations. Retrieved March 31, 2023, from http://faculty.sfasu.edu/judsontw/ode/html-20220730/firstlook01.html

Robb, Amanda. "The 10% Rule in Energy Pyramids." *Study.com | Take Online Courses. Earn College Credit. Research Schools, Degrees & Careers*, https://study.com/academy/lesson/the-10-energy-rule-in-a-food-chain.html.

Winkel, B. (n.d.). *Simiode*. Simode. Retrieved March 31, 2023, from https://cauchy.simiode.org/resources/2505/download/6-21-S-AcornsRodentsSnakes-StudentVersion.pdf

Appendice (code, in python):

```
##### import numpy as np

import matplotlib.pyplot as plt

#Our parameters defined

a = 0.0001

b = 0.001

c = 0.001

d = 0.012

f = 0.08

g = 0.001

h = 0.0001

m = 0.2

#time start and end

t_start = 0

t_end = 1000

step = 0.1
```

```
def dA_dt(S,R,A):

    return (g*A) * (1+m)  - ((R * h * A))

def dR_dt(S,R,A):

    return (R*a*A) - ((b*S * R)) - (f * R)

def dS_dt(S,R,A):

    return S*((c * R)) - (d*S)



time_arrplaceHolder = np.arange(t_start,t_end,step)

time_arrplaceHolder.size

Tree_arrplaceHolder = np.zeros(time_arrplaceHolder.size)

Rodent_arrplaceHolder = np.zeros(time_arrplaceHolder.size)

Snake_arrplaceHolder = np.zeros(time_arrplaceHolder.size)



Tree_arrplaceHolder[0] = 800

Rodent_arrplaceHolder[0] = 100

Snake_arrplaceHolder[0] = 10

for i in range(len(time_arrplaceHolder) - 1):
```

```
    Tree_arrplaceHolder[i + 1] = Tree_arrplaceHolder[i] +
dA_dt(Snake_arrplaceHolder[i],Rodent_arrplaceHolder[i],Tree_arrplaceHolder[i])

    Rodent_arrplaceHolder[i + 1] = Rodent_arrplaceHolder[i] +
dR_dt(Snake_arrplaceHolder[i],Rodent_arrplaceHolder[i],Tree_arrplaceHolder[i])

    Snake_arrplaceHolder[i + 1] = Snake_arrplaceHolder[i] +
dS_dt(Snake_arrplaceHolder[i],Rodent_arrplaceHolder[i],Tree_arrplaceHolder[i])

Rodent_arrplaceHolder

plt.figure(figsize = (12, 8))

plt.plot(time_arrplaceHolder, Tree_arrplaceHolder, 'g--', label='Acorns')

plt.plot(time_arrplaceHolder, Rodent_arrplaceHolder, 'r--', label='Rodent')

plt.plot(time_arrplaceHolder, Snake_arrplaceHolder, 'b--', label='Snake')

plt.xlabel('t')

plt.ylabel('Population')

plt.grid()

plt.legend(loc='lower right')

plt.show()


    #Note: I asked guidance and learned from Henry, Li, Royce and Ethan
```