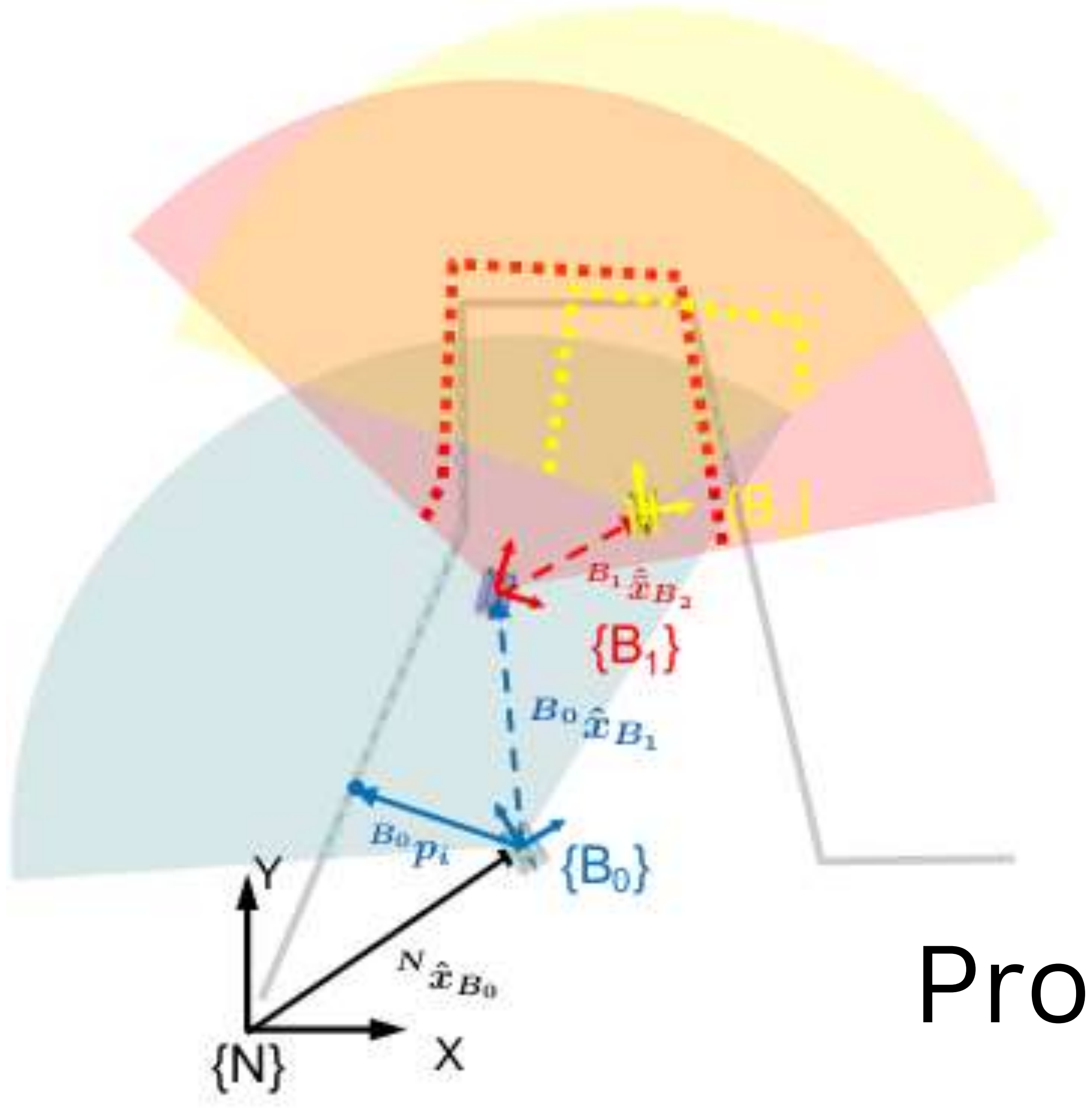
Posed Based EKF SLAM (PEKF-SLAM) Using ICP laser scans matching



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Gebrecherkos G.

Supervisors:

Prof. Pedro Ridao Rodriguez

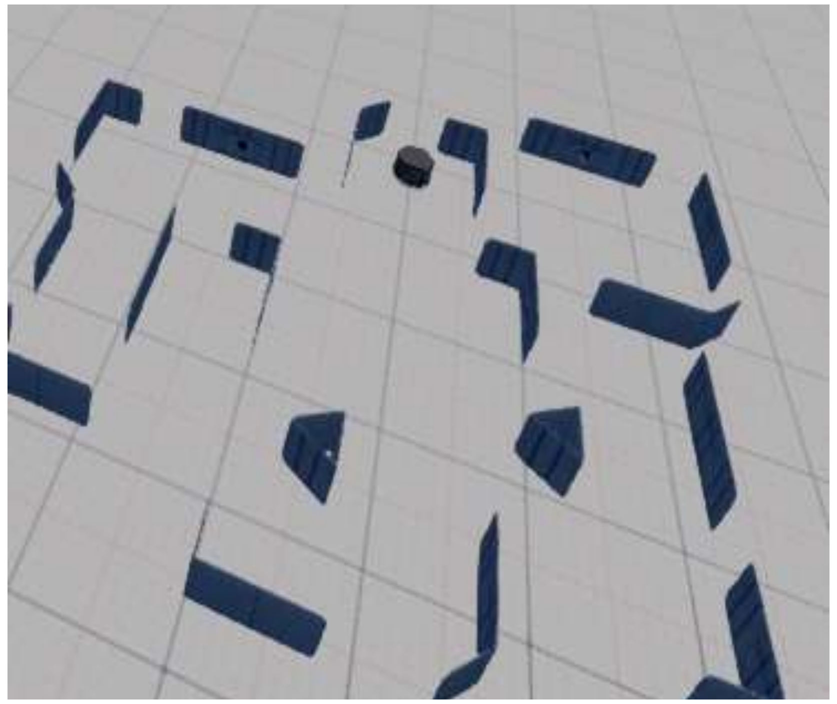
Roger Pi Roig

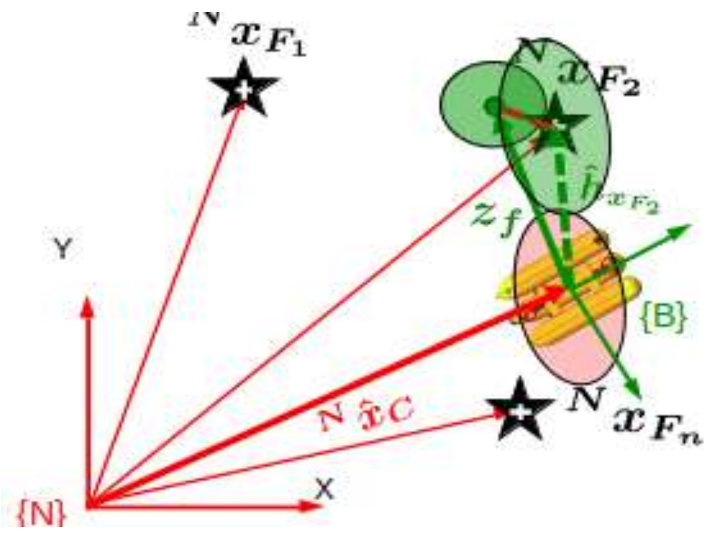
CONTENTS

1. Introduction-SLAM

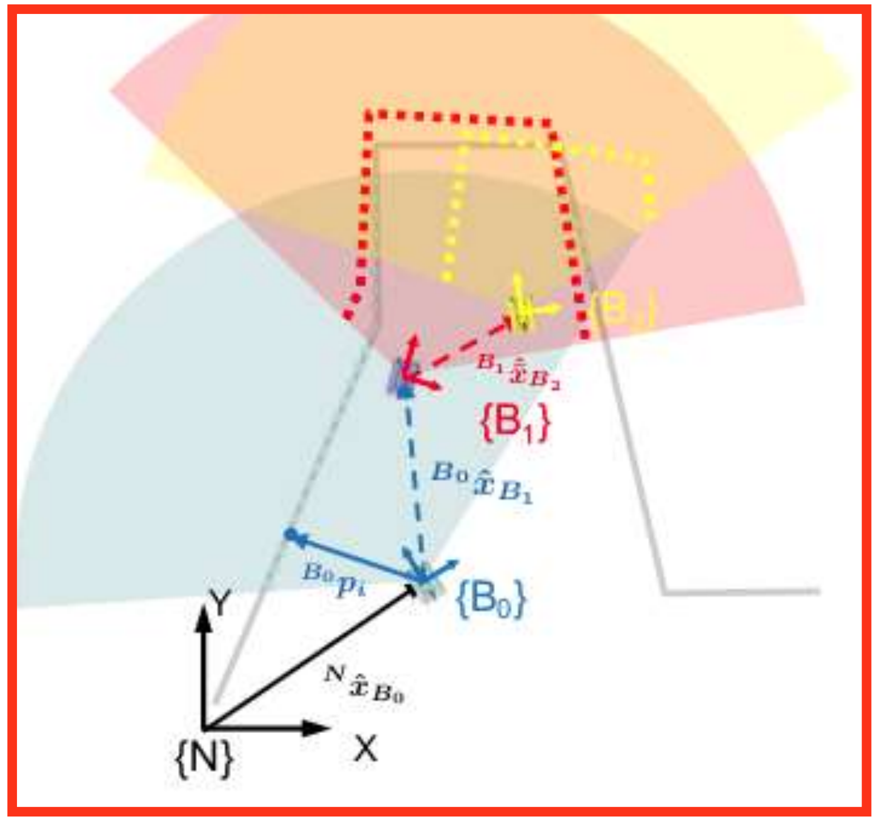
"Where am I?"
"What does the world around me look like?"
"Can I figure both out at the same time?

