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BLUEPRINT FOR

DEPARTMENTAL MODEL QUESTION PAPER-2022

MATHEMATICS (35)

TIME: 3 hours 15 minute **Max. Mark: 100**

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		Hrs.	A	B	C	D	E		MARKS
			1M	2M	3M	5M	4M	6M	
1	RELATIONS AND FUNCTIONS	11	2	1	1	2			17
2	INVERSE TRIGONOMETRIC FUNCTIONS	8	2	2	1				9
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7	INTEGRALS	21	2	3	3	1		1	28
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11	THREE DIMENSIONAL GEOMETRY	12	1	2	1	1			13
12	LINEAR PROGRAMMING	7	1					1	7
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	Total	150	15	18	18	12	3	3	195

MODEL QUESTION PAPER-1

MATHEMATICS (35)

TIME: 3 Hours 15 Minutes **MAX. MARKS: 100**

Instructions:

i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii) Use the graph sheet for the question on Linear programming in PART E.

PART - A

Answer any TEN questions: **10 × 1=10**

1. A relation R on set $A=\{1,2,3\}$ is defined as $R=\{(1,1),(1,2),(3,3)\}$ is not symmetric. Why?
2. Define binary operation.
3. Write the domain of $\operatorname{cosec}^{-1}x$.
4. Find the set of values of x such that $2\tan^{-1}(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$
5. Define a scalar matrix.
6. If A is a square matrix with $|A| = 8$,find the values of $|AA'|$.
7. Find $\frac{dy}{dx}$ if $x - y = \pi$
8. Find $\frac{dy}{dx}$, if $y = e^{\sin^{-1}x}$.
9. Find $\int\sqrt{1+\sin 2x}dx$
10. Find: $\int\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$ dx
11. Define negative of a vector.
12. If $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear then find a.
13. Write the direction cosines of z – axis.
14. Define an objective function of a LPP.
15. If P(A)= ½ , P(B)=0 then find P(A/B).

PART - B

Answer any TEN questions. **$10 \times 2 = 20$**

16. Find gof and fog, if $f(x) = 8x^3$ and$(x) = x^{\frac{1}{3}}$.

17. Find the principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

18. Show that $\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}x,\ x \geq 1$ or $x \leq -1$.

19. Find AB.if $A=\begin{bmatrix}0 & -1\\0 & 2\end{bmatrix},\ B=\begin{bmatrix}3 & 5\\0 & 0\end{bmatrix}$ and write the conclusion.

20. Find the area of the triangle whose vertices are (1, 0), (6, 0) and (4, 3).

21. Find $\frac{dy}{dx}$, if $ax + by^2 = cosy$.

22. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

23. Find the second order derivatives of $\tan^{-1}x$ with respect to x.

24. Find two numbers whose sum is 24 and whose product is as large as possible.

25. Find$\int \frac{\cos 2x - \sin 2\alpha}{\cos x - \sin \alpha}\, dx$

26. Find: $\int x^2 e^{x^3} dx$

27. Evaluate: $\int_0^{\pi}(\sin^2\frac{x}{2} - \cos^2\frac{x}{2})dx$

28. Find the order and degree of the differential equation

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$$

29. Find$|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$.

30. Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a} = -2\hat{i} - 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - \hat{k}$.

31. Find the Cartesian equation of the line parallel to the y-axis and passing through the point (1, 1, 1)

32. Find the angle between the pair of lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

33. Mother ,Father , and son line up at random for a family picture. Find P(E/F) where E: Son on one end, F: Father in middle.

PART – C

Answer any TEN questions. **$10 \times 3 = 30$**

34. Determine whether the Relation R in the set $\mathbb{Z}$ of all integers defined as $R = \{(x, y): x - y \text{ is an integer}\}$ is reflexive, symmetric & transitive.

35. Express $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), 0 < x < \pi$ in the simplest form.

36. For any square matrix A with real number, prove that $A + A'$ is a symmetric and $A - A'$ is a skew symmetric matrix.

37. Examine the consistency of the system of equations

$x + 2y = 2$ and $2x + 3y = 3$.

38. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ prove that $\frac{dy}{dx} = \tan\frac{\theta}{2}$.

39. Find the derivative of $(\log x)^{\log x}$ with respect to x.

40. Verify mean value theorem if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b] where a = 1 and b = 3. Find $c \in (1, 3)$ for which $f'(c) = 0$.

41. Find the equation of the normal to the curve $2y + x^2 = 3$ at the point (1,1).

42. Find $\int \frac{x^2+1}{x^2-5x+6}dx$

43. Find $\int \frac{x}{(x+1)(x+2)}dx$

44. Find $\int x\tan^{-1}x\,dx$

45. Find the area of the region bounded by $x^2=4y$, y=2,y=4 and the y-axisin the first quadrant.

46. Form the D .E. representing the given family of curves y = asin(x+b) by eliminating arbitrary constants.

47. Find the general solution of the differential equation $\frac{dy}{dx}+y=1\ (y\neq 1)$.

48. Prove that $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$.

49. Derive the formula for position vector of a point which divides the join of twopoints A and B internally in the ratio $m:n$.

