Progressive Smoothing for Motion Planning in Real-Time NMPC

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European Control Conference 2024
June 26, 2024

Outline

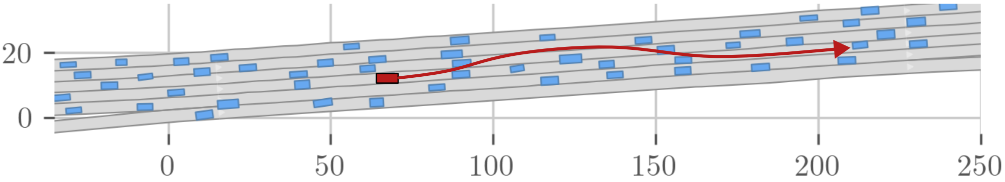
1. Preliminaries: motion planning, NMPC, SQP
2. Problem Statement
3. Method - Progressive Smoothing
4. Results and Conclusion

Goal:

- Driving a collision-free trajectory
- Optimizing some objective
  - Energy efficiency
  - Reaching a goal state
  - Time optimality

Assumption:

- Trajectories of surrounding vehicle are deterministic



NMPC is an advanced control strategy that uses a nonlinear model of the system to predict future states and optimize control inputs over a finite time horizon in each iteration.

Optimization Problem:

- At each time step t, solve the following optimization problem:

$$
\begin{aligned}
\min_{u(t),\dots,u(t+N-1)} \quad & \sum_{k=t}^{t+N-1} \left(\|x(k+1) - x_{\text{ref}}(k+1)\|_Q^2 + \|u(k)\|_R^2 \right) \\
\text{subject to} \quad & x(k+1) = f(x(k), u(k)), \ \forall k = t, \dots, t+N-1 \\
& x(t) = x_{\text{current}} \\
& h_{\text{obs}}(x(k)) \geq 0, \ \forall k \\
& x_{\min} \leq x(k) \leq x_{\max}, \ \forall k \\
& u_{\min} \leq u(k) \leq u_{\max}, \ \forall k
\end{aligned}
$$

- $x(k)$ is the state vector, $u(k)$ is the control input vector, $f(x, u)$ is the nonlinear system model, $x_{\text{ref}}(k)$ is the reference trajectory, Q and R are weighting matrices, N is the prediction horizon

Assumption

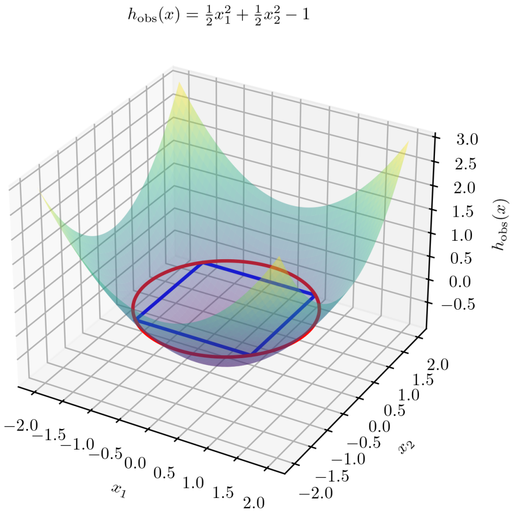
- The ego vehicle is a point and the obstacle is accordingly over-approximated
- The obstacle shape is rectangular

Goal

- A function $h_{\mathrm{obs}}(x)$ whose sublevel set at 0 contains the obstacle shape

Idea

- Linear transformation $x \rightarrow x'$ of rectangular obstacle to $\mathcal{O} = \{x'_1, x'_2 \in \mathbb{R} | \text{-}1 \leq x_{\{1,2\}} \leq 1\}$
- Obstacle constraint $0 \leq h_{\mathrm{obs}}(x;p) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 - 1$