SLAM – Simultaneous Localization and Mapping

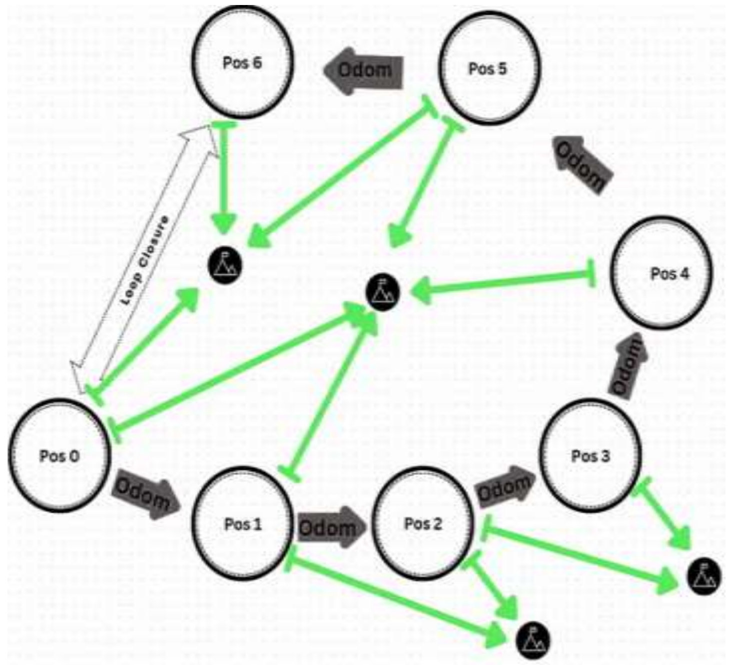




Feature EKF SLAM



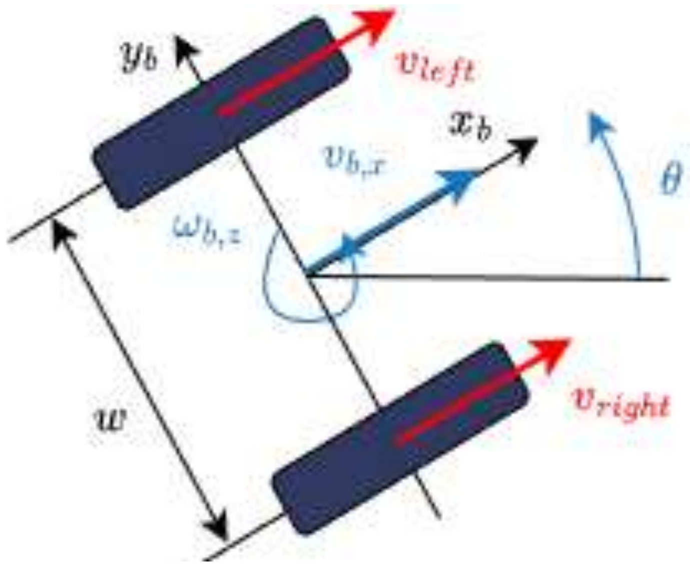
Pose EKF SLAM



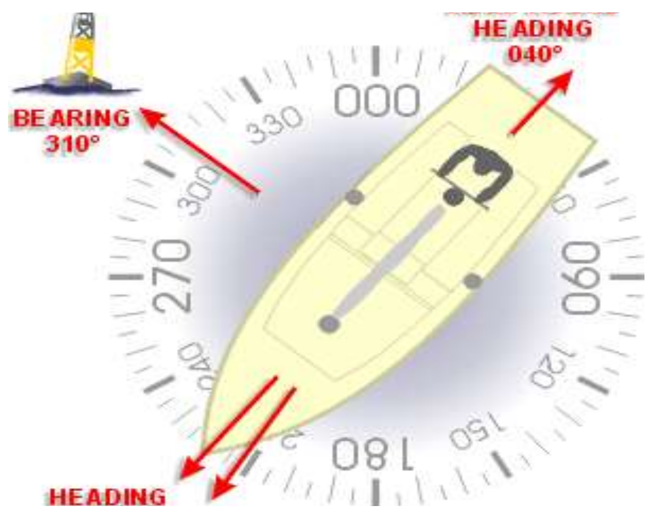
Graph SLAM

2. Problem Statement

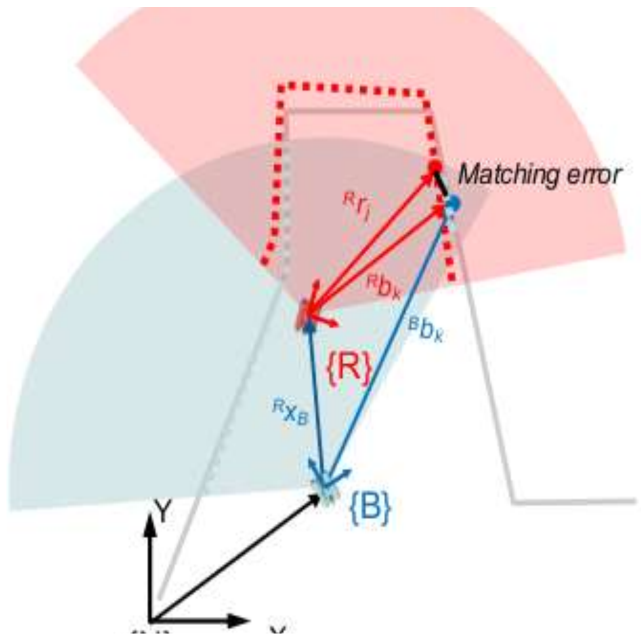
A Full SLAM Pipeline using:



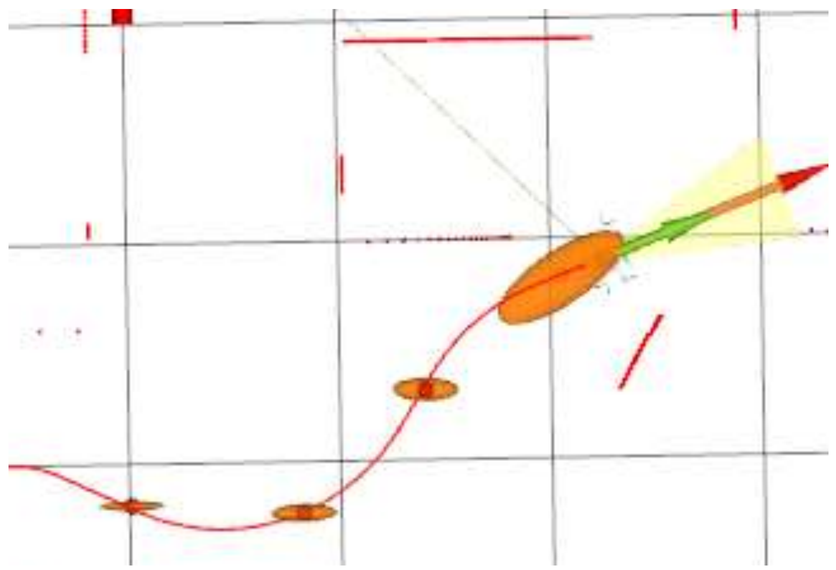
1. Motion Model : Odometry



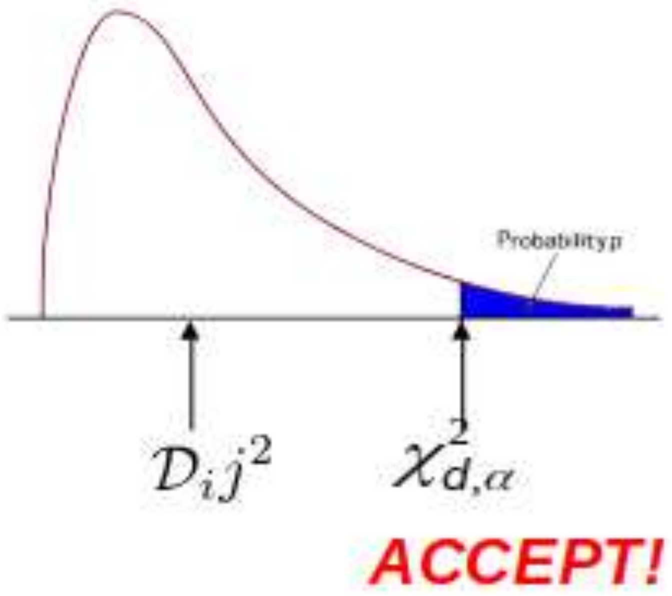
2. Measurement Model: Compass (heading)



3. Observation Model: ICP (relative pose)

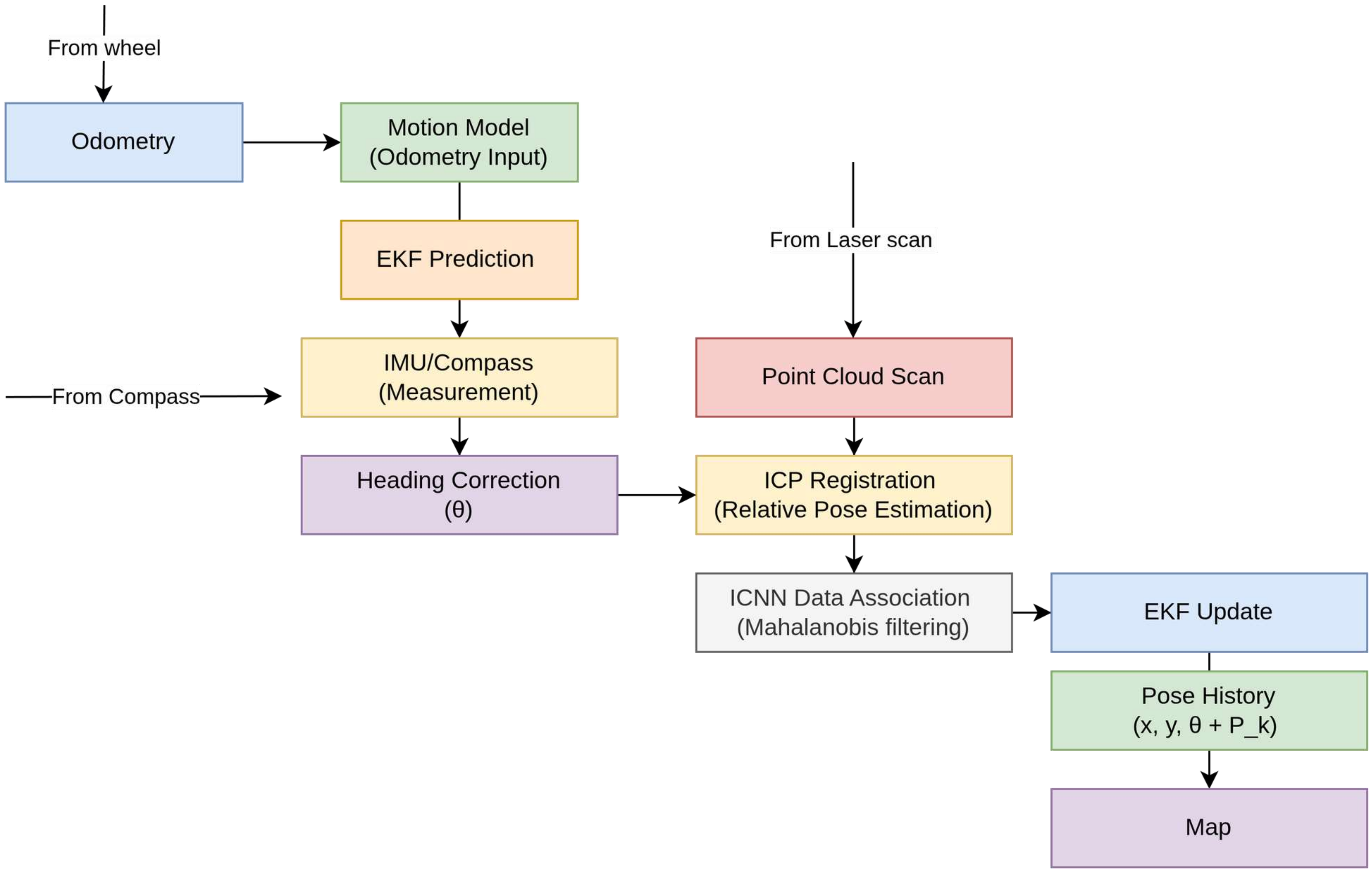


4. Estimation Method: EKF (Pose-Based)



5. Data Association: ICNN

3. System Overview

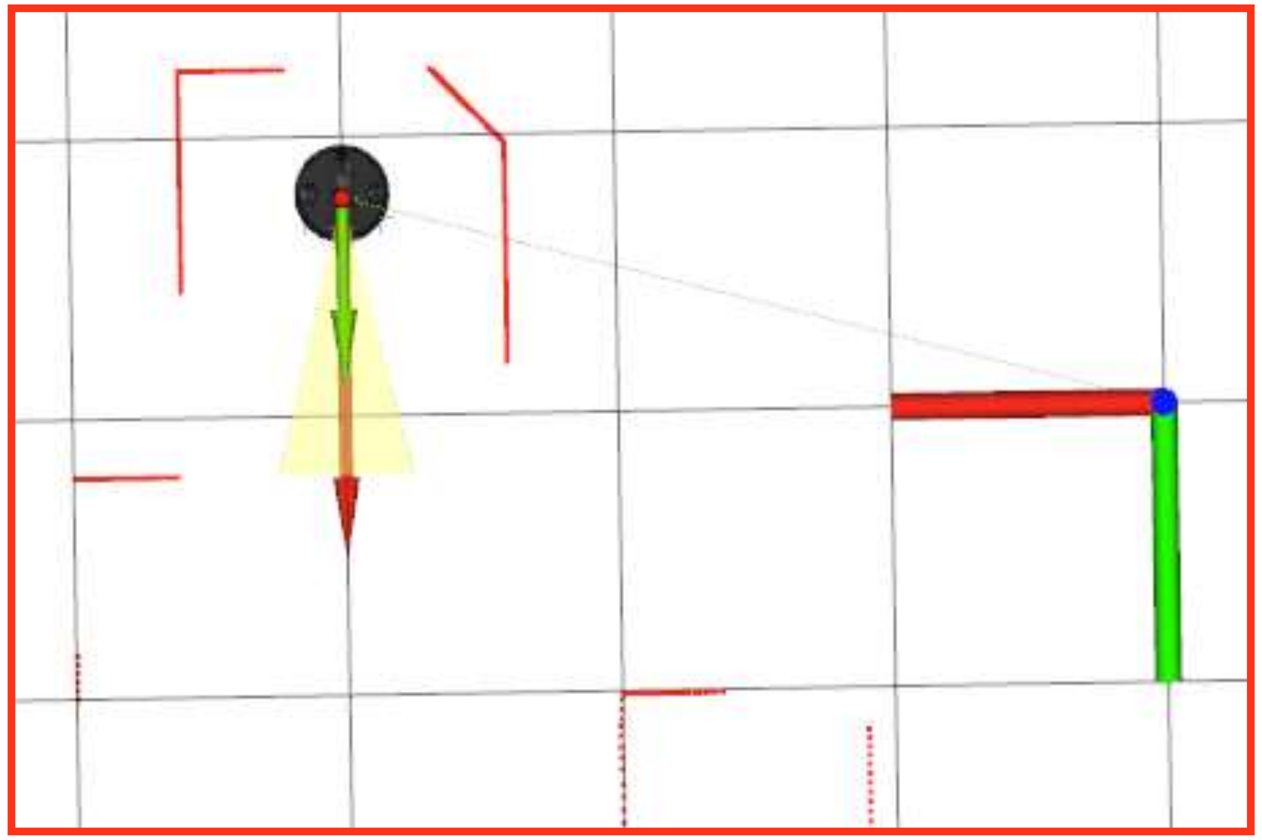


4. State Representation

1. State Vector: N^xk (Global frame {N }):

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{N}\hat{\mathbf{x}}_{B_0} \\ \mathbf{N}\hat{\mathbf{x}}_{B_1} \\ \vdots \\ \mathbf{N}\hat{\mathbf{x}}_{B_k} \end{bmatrix} \in \mathbb{R}^{3(k+1)\times 1}$$