50. Find the distance between the lines

$\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and

$\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$.

51. A man is known to speak truth 3 out of 4 times. He throws a die and reports it is six Find the probability that it is actually a six.

PART – D

Answer any SIX questions. **$6\times 5=30$**

52. State whether the function $f:R\to R$ defined by $f(x)=3-4x$ is one-one, onto or bijective. Justify your answer.

53. Let $Y = \{n^2 : n \in N\} \subset N$. Consider $f : N \to Y$ as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

54. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$. Calculate AC, BC and $(A+B)C$. Also, verify that $(A+B)C = AC + BC$.

55. Solve the following system of linear equations by matrix method: $x-y+2z=7$, $3x+4y-5z=-5$ and $2x-y+3z=12$.

56. If $y = 3\cos(\log x) + 4\sin(\log x)$, prove that $x^2y_2 + xy_1 + y = 0$.

57. The length x of a rectangle is decreasing at the rate of 3cm/min and the width y is increasing at the rate of 2cm/min.

When $x = 10$cm and $y = 6$cm, find the rate of change of

(i) the perimeter and (ii) the area of the rectangle.

58. Find $\int \frac{dx}{\sqrt{x^2-a^2}}$ with respect to x and hence evaluate $\int \frac{dx}{\sqrt{x^2+6x-7}}$

59. Find the area enclosed by the circle $x^2 + y^2 = a^2$.

60. Find the general solution of $\frac{dy}{dx} + \sec x\, y = \tan x$.

61. Derive the equation of the plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

62. An unbiased die is thrown twice . let the event A be 'odd number on the first throw' and B the event 'odd number on the second throw' check the independency of events A and B.

63. Five cars are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

i) all the five cards are spade?

ii) only three cards are spade?

iii) none is spade?

PART – E

Answer any ONE question. **$1 \times 10 = 10$**

64(a). Solve LPP graphically: Minimize and maximize $Z = 3x + 9y$, subject to the constrains $x + 3y \leq 60$; $x + y \geq 10$; $x \leq y$; $x \geq 0$ and $y \geq 0$.

64(b). Prove that $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + bc & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

65(a). Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if\ f(2a-x) = f(x) \\ 0, if & if\ f(2a-x) = -f(x). \end{cases}$

and hence evaluate $\int_0^{2\pi} cos^5 x dx$.

65(b). Find the values of a and b such that

$f(x) = \begin{cases} 5 & if\ x \leq 2 \\ ax + b & if\ \ 2 < x < 10 \\ 21 & if\ x \geq 10 \end{cases}$ is continuous function.

66(a). Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

66(b). Show that the matrix A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation A^2-4A+I =O, where I is 2 x 2 identity matrix and 2 x 2 zero matrix. Using this equation, find A^{-1}.

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ANSWERS TO MODEL QUESTION PAPER-1

PART-A

1. R on set $A = \{1,2,3\}$ is not symmetric as $(1,2) \in R$ but $(2,1) \notin R$.

2. A binary operation * on a set A is a function $* : A \times A \rightarrow A$.

Defined by $*(a, b) = a * b$.

3. Domain is $R - (-1,1)$

4. The set of values of x is $[-1,1]$.

5. A <u>Diagonal matrix</u> is said to be *scalar matrix* if all its <u>diagonal elements are *equal*.</u>

7. $y = x - \pi, \quad \therefore \frac{dy}{dx} = 1.$

8. Given $y = e^{\sin^{-1}x} \Rightarrow \frac{dy}{dx} = e^{\sin^{-1}x}\left(\frac{1}{\sqrt{1-x^2}}\right).$

9. $$I = \int \sqrt{(\sin^2 x + \cos^2 x + 2\sin x \cos x)}\, dx$$

$$= \int \sqrt{(\sin x + \cos x)^2}\, dx = \int (\sin x + \cos x)\, dx = -\cos x + \sin x + C.$$

10. $\int \frac{sinx}{2} dx = \frac{1}{2}(-cosx) + C.$

11. A vector whose magnitude is the same as that of a given vector, but direction is opposite to that of it, is called negative of the given vector.

12. Given vectors are collinear $\Rightarrow \frac{a}{2} = \frac{6}{-3} = \frac{-8}{4} = -2 \Rightarrow a = -4.$

13. Let y-axis made an angles with x, y and z axes $90^\circ, 0^\circ$ and 90° respectively.

$\therefore$ direction cosines of z – axis $\cos 90^\circ$, $\cos 90^\circ$, $\cos 90^\circ$ ie 0,0,1

14. Linear function Z = ax + by, where a and b are constants, which has to be maximized or minimized is called a linear objective function of an LPP is called objective function.