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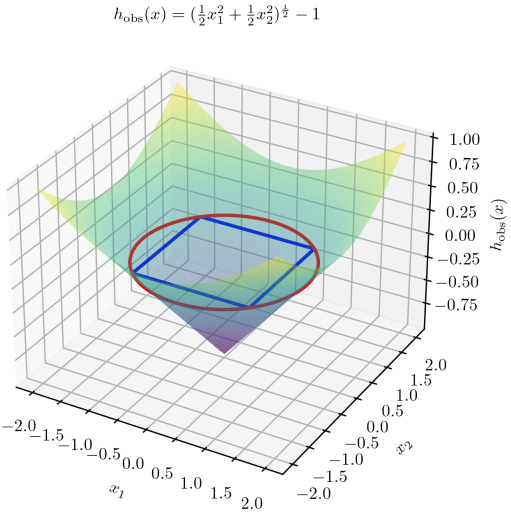

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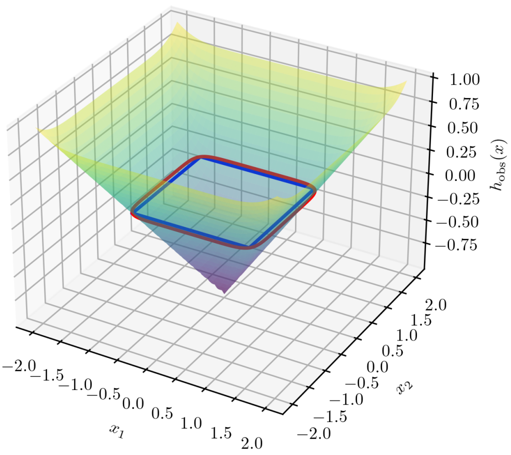
$h_{\mathrm{obs}}(x) = (\frac{1}{2}x_1^8 + \frac{1}{2}x_2^8)^{\frac{1}{8}} - 1$



SQP is an iterative method for nonlinear optimization that solves a series of quadratic programming (QP) subproblems to approach the solution of the original nonlinear problem.

Nonlinear Optimization Problem:

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{subject to} \quad & h_i(x) \leq 0, \ i = 1, \dots, m \\ & g_j(x) = 0, \ j = 1, \dots, p \end{aligned}$$

Quadratic Programming Subproblem:

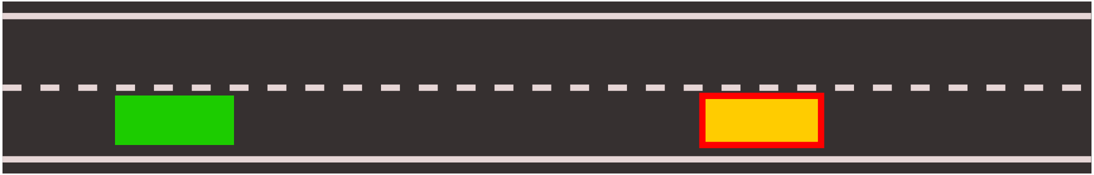
- At each iteration k, solve the QP subproblem:

$$\begin{aligned} \min_{\Delta x} \quad & \frac{1}{2} \Delta x^T \nabla^2 L(x_k, \lambda_k) \Delta x + \nabla f(x_k)^T \Delta x \\ \text{subject to} \quad & \nabla h_i(x_k)^T \Delta x + h_i(x_k) \leq 0, \ i = 1, \dots, m \\ & \nabla g_j(x_k)^T \Delta x + g_j(x_k) = 0, \ j = 1, \dots, p \end{aligned}$$

- Where $L(x, \lambda)$ is the Lagrangian, Δx_k is the Newton step with $x_{k+1} = x_k + \alpha_k \Delta x_k$