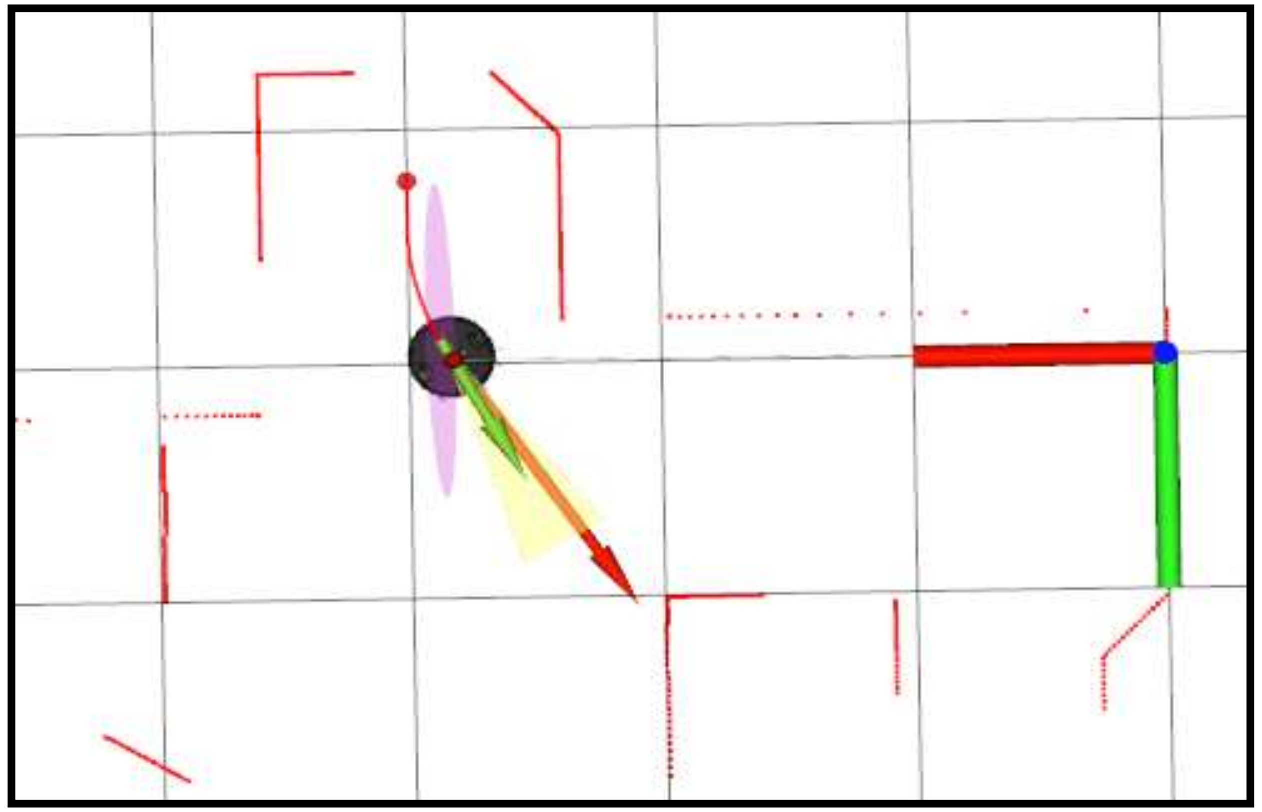
$$\mathbf{N}\hat{\mathbf{x}}_{B_i} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \quad \text{a 2D pose in } SE(2)$$



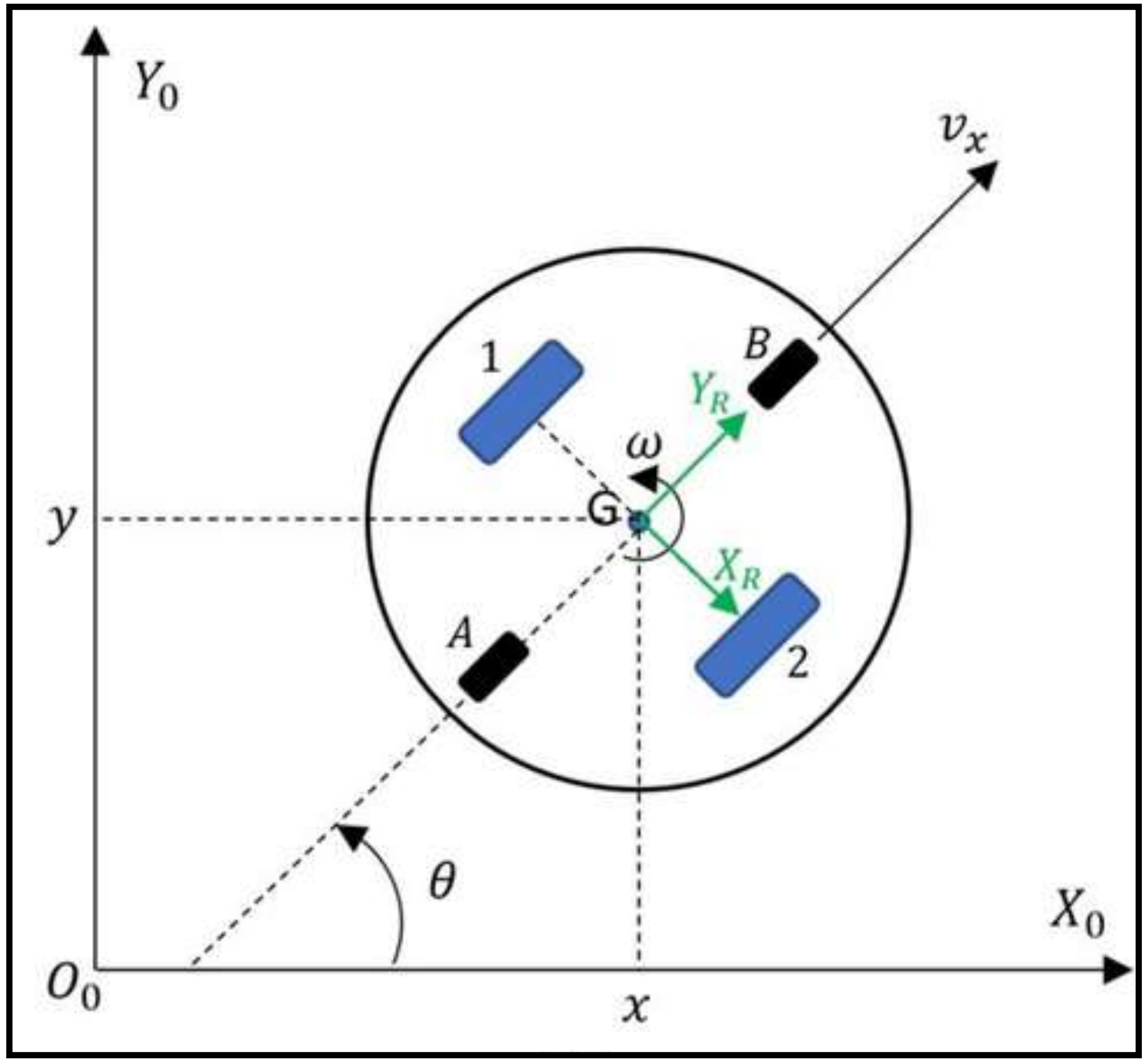
2. Covariance Matrix: NPk

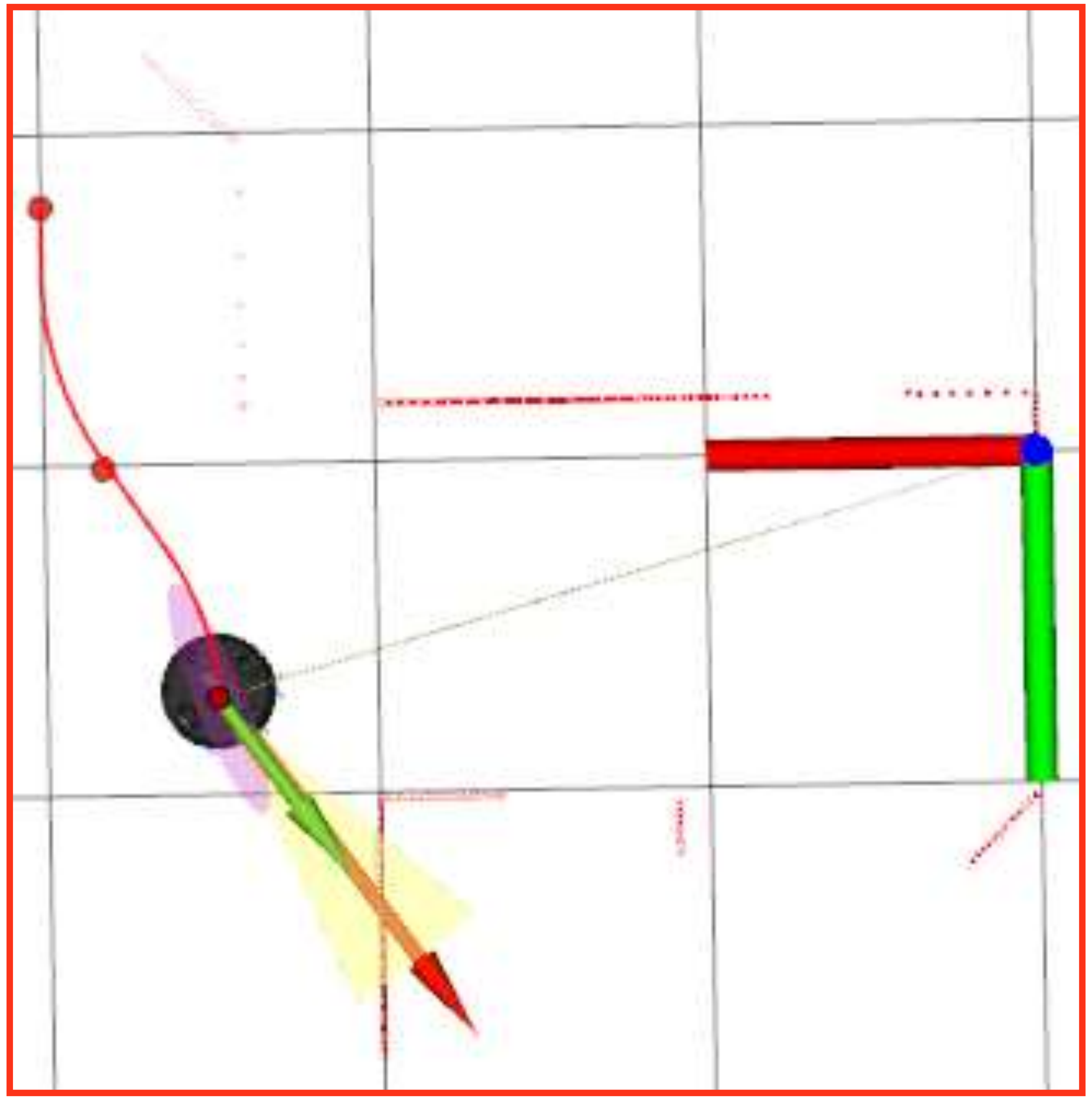
Pose covariances, Cross-covariances

$$\mathbf{NP}_k = \begin{bmatrix} \mathbf{NP}_{B_0} & \mathbf{NP}_{B_0B_1} & \cdots & \mathbf{NP}_{B_0B_k} \\ \mathbf{NP}_{B_1B_0} & \mathbf{NP}_{B_1} & \cdots & \mathbf{NP}_{B_1B_k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{NP}_{B_kB_0} & \mathbf{NP}_{B_kB_1} & \cdots & \mathbf{NP}_{B_k} \end{bmatrix}$$