15. $P(A/B).=\frac{P(A\cap B)}{P(B)}$ as $P(B)=0$ $P(A/B)$.IS not defined.

PART – B

16. Given $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

$$(gof)(x) = g(f(x)) \quad = g(8x^3) = (8x^3)^{\frac{1}{3}} = 2x$$

$$(fog)(x) = f(g(x)) = f\left(x^{\frac{1}{3}}\right) = 8\left(x^{\frac{1}{3}}\right)^3 = 8x\ .$$

17. $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

18. Let $\sin^{-1}\frac{1}{x} = \theta \Rightarrow \sin\theta = \frac{1}{x} \Rightarrow x = \operatorname{cosec}\theta$

$\theta = \operatorname{cosec}^{-1}x \Rightarrow \sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}x$.

19. $AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0-0 \\ 0+.0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Thus, if the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix. (If AB = 0 but not necessary that A= O or $B = O$)

20. The area of the triangle is given by $\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}$ [1(0-3 -0 +1(18-0)]= $\frac{1}{2}$ (-3+18) = $\frac{15}{2}$ sq. units.

21. $ax + by^2 = cosy$

Differentiate with respect to x

$\Rightarrow a(1) + b2y\frac{dy}{dx} = -siny\frac{dy}{dx} \quad \Rightarrow 2by\frac{dy}{dx} + siny\frac{dy}{dx} = -a$

$\Rightarrow \frac{dy}{dx}(2by + siny) = -a \Rightarrow \frac{dy}{dx} = \frac{-a}{2by+siny}$.

22. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ Put $x = tan\theta$

$$y = \sin^{-1}\left(\frac{2tan\theta}{1+tan^2\theta}\right) = \sin^{-1}(sin2\theta) = 2\theta = 2\tan^{-1}x$$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}.$

23. $y = \tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{1}{1+x^2} = (1+x^2)^{-1}$

$\therefore \frac{d^2y}{dx^2} = -(1+x^2)^{-2}2x = -\frac{2x}{(1+x^2)^2}.$

24. Given $x + y = 24 \Rightarrow y = 24 - x$.............(1)

Product $P = xy$ is maximum

$\Rightarrow P = x(24 - x)$ is maximum $\Rightarrow P = 24x - x^2$ is to be maximum

$\Rightarrow \frac{dP}{dx} = 0 \quad \frac{dP}{dx} = 24 - 2x = 0 \Rightarrow 24 = 2x \Rightarrow x = 12.$

And $\frac{d^2P}{dx^2} = -2.$ $\quad \Rightarrow$ *P has maximum value at* $x = 12.$

Substitute in equation (1) $\quad y = 24 - 12 = 12$

$\therefore x = 12$ and $y = 12.$

25. $I = \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos\alpha} dx = 2\int \frac{\cos^2 x - \cos^2 \alpha}{\cos x - \cos\alpha} dx = 2\int (\cos x + \cos\alpha)\, dx$

$= 2(\sin x + x\cos\alpha) + C$

26. Put $x^3 = t \Rightarrow 3x^2dx = dt \therefore x^2dx = \frac{dt}{3}$

$$\therefore I = \int e^t \frac{dt}{3} = \frac{e^t}{3} + C = \frac{e^{x^3}}{3} + C.$$

27. $I = \int_0^{\pi} (-\cos x)\, dx = -\sin x\Big]_0^{\pi}$

$= -(\sin\pi - \sin 0) = 0$

28. Order is 2 and Degree is 1.

29. Given, $|\vec{a}| = 1$ and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

$\Rightarrow \vec{x}\cdot\vec{x} + \vec{x}\cdot\vec{a} - \vec{a}\cdot\vec{x} - \vec{a}\cdot\vec{a} = 12$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12 \Rightarrow |\vec{x}|^2 - 1 = 12$

$|\vec{x}|^2 = 13 \quad \Rightarrow |\vec{x}| = \sqrt{13}.$

30. The area of a triangle with $\vec{a}$ and $\vec{b}$ as its adjacent sides is given by$A = \frac{1}{2}|\vec{a} \times \vec{b}|$.

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= (0-10)\hat{\imath} - \big(2-(-5)\big)\hat{\jmath} + (4-0)\hat{k} = -10\hat{\imath} - 7\hat{\jmath} + 4\hat{k}.$$

$\therefore$ area of triangle $\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}\sqrt{100+49+16} = \frac{1}{2}\sqrt{165}$ sq units.

31. DRs of y- axis is 0, b, 0

and line passes through (1, 1, 1)

$\therefore$ equation of the line is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \Rightarrow \frac{x-1}{0} = \frac{y-1}{b} = \frac{z-1}{0}$.