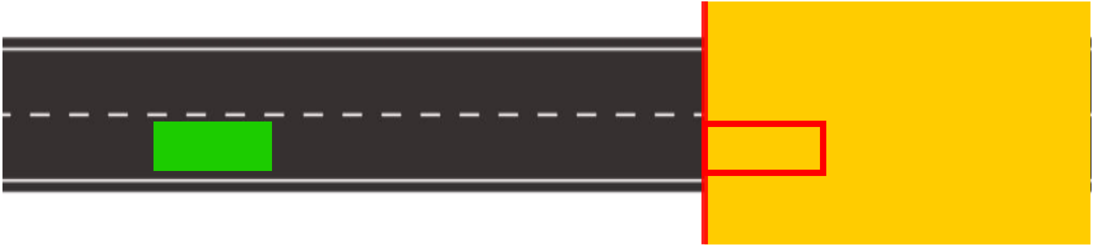
Problem Statement

Obstacle constraints



Problem Statement

Linearization of rectangle - ∞-norm



Problem Statement

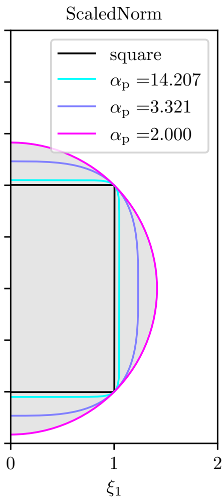
Over-approximation using 2-norm

Idea

- set $\alpha_p = 2$ to obtain smooth 2-norm
- linearize NLP and perfrom 1 SQP iteration

- set $\alpha_p \leftarrow \alpha_p + \Delta\alpha_p$, with $\Delta\alpha_p > 0$ to obtain tighter norm
- linearize NLP, initialize with previous solution and perfrom 1 SQP iteration

- ... repeat until $\alpha_p \geq \alpha_{p,\max}$



Problem with real-time iterations:

- We only want to perform one iteration at each sampling time

Problem, when taking the outer approximation, as shown before:

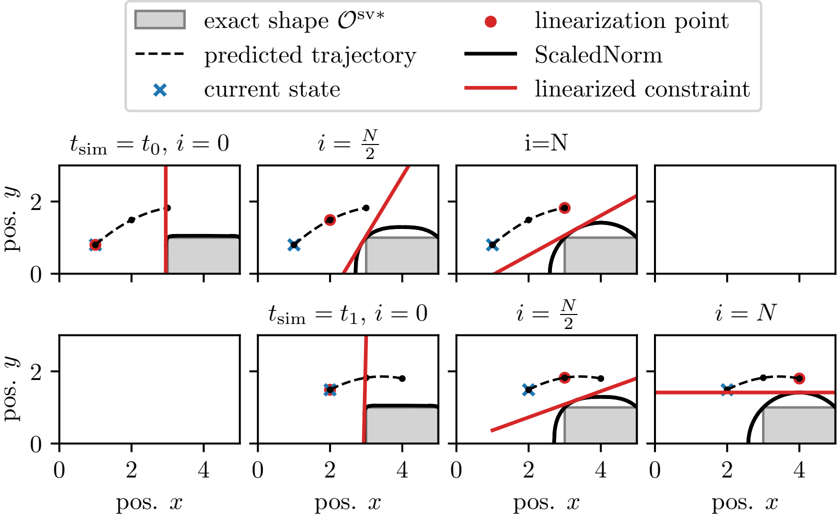
- The starting point at the next iteration would be infeasible

Problem, when taking the inner approximation, not shown before:

- The starting point at the next iteration would be a bad initial guess

Method

Homotopy along the horizon - progressive smoothing



Alternative formulations for progressive smoothing:
The non-smooth ∞-norm can also be expressed by the maximum function

$$0 \geq h_{\text{obs}}(x) = \max_{i=1,2} |x_i| - 1$$

Other smoothing formulations that approximate the maximum:

- LogSumExp smoothing
- Boltzmann smoothing

Alternative fixed shape formulations from literature:

- covering circles[1]
- "ReLU^2"[2]
- constant higher-order norms[3]