5. Motion Model



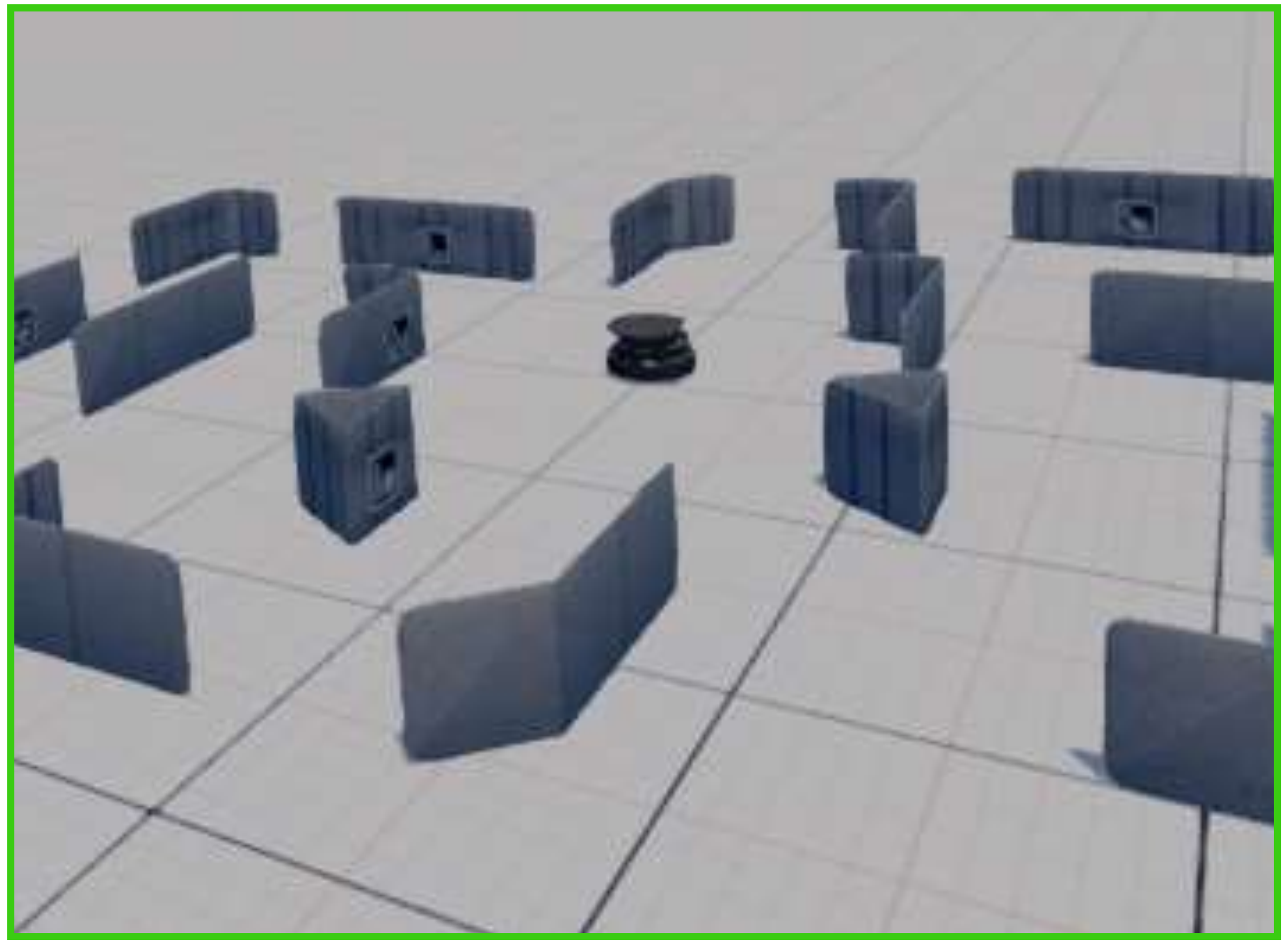


$$\mathbf{u}_k = \begin{bmatrix} v \cdot \Delta t \\ \omega \cdot \Delta t \end{bmatrix}$$

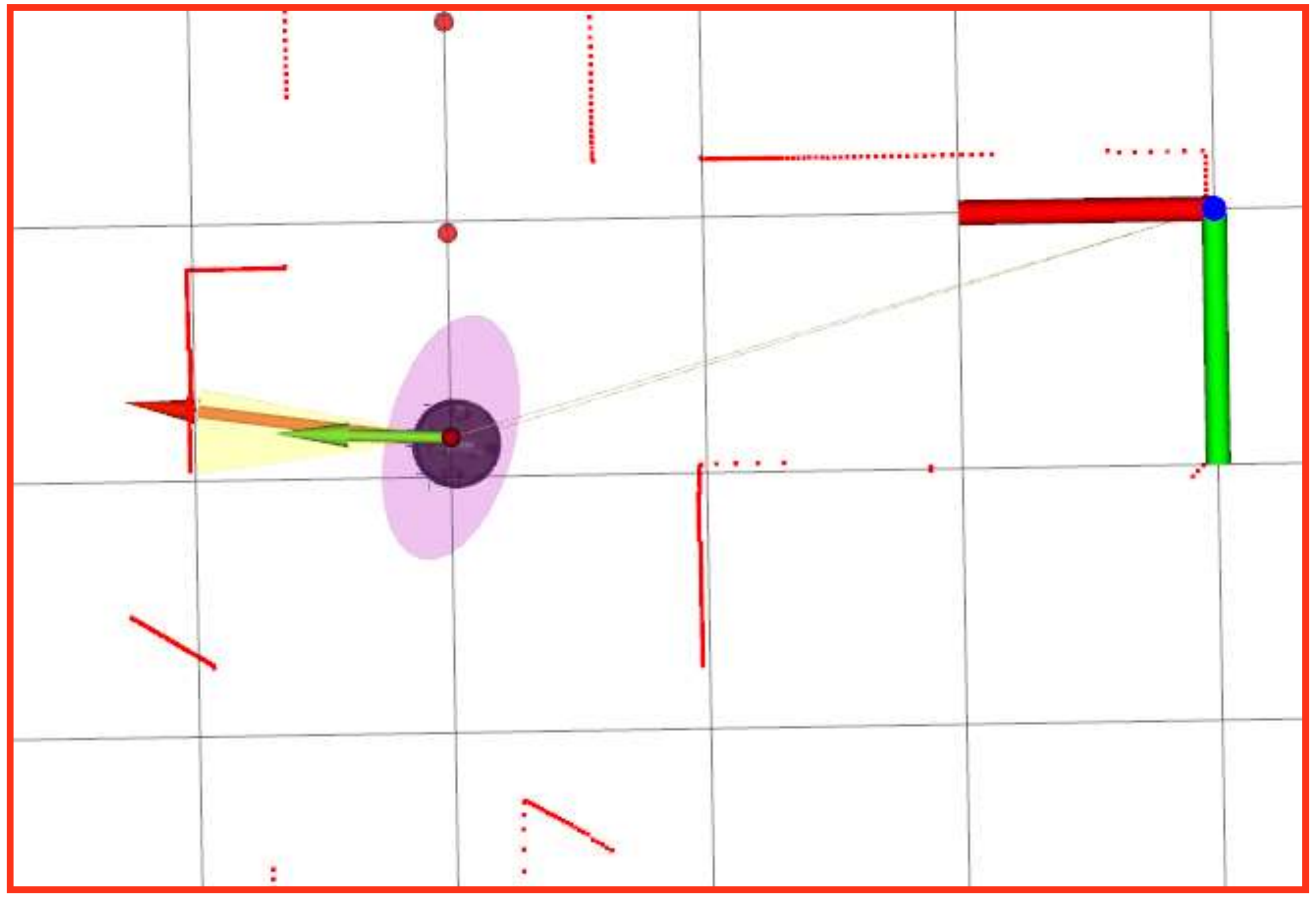
$$\mathbf{x}_k = \mathbf{x}_{k-1} \oplus \mathbf{u}_k$$

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + v\Delta t \cos(\theta_{k-1}) \\ y_{k-1} + v\Delta t \sin(\theta_{k-1}) \\ \theta_{k-1} + \omega\Delta t \end{bmatrix}$$

6. Scan Acquisition



Lidar



Map Representation: M

$$\mathcal{M} = [\mathbf{B_0 S_0}, \quad \mathbf{B_1 S_1}, \quad \cdots, \quad \mathbf{B_k S_k}]$$

$$\mathbf{B_k S_k} = [{}^{B_k}\mathbf{p}_0, \quad \cdots, \quad {}^{B_k}\mathbf{p}_i, \quad \cdots, \quad {}^{B_k}\mathbf{p}_{np}]$$

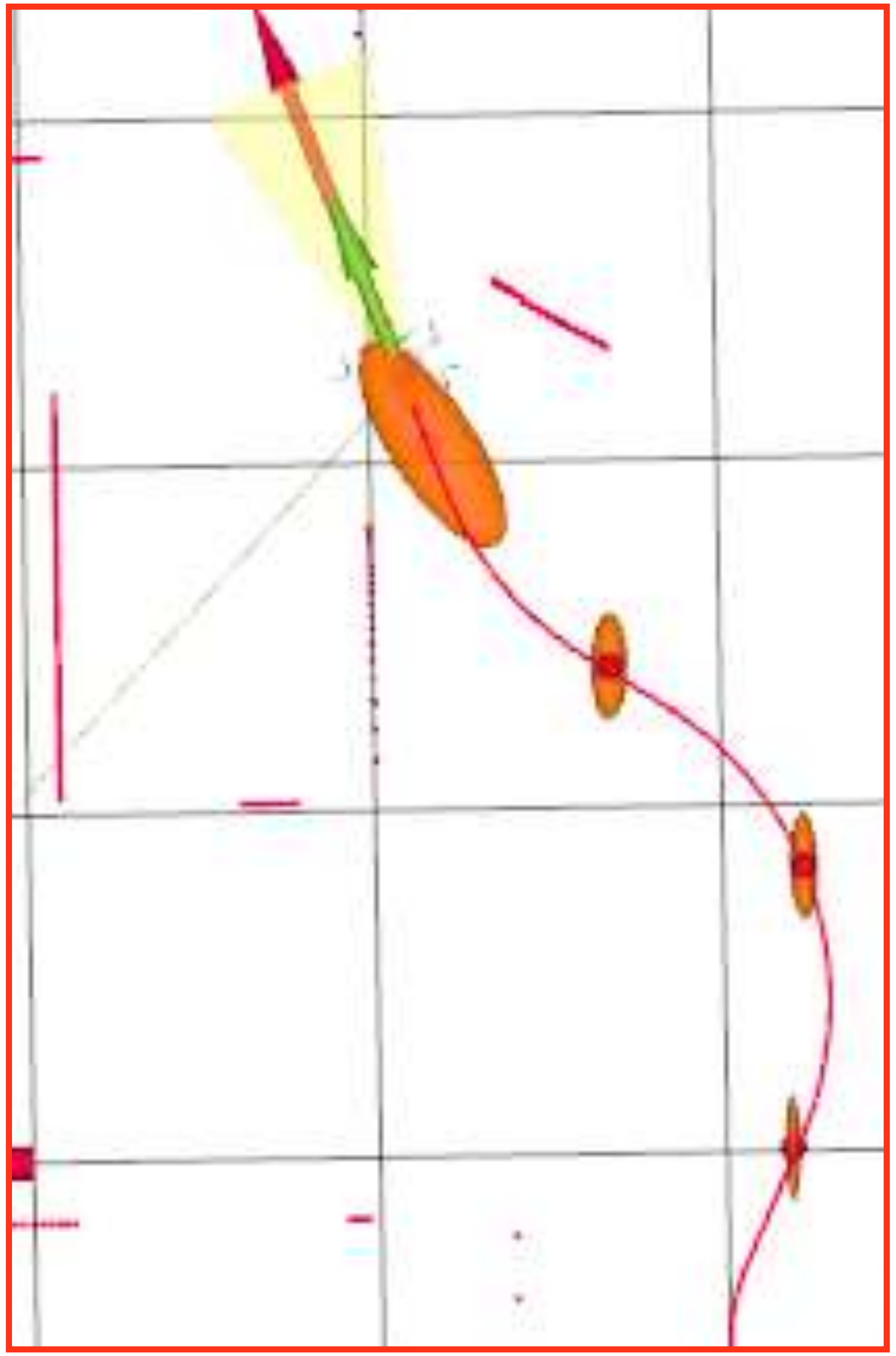
7. New Pose Addition

1. State vector Expansion

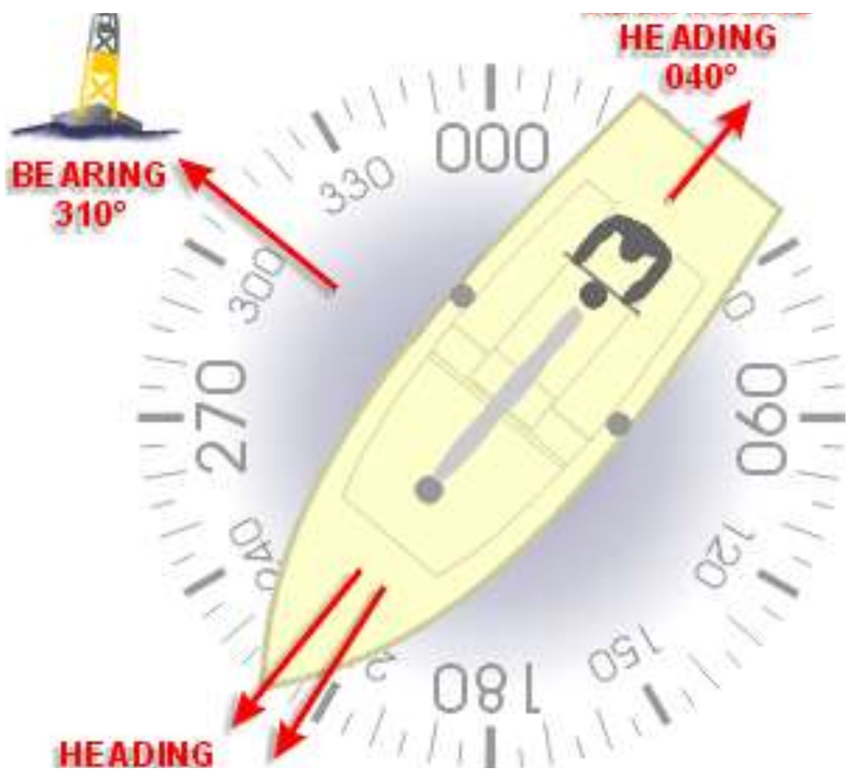
$$\mathbf{N}\hat{\mathbf{x}}_k^+ = \begin{bmatrix} \mathbf{N}\hat{\mathbf{x}}_{B_0} \\ \mathbf{N}\hat{\mathbf{x}}_{B_1} \\ \vdots \\ \mathbf{N}\hat{\mathbf{x}}_{B_{k-1}} \\ \mathbf{N}\hat{\mathbf{x}}_{B_k} \end{bmatrix} \in \mathbb{R}^{3(k+1)\times 1}$$

