Assignment Quantitative Finance 2023-2024

- Due date: Friday December 22 (17:00), 2023
- You have to work in groups consisting of three or four persons.
- You can use your favorite programming language.
- Your report should be concise and may be written in Dutch or English and should include your code.
- For administrative use, be aware to put your full names and administration numbers on the first page of your document.
- If you need extra assumptions in order to answer a question, motivate why you need them, and state these assumptions clearly.
- Fraud and plagiarism are incompatible to a correct academic attitude. Therefore, a suspicion of fraud or plagiarism will be reported to the Examination Committee.
- This assignment determines 20% of your final grade.
- In case of questions: send an email to `r.vdnakker@tilburguniversity.edu`.

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1 Question 1 (7 pts)

Consider the following option on the AEX-index. If you buy the option, at time $t = 0$, you get the following payoff at expiration date $T = 5$ (years):

$$C_T = \max \left\{ K - \frac{1}{T} \sum_{t=1}^{T} S_t,\ 0 \right\},$$

where $K = 740$ and where S denotes the level of the AEX-index.

(a) Assume that the level of the AEX-index is described by a geometric Brownian motion. Estimate the parameters (of the geometric Brownian motion) on basis of historical data (see, for example, link). [2pts]

(b) Assuming the Black-Scholes model for the financial market, use Monte-Carlo simulations to obtain a confidence interval for the price, at $t = 0$, of the option. [2pts]

(c) Use the bump-and-reprice method to obtain an estimate and a confidence interval for the delta of this option at $t = 0$. Consider 'non-common random numbers' as well as 'common random numbers'. [3pts]

2 Question 2 (13 pts)

Consider, the Black-Scholes market with $\mu = 10\%$, $\sigma = 25\%$, $S_0 = 100$, $B_0 = 1$, and $r = 3\%$. We consider a European put option with strike price $K = 100$ and maturity $T = 2$.

(a) Compute the delta of the option at $t = 0$. Also obtain numerical approximations by using i) bump-and-reprice, ii) the pathwise method, and iii) the likelihood ratio method. [2pts]

(b) Compute the gamma of the option at $t = 0$. Obtain, if possible, numerical approximations by using i) bump-and-reprice, ii) the pathwise method, and iii) the likelihood ratio method. [4pts]

(c) Assume that a financial institution has sold 1,000 puts (and assume that these put options are not directly traded on financial markets) and would like to hedge the risk. In this exercise we acknowledge that we cannot readjust hedging positions in continuous time and will only allow for readjustments at discrete points in time. We will consider readjustments at a time grid with stepsize $\Delta = 0.01$. The strategy is as follows:

- At each readjustment moment we rebalance the position in the stock such that the total portfolio (consisting of the short position in the option, the stocks, and the position in B) is delta-neutral.
- You take your position in the money-market-account such that the readjustments of the position in the stock can be done without net cashflow. At $t = 0$ you can use the proceedings of selling $1,000$ options to set up your hedge.
- Consequently, your total portfolio should only exhibit a net cashflow at maturity T.

Report the precise positions in the stock and money-market account (at $t = 0$). Next, generate at least 2,500 simulations of this strategy. Plot, for each point on the time grid, the 5%, 50%, and 95% quantiles of the positions in S and B. Also plot, for each point on the time grid, the 5%, 50%, and 95% quantiles of the gamma of the portfolio. Finally, present the mean and standard deviation of the total portfolio value at maturity. What price would you charge for the option? [3pts]

(di) Suppose now that a call option with strike price $K = 120$ and $T = 5$ is also traded. Repeat exercise (c) using delta-gamma hedging. Interpret the results. [3pts]

(dii) Suppose now that a second stock is traded. The stock above is now denoted by $S_t^{(1)}$ and the new stock by $S_t^{(2)}$. Each stock is asssumed to follow a Geometric Brownian motion. We assume that there is a driving bivariate Brownian motion with correlation per unit-of-time is equal to 0.6 In equations:

$$\mathrm{d}S_u^{(j)} = \mu_j S_u^{(j)} \mathrm{d}u + \sigma_j S_u^{(j)} \mathrm{d}W_u^{(j)}, \quad j = 1, 2,$$

where $S_0^{(j)} = s_0^{(j)}$ and where $W = (W^{(1)}, W^{(2)})$ is a bivariate Brownian motion with $W_1 \sim N_2(0, \Sigma)$ where $\Sigma_{11} = \Sigma_{22} = 1$ and $\Sigma_{12} = 0.6$. Assume $\mu_2 = 6\%$, $\sigma_2 = 14\%$, and $s_0^{(2)} = 110$. Repeat exercise (c) using delta-gamma hedging. Interpret the results. [1pts]