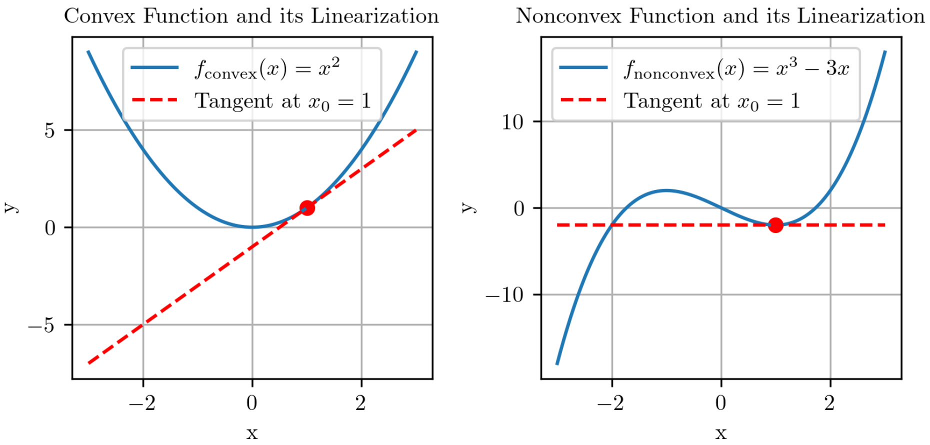
[1] Julius Ziegler et al. "Trajectory planning for Bertha - A local, continuous method". In: *IEEE Intelligent Vehicles Symposium, Proceedings*. June 2014, pp. 450–457. ISBN: 978-1-4799-3638-0. DOI: 10.1109/IVS.2014.6856581.

[2] Ajay Sathya et al. "Embedded nonlinear model predictive control for obstacle avoidance using PANOC". In: *European Control Conference (ECC)*. 2018, pp. 1523–1528. DOI: 10.23919/ECC.2018.8550253.

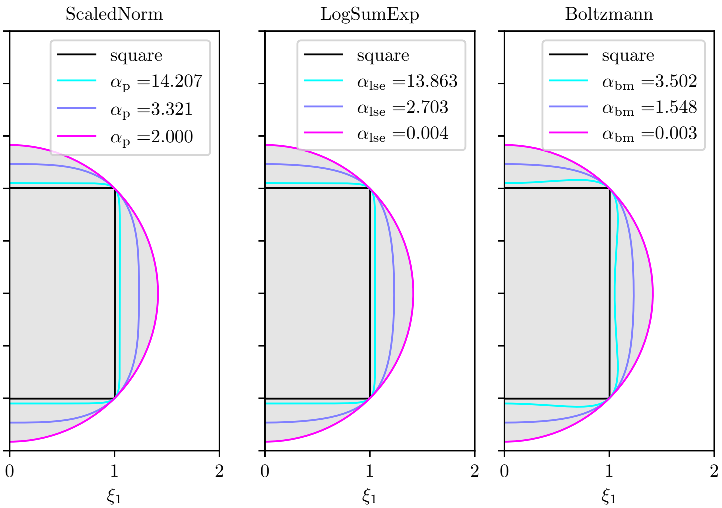
[3] Andreas Schimpe and Frank Diermeyer. "Steer with Me: A Predictive, Potential Field-Based Control Approach for Semi-Autonomous, Teleoperated Road Vehicles". In: *IEEE 23rd International Conference on Intelligent Transportation Systems (ITSC)*. 2020, pp. 1–6. DOI: 10.1109/ITSC45102.2020.9294702.

Why convexity?

Linearization is always safe



Scaled Norm, LogSumExp are convex ✓, Boltzmann can be nonconvex ✗



Let $o(\xi, \alpha)$ be the obstacle constraint with homotopy parameter α and position coordinates ξ.

Definition

A homotopy $o(\xi, \alpha) : \mathbb{R}^n \times [\underline{\alpha}, \overline{\alpha}] \to \mathbb{R}$ is monotonously *tightening* with an increasing α, if for all $\alpha_2 \geq \alpha_1$

$$\{\xi \in \mathbb{R}^n | o(\xi, \alpha_2) \leq 1\} \subseteq \{\xi \in \mathbb{R}^n | o(\xi, \alpha_1) \leq 1\}.$$

The property of *over-approximation* is used to describe whether for any value of the homotopy parameter $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, the smooth shape is over-approximating the rectangular shape $\mathcal{B}$.

Definition

A homotopy $o(\xi, \alpha) : \mathbb{R}^n \times [\underline{\alpha}, \overline{\alpha}] \to \mathbb{R}$ is an *over-approximation* of $\mathcal{B}$, if for $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ it holds that

$$\mathcal{B} \subseteq \{\xi \in \mathbb{R}^n | o(\xi, \alpha) \leq 1\}.$$

Properties

Over-approximation and monotonous tightening

Why over-approximation?