2. Covariance Matrix Expansion

$$\mathbf{NP}_k^+ = \left[\begin{array}{ccc|c} \mathbf{NP}_{B_0} & \cdots & \mathbf{NP}_{B_0,B_{k-1}} & \mathbf{N\bar{P}}_{B_0,B_k} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{NP}_{B_{k-1},B_0} & \cdots & \mathbf{NP}_{B_{k-1}} & \mathbf{N\bar{P}}_{B_{k-1},B_k} \\ \hline \mathbf{N\bar{P}}_{B_k,B_0} & \cdots & \mathbf{N\bar{P}}_{B_k,B_{k-1}} & \mathbf{N\bar{P}}_{B_k} \end{array}\right] \in \mathbb{R}^{3(k+1)\times 3(k+1)}$$



8. IMU Heading Correction



- $\hat{\theta}$ — predicted heading from the state xk
- vk ∼ N (0, Rk) — additive zero-mean Gaussian noise from the IMU
- θIMU — measured yaw angle from the IMU
- Hk — observation matrix
- Rk — measurement noise covariance
- Pk — current pose covariance

Measurement

$z_k = h(x_k) + v_k = \theta_k + v_k$

↓

Observation Matrix

$(H_k \in R1{\times}n)$

$Hk = [0\ 0 \cdot \cdot \cdot 1]$

↓

Innovation (Residual)

$y_k = \mathrm{wrap}(\theta_{IMU} - \hat{\theta}_k)$

↓

Kalman Gain

$K_k = P_k H_k^\top (H_k P_k H_k^\top + R_k)^{-1}$

↓

State Update

$x_k \leftarrow x_k + K_k \tilde{y}_k$

↓

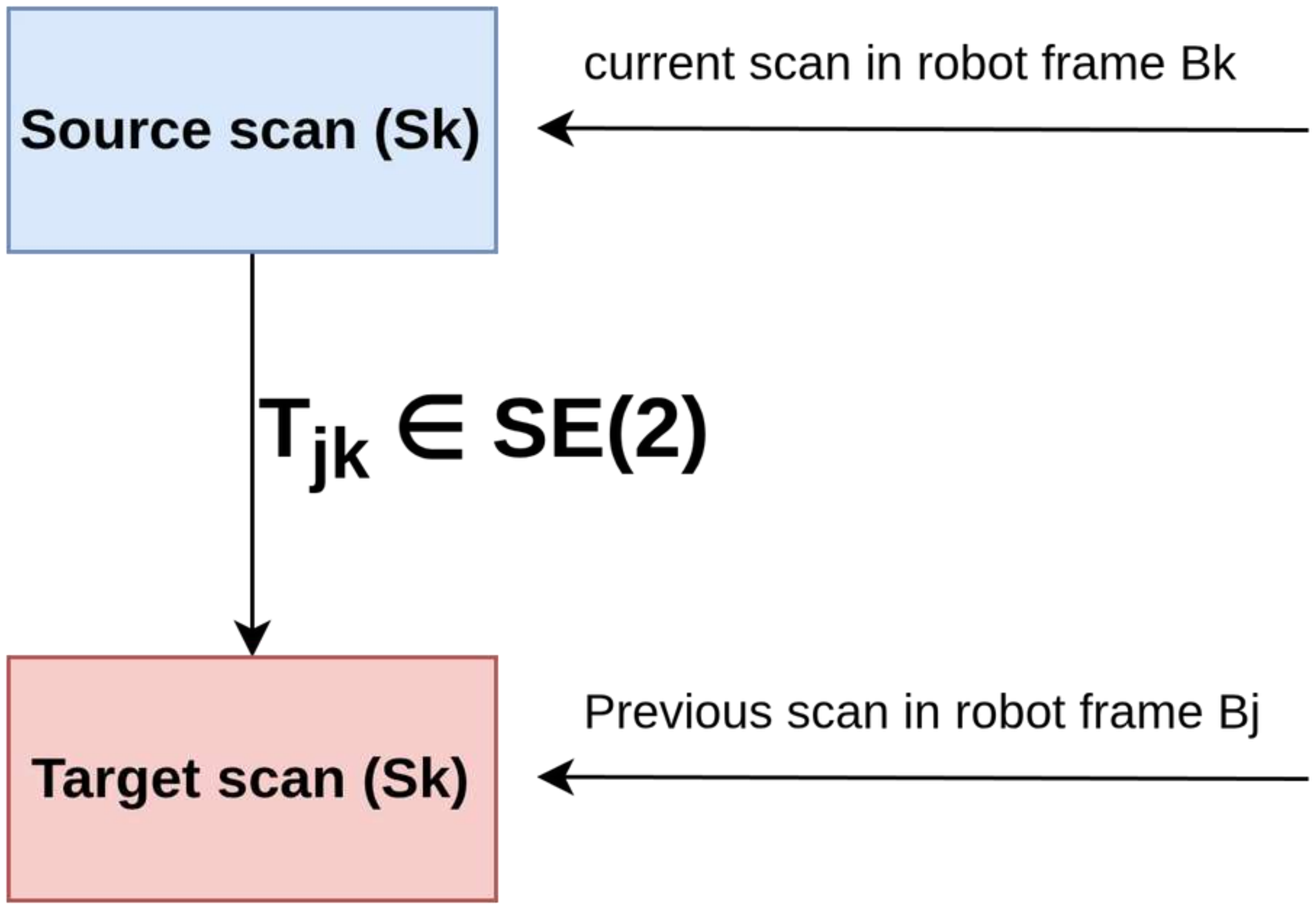
Covariance Update

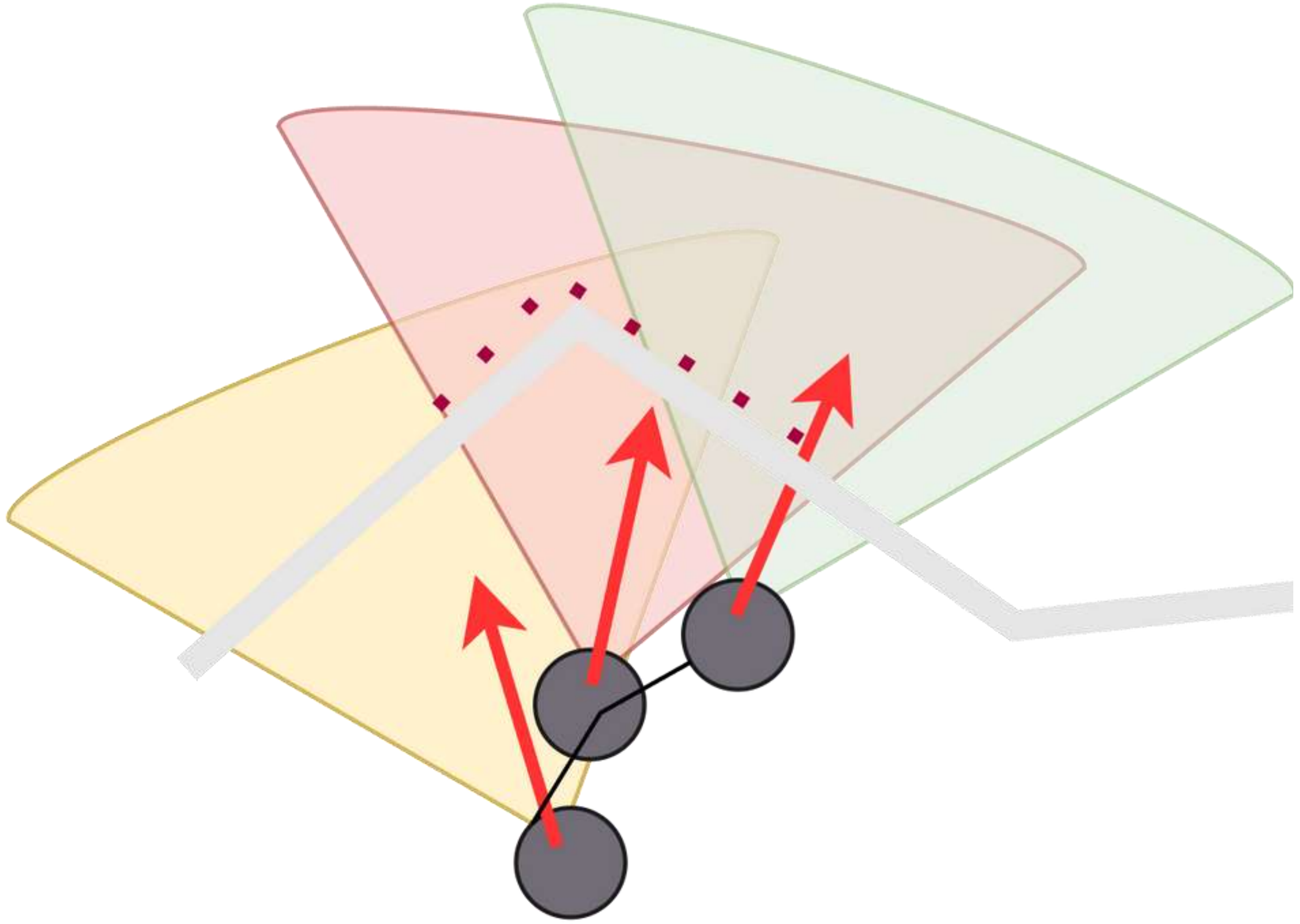
$P_k \leftarrow (I - K_k H_k) P_k (I - K_k H_k)^\top$

9. ICP Scan Match

ICP (Iterative Closest Point)

Aligning (2D or 3D) point clouds by estimating the rigid transformations.

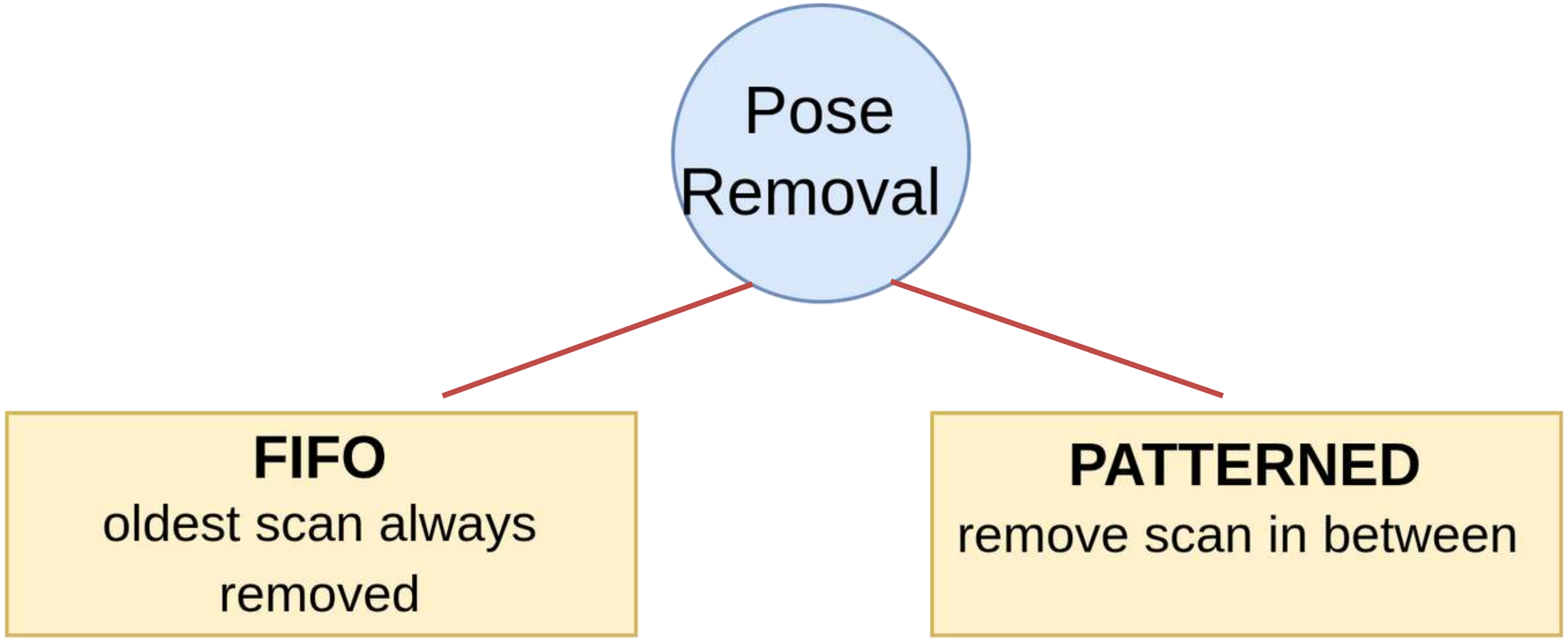




Goal: Find the transformation Tjk that minimizes this total squared distance:

$$\sum_i \|\mathbf{p}_j^{(i)} - \mathbf{T}_{jk} \cdot \mathbf{p}_k^{(i)}\|^2$$

10. ICP Trade-OFF



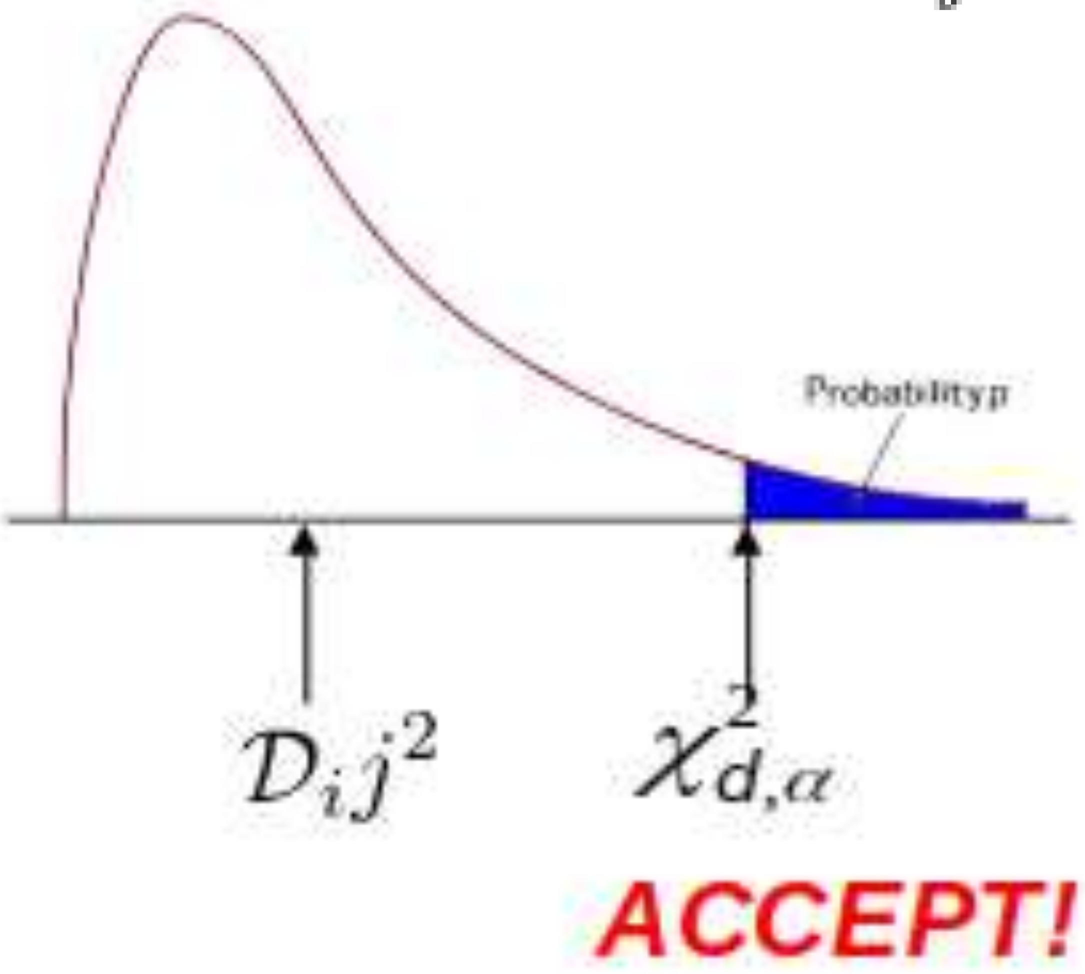
Slightly Better
Still not Optimal
Might Remove Useful or Good Scan

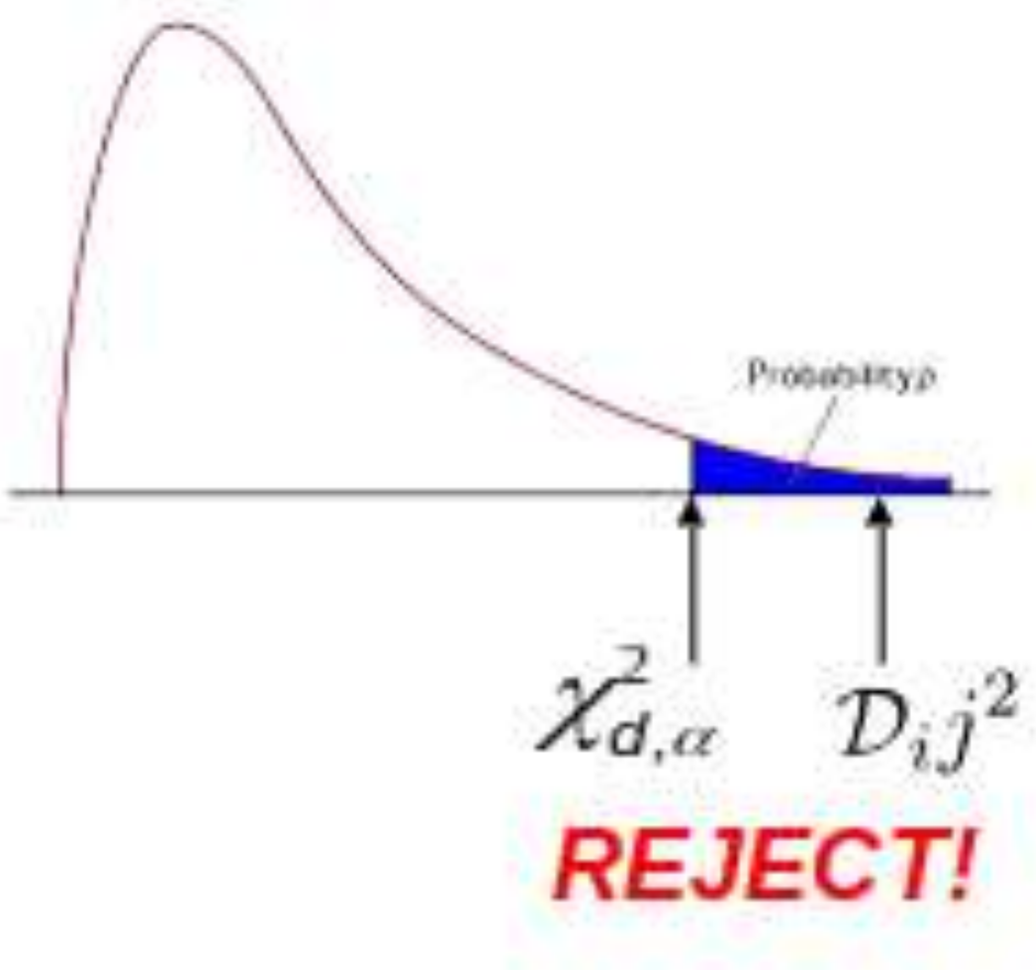
11. Data Association

Strategy: ICNN (Individual Compatibility Nearest Neighbour)

Tool : Mahalanobis Distance

Test: Chi Square Test $D_{ij}^2 < \chi^2(\alpha, \mathrm{dof})$





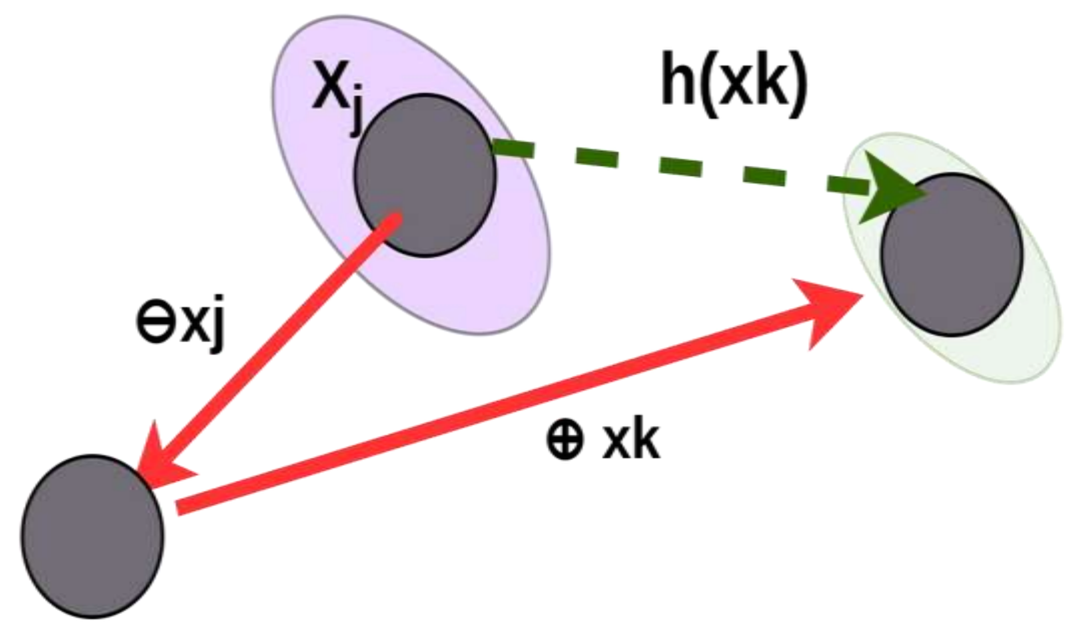
α is the confidence level (e.g., 0.95)

dof = degrees of freedom (3 for 2D pose)

12. Observation Jacobian

Relative pose prediction

$$h(\mathbf{x}_k) = \ominus \mathbf{x}_j \oplus \mathbf{x}_k$$



Observation Jacobian with Respect to Pose

$$H_k = \left.\frac{\partial h(N\hat{\mathbf{x}}_k, \mathbf{v}_k)}{\partial N\hat{\mathbf{x}}_k}\right|_{N\hat{\mathbf{x}}_k = N\hat{\mathbf{x}}_k^-, \mathbf{v}_k = 0}$$

$$H_k = \begin{bmatrix} \frac{\partial h(\cdot)}{\partial x_k} \\ \frac{\partial h(\cdot)}{\partial y_k} \\ \frac{\partial h(\cdot)}{\partial \theta_k} \end{bmatrix} = \begin{bmatrix} J_{1x} & J_{2x} & \cdots & J_{n_p x} \\ J_{1y} & J_{2y} & \cdots & J_{n_p y} \\ J_{1\theta}\mathbb{J}_\theta & J_{2\theta}\mathbb{J}_\theta & \cdots & J_{n_p\theta}\mathbb{J}_\theta \end{bmatrix}$$

Observation Jacobian with Respect to Noise

$$V_k = \left.\frac{\partial h(N\hat{\mathbf{x}}_k, \mathbf{v}_k)}{\partial \mathbf{v}_k}\right|_{N\hat{\mathbf{x}}_k = N\hat{\mathbf{x}}_k^-, \mathbf{v}_k = 0}$$

$$V_k = \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix} \in \mathbb{R}^{2n_p \times 2n_p}$$