Safety

Why monotonous tightening?

Recursive feasibility

"Scaled Norm" has both properties✓

Homogeneity Property

- Consider a function $h : \mathbb{R}^{n_x} \to \mathbb{R}$
- One of the fundamental properties of a function is **homogeneity** (also known as **absolute scalability**).

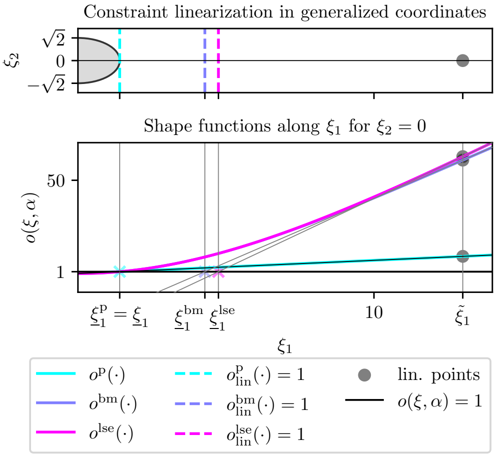
Homogeneity Property

For any vector $x \in \mathbb{R}^{n_x}$ and any scalar $\beta \in \mathbb{R}$:

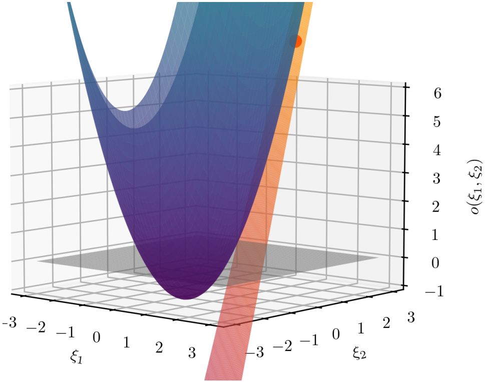
$$h(\beta x) = |\beta| h(x)$$

- This property indicates that scaling a vector by a scalar β scales the function of the vector by $|\beta|$.
- Intuitively, this means that the length (or size) of a vector scales linearly with the absolute value of the scalar.
- Consequently, a linearization at x_0 is exact along a beam through 0 and x_0

Why Homogeneity? - Linearizations are "better"



Linearization of quadratic function:



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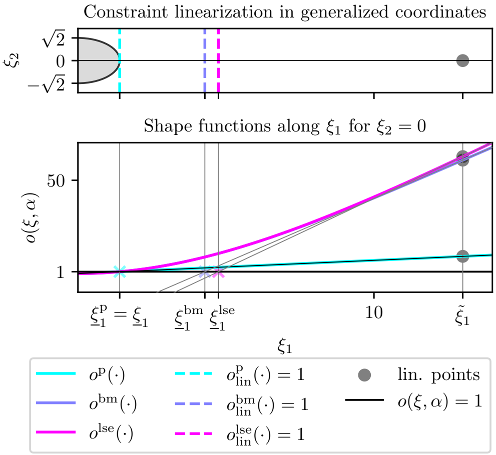
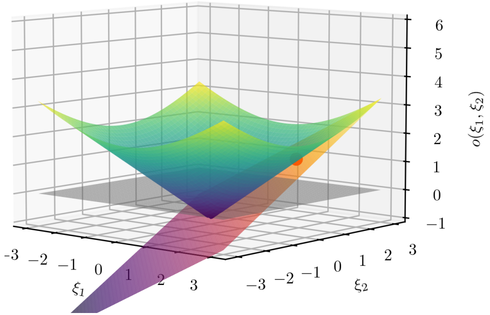


Linearization of norm function:

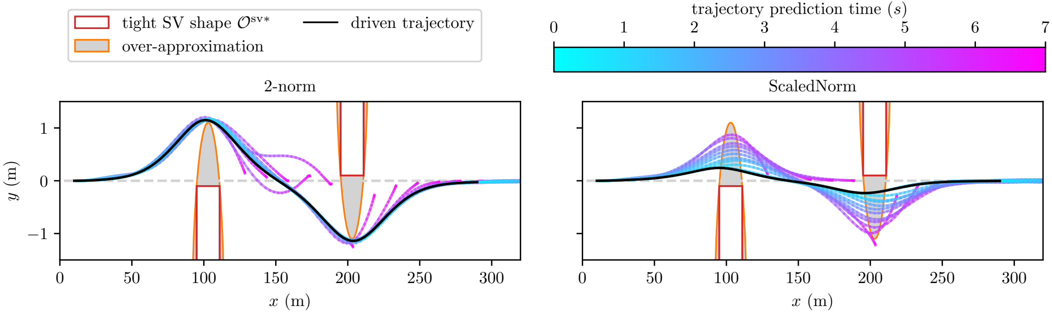


Properties

Comparison

Property	*ScaledNorm*	*LogSumExp*	*Boltzmann*	p-norm	$ReLU^2$	cov. circles
Progressive Smoothing	✓	✓	✓	✗	✗	✗
Convexity	✓	✓	✗	✓	✓	✓
Over-approximation	✓	✓	✓	✓	✓	✓
Homogeneity	✓	✗	✗	✓	✗	✓
Exact slack penalty	✓	✓	✓	✓	✗	✓

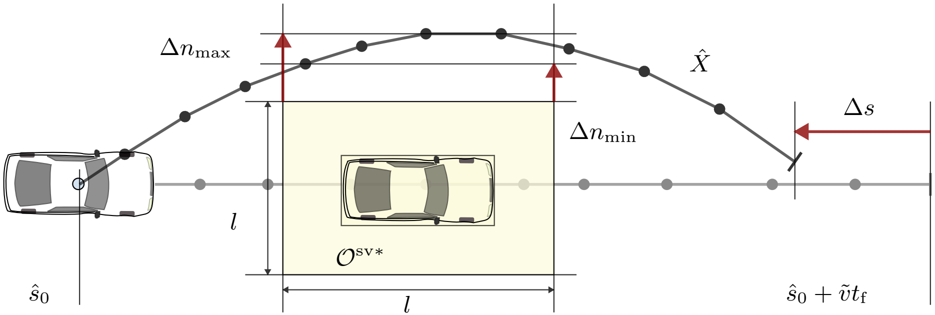
Table: Properties of the considered obstacle formulations.



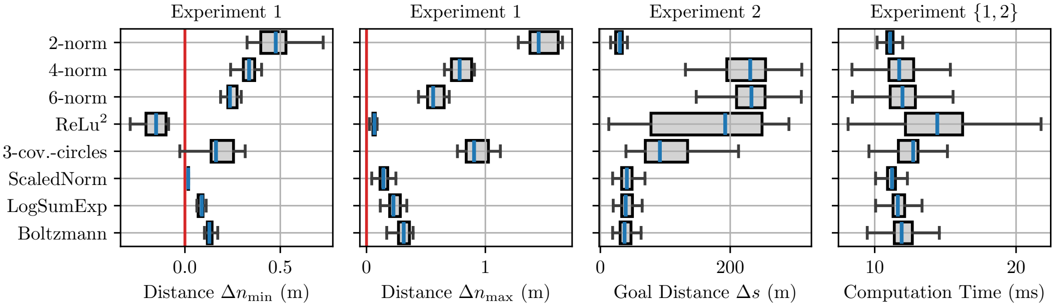
tight SV shape $\mathcal{O}^{\text{sv}*}$
over-approximation
driven trajectory
trajectory prediction time (s)
0
1
2
3
4
5
6
7
2-norm
ScaledNorm
y (m)
1
0
−1
0
50
100
150
200
250
300
x (m)

Results

Randomized scenarios - setup



- Experiment 1: Overtaking with low lateral distance
- Experiment 2: Overtaking smoothly



Conclusion

- Representing obstacles by higher-order norms yields good properties for SQP
- Progressing the smoothness along the horizon leads to tighter approximations in closed-loop
- Progressing the smoothness along the horizon is less prone to get stuck in bad local minima
- The smoothing parameter schedule can be tuned for a specific problem

Thanks to the Coauthors!

Thank you for your attention!