13. EKF Update

Residual (Innovation)

$$\tilde{y}_k = \mathbf{z}_k - h(\hat{\mathbf{x}}_k)$$

Kalman Gain

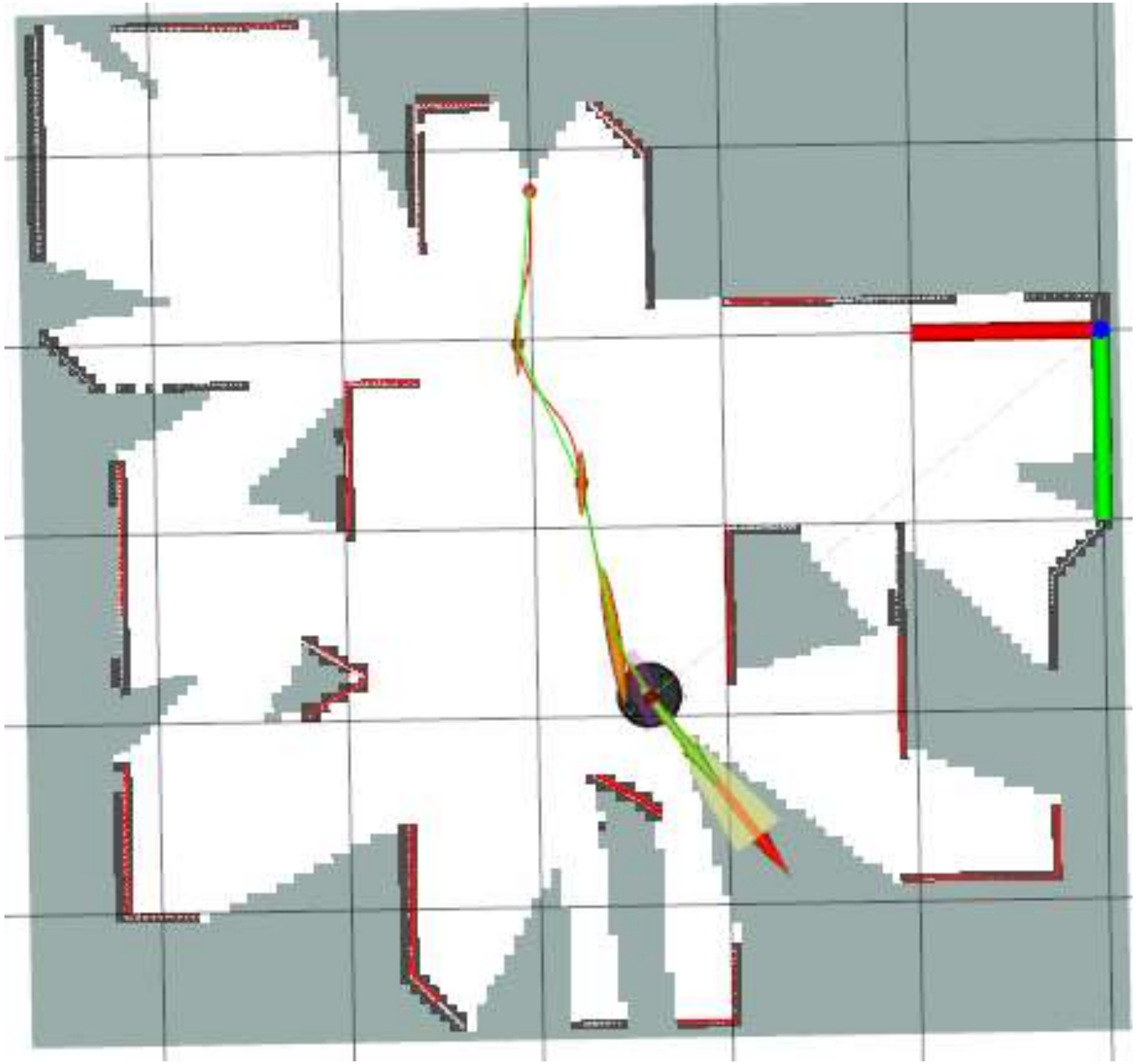
$$K_k = P_k H_k^\top (H_k P_k H_k^\top + R_k)^{-1}$$

State Update

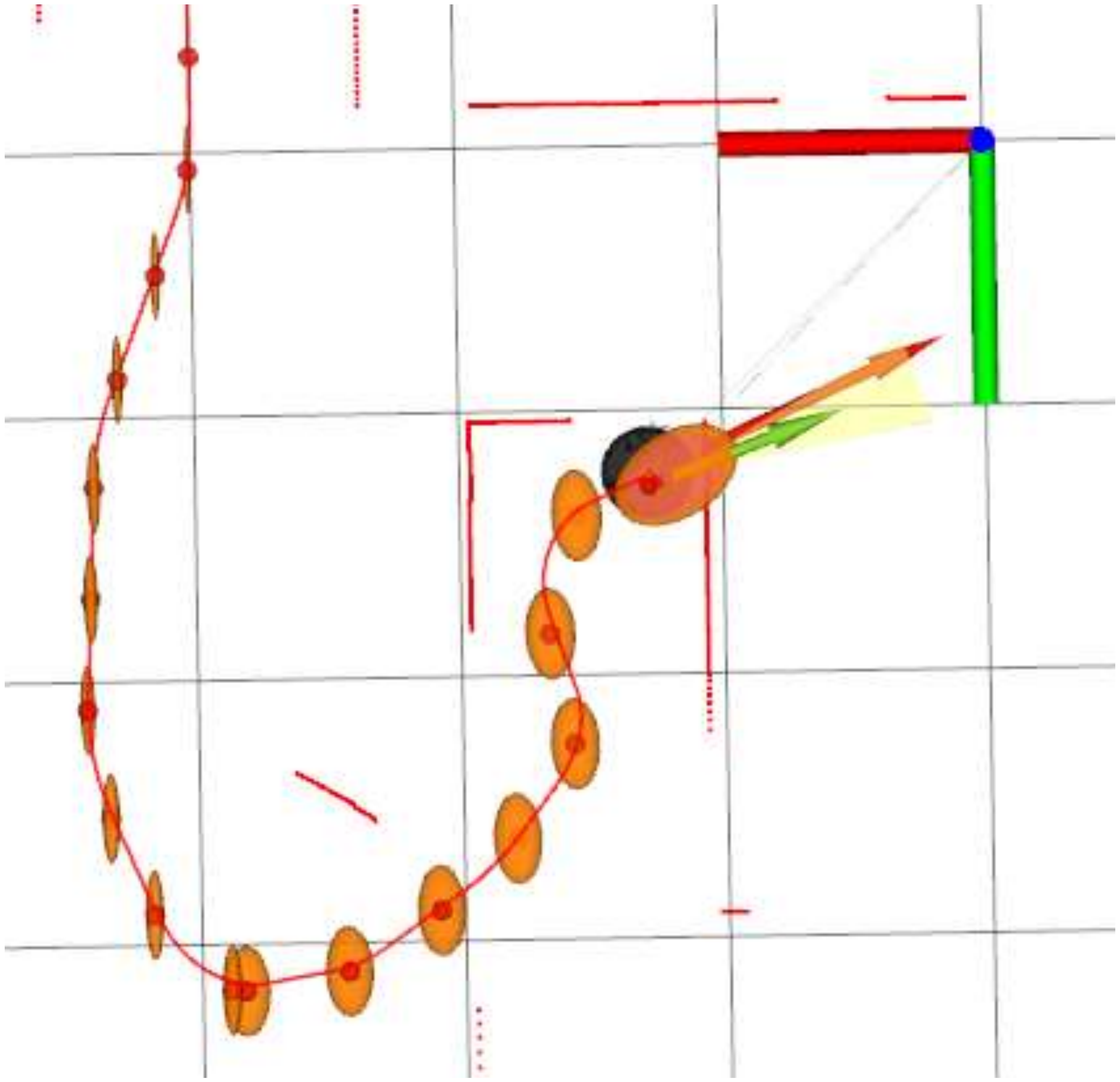
$$\mathbf{x}_k \leftarrow \hat{\mathbf{x}}_k + K_k \tilde{y}_k$$

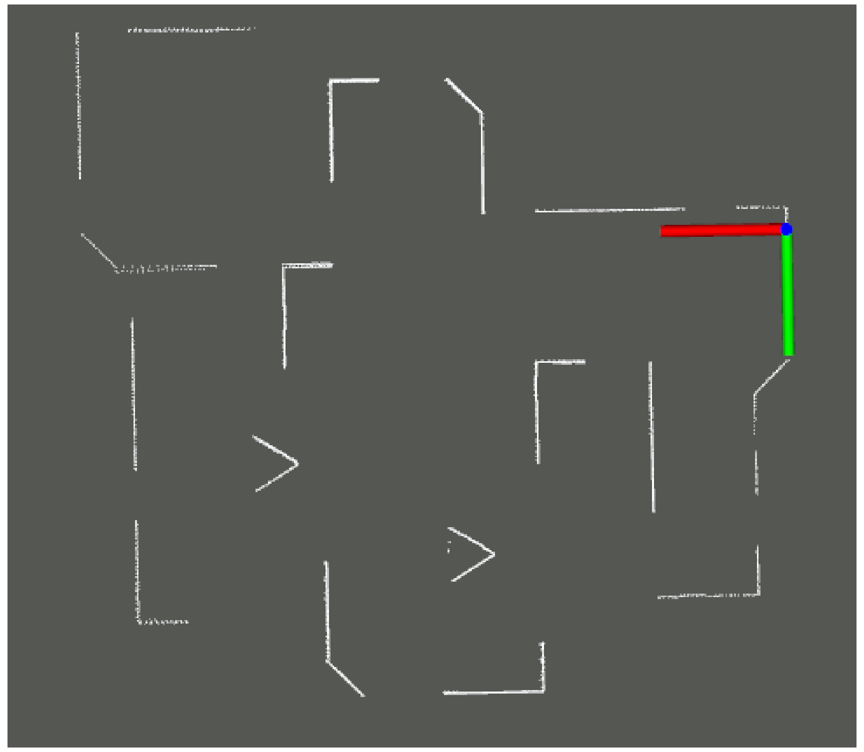
Covariance Update

$$P_k \leftarrow (I - K_k H_k) P_k (I - K_k H_k)^\top$$



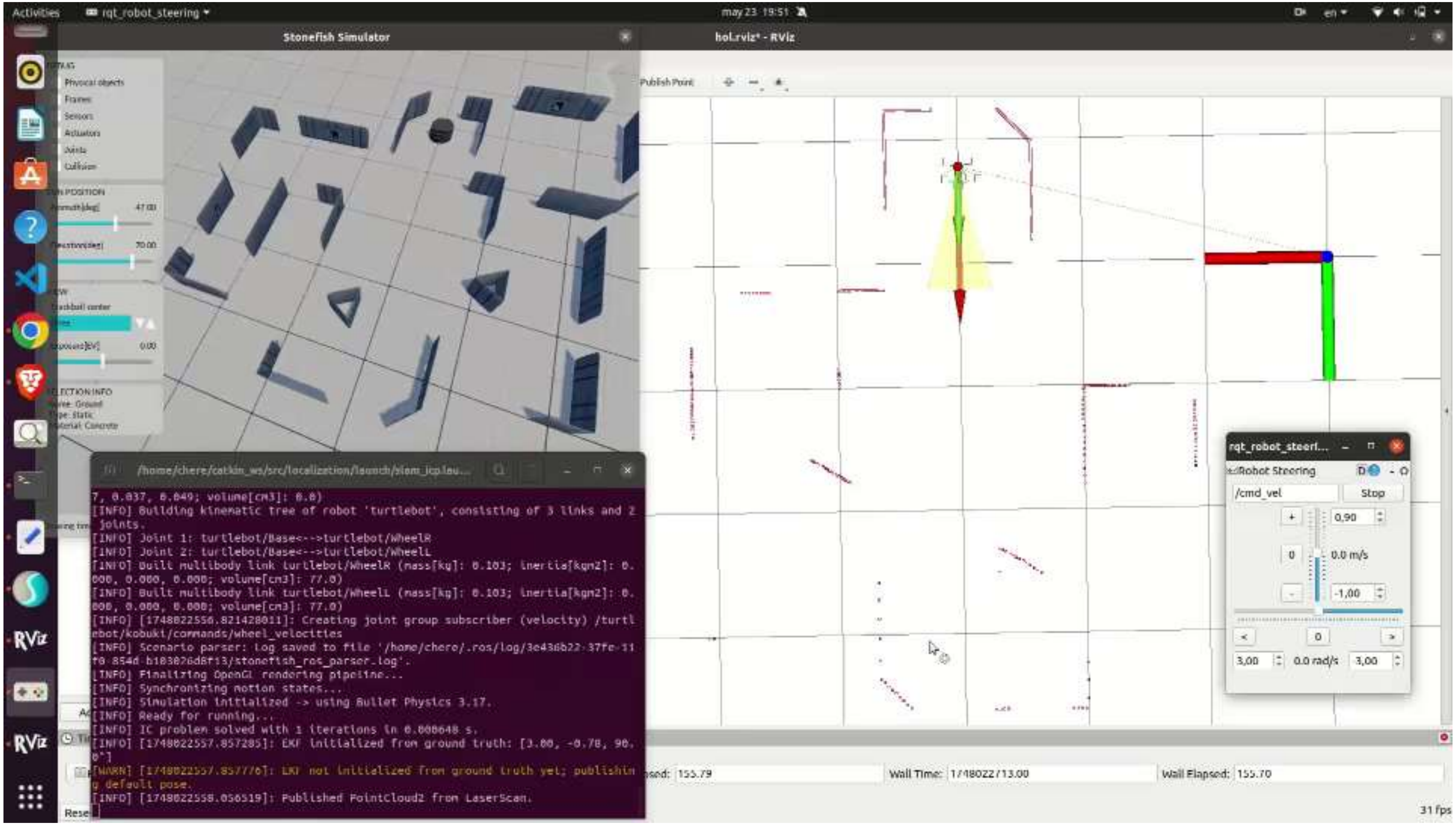
14. Mapping & Visualization



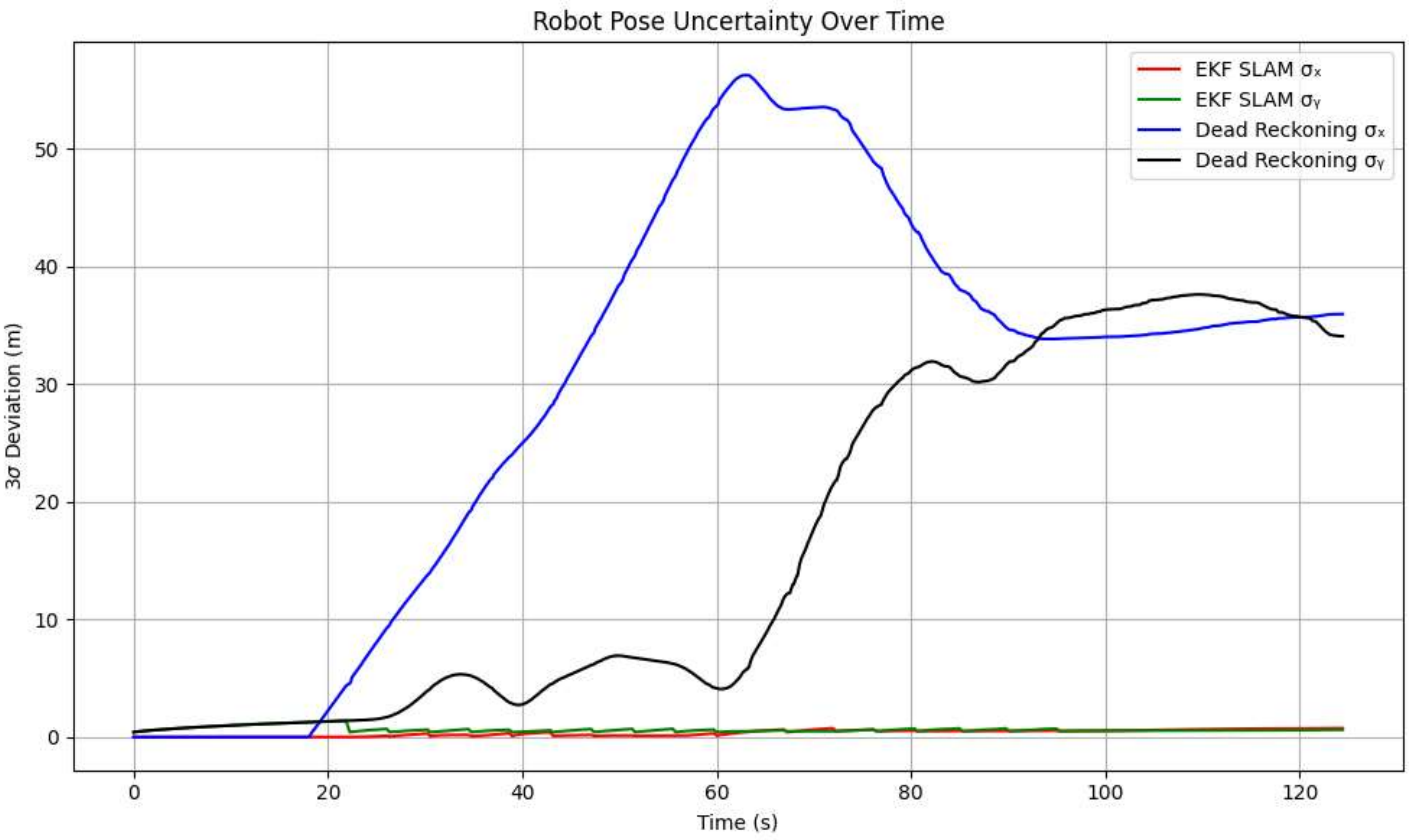


Perfectly Aligned Scans

Mis- Aligned Scans

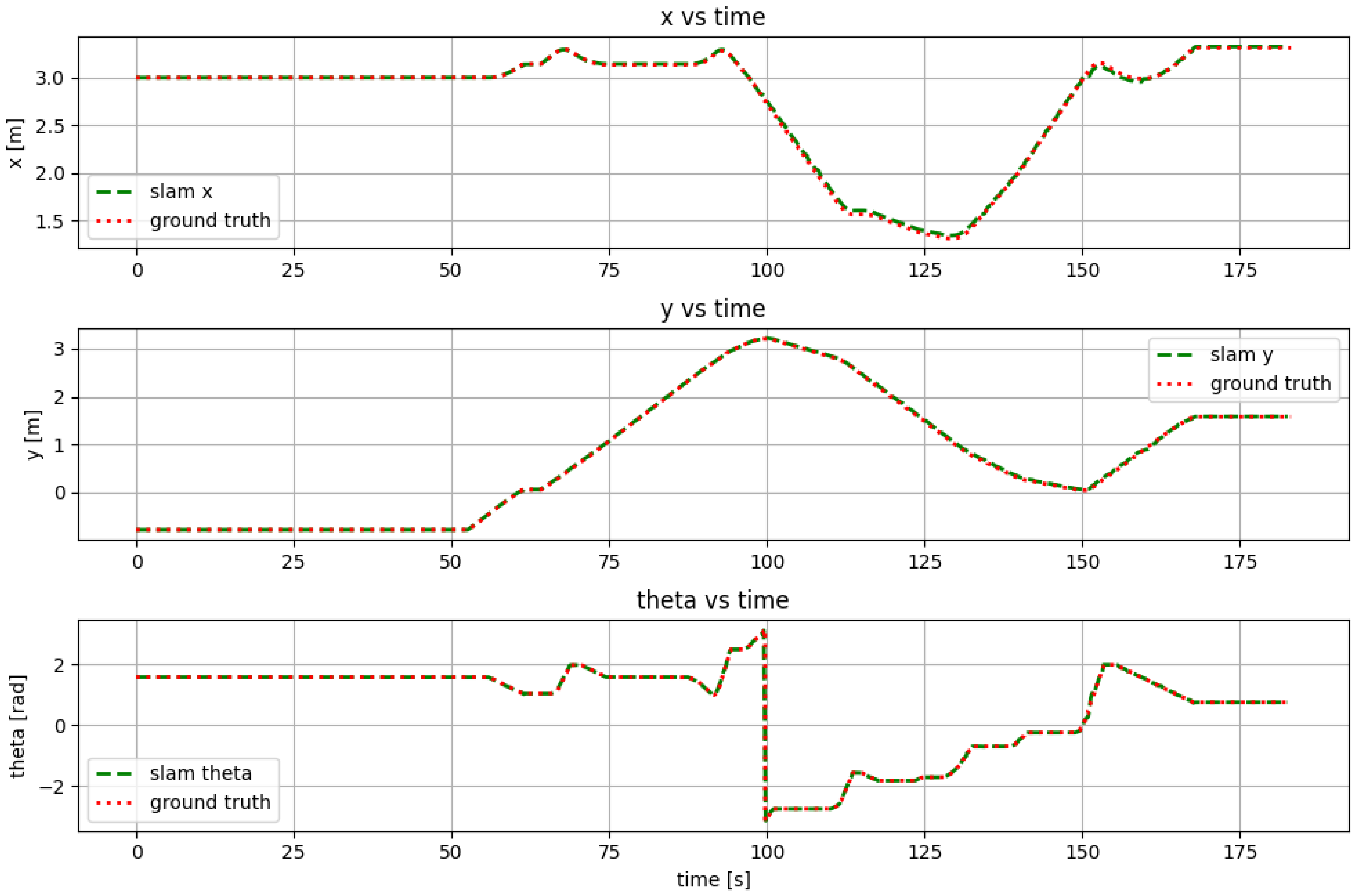

Activities
rqt_robot_steering
Stonefish Simulator
hol.rviz* - RViz
Publish Point
[INFO] Building kinematic tree of robot 'turtlebot', consisting of 3 links and 2 joints.
[INFO] Joint 1: turtlebot/Base<-->turtlebot/WheelR
[INFO] Joint 2: turtlebot/Base<-->turtlebot/WheelL
[INFO] Finalizing OpenGL rendering pipeline...
[INFO] Synchronizing motion states...
[INFO] Simulation initialized -> using Bullet Physics 3.17.
[INFO] Ready for running...
rqt_robot_steeri...
/cmd_vel
Stop
0.0 m/s
0.0 rad/s
Wall Time:
Wall Elapsed:
31 fps

16. Experiment & Result



ICP slam Vs DR IMU (3 sigma uncertainity)

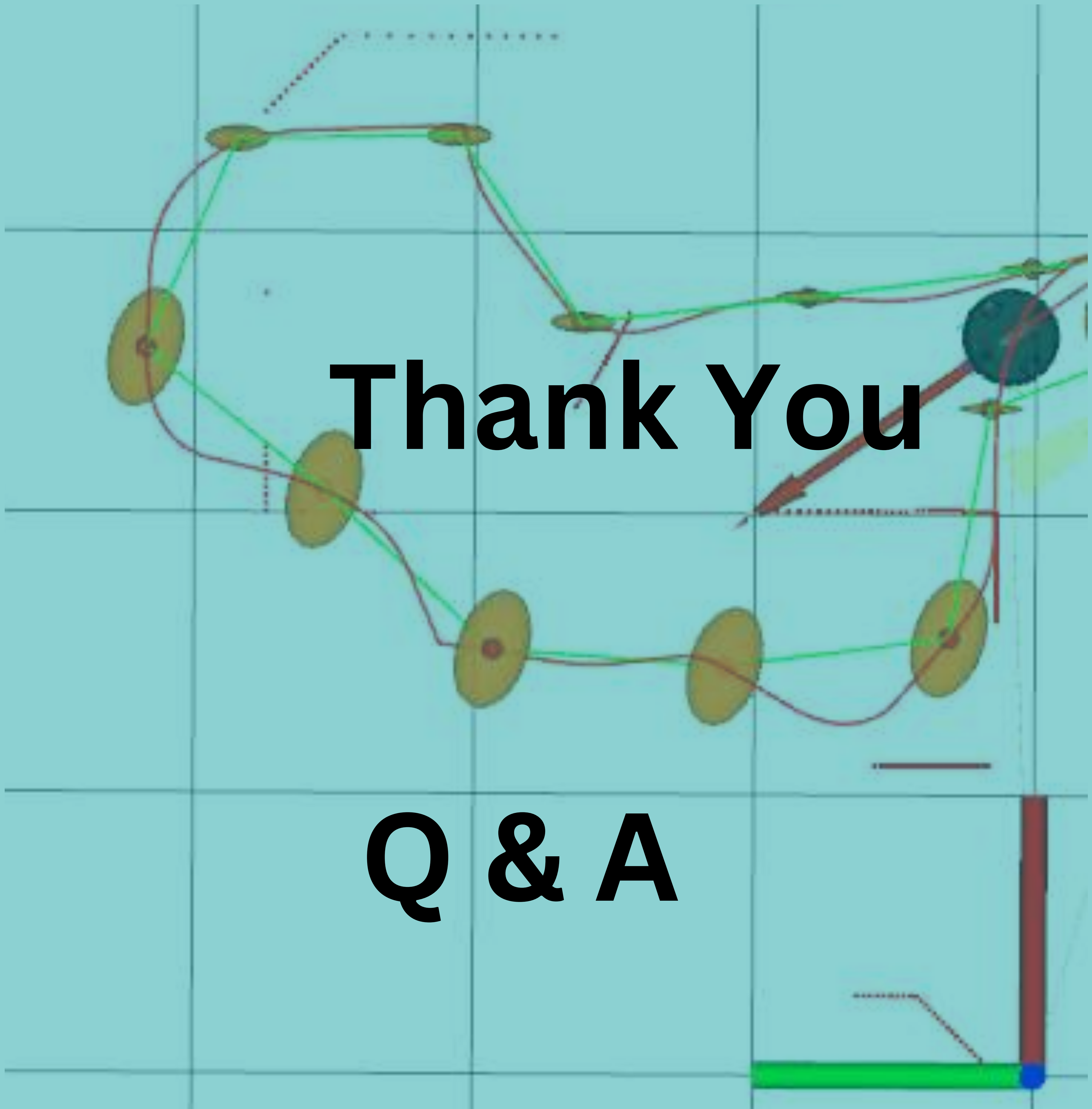
16. Experiment & Result



ICP slam Vs DR IMU (x, y, Theta)

17. Conclusion

- **PEKF SLAM system clearly outperforms dead reckoning.**
- **Compensates for odometry drift through IMU corrections and scan registration.**
- **Maintains a global map and pose history.**
- **ICNN filters out bad scan matches, improving robustness.**
- **Trustworthy SLAM in Noisy and Featureless Environment.**



Thank You

Q & A