32. Let$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$....(1) and$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$....(2)

$\because$ DR of line (1): 3, 5, 4 = a_1, b_1, c_1 and

DR of line (2): 1, 1, 2 = a_2, b_2, c_2

$\because |\overrightarrow{b_1}| = \sqrt{a_1^2 + b_1^2 + c_1^2} = \sqrt{3^2+5^2+4^2} = \sqrt{9+25+16} = \sqrt{50} = 2\sqrt{5}$

$|\overrightarrow{b_1}| = \sqrt{a_2^2 + b_2^2 + c_2^2} = \sqrt{1^2+1^2+2^2} = \sqrt{1+1+4} = \sqrt{6}$

and $\overrightarrow{b_1} . \overrightarrow{b_2} = 3(1) + 5(1) + 4(2) = 3 + 5 + 8 = 16$

Wkt $\cos\theta = \left|\frac{\overrightarrow{b_1} . \overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}\right| = \left|\frac{16}{2\sqrt{5}\,x\,\sqrt{6}}\right| = \left|\frac{\cancel{16}^{8}}{\cancel{2}\sqrt{5}\,x\,\sqrt{6}}\right| = \frac{8}{\sqrt{30}}$

$\Rightarrow \boldsymbol{\theta = \cos^{-1}\left(\frac{8}{\sqrt{30}}\right)}$

33. S={ MFS, MSF,FMS, FSM ,SMF, SFM}

E: {MFS, FMS, SMF, SFM}

F: {MFS, SFM} and E∩F= { MFS,SFM}

P(E/F)= $\frac{P(E\cap F)}{P(F)} = \frac{2/8}{2/8} = 1$.

PART - C

34. Let $\mathbb{Z} = \{0, \pm1, \pm2, \ldots\ldots\}$ and $R = \{(x, y): x - y \text{ is an integer}\}$

Reflexive: $\forall x \in \mathbb{Z}, x - x = 0$ is an integer$\Rightarrow (x, x) \in R. \Rightarrow$ R is reflexive.

Symmetric: Let $(x, y) \in R \Rightarrow x - y$ is an integer

$\Rightarrow y - x$ is also an integer $\Rightarrow (y, x) \in R \Rightarrow$ R is symmetric

Transitive: Let $(x, y) \in R$ and$(y, z) \in R. \Rightarrow x - y$ & $y - z$ are integers

$\Rightarrow (x - y) + (y - z) = x - z$ is an integer $\Rightarrow (x, z) \in R \Rightarrow$ R is transitive.

35. $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\left[\frac{\cos x\left(1 - \frac{\sin x}{\cos x}\right)}{\cos x\left(1 + \frac{\sin x}{\cos x}\right)}\right]$

$= \tan^{-1}\left[\frac{1 - \tan x}{1 + \tan x}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4}\right) - x\right] = \frac{\pi}{4} - x.$

36. Let $B = A + A'$,then $B' = (A + A')' = (A' + (A')'$ $(\because (A + B)' = A' + B')$

$= A^1 + A = B$ $(\because (A')' = A)$

Therefore $B = A + A'$ is a symmetric matrix

Now let $C = A - A'$,then $C' = (A - A')' = (A' - (A')'$

$(\because (A - B)' = A' - B')$

$= A^1 - A = -(A - A') = -C$

Therefore $C = A - A'$ is a skew symmetric matrix.

37. The given system of equations can be written in form AX=B, where

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Here $|A| = 3 - 4 = -1 \neq 0.$

$\therefore$ A is non-singular and has a unique solution.

$\therefore$ System of equations is consistent.

38. $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(1 + \cos\theta)$ and $\frac{dy}{d\theta} = a(\theta + \sin\theta) = a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1+\cos\theta)}$

$$\Rightarrow \frac{dy}{dx} = \frac{2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2a\cos^2\frac{\theta}{2}} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}.$$

39. $y = (\log x)^{\log x}$ Taking logarithm on both sides

$\log y = \log(\log x)^{\log x} \Rightarrow \log y = \log x \log(\log x)$

Differentiating w.r.t x on both sides

$$\frac{1}{y}\frac{dy}{dx} = \log x \frac{d}{dx}(\log(\log x)) + \log(\log x)\frac{d}{dx}(\log x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\log x}{x\log x} + \frac{\log(\log x)}{x}$$

$$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{\log(\log x)}{x}\right] = (\log x)^{\log x}\left[\frac{1}{x} + \frac{\log(\log x)}{x}\right].$$

40. Given $f(x) = x^3 - 5x^2 - 3x$ $x \in [1, 3]$ which is a polynomial function.

Since a polynomial function is continuous and derivable at all $x \in R$.

1) f(x) is continuous on [1, 3]

2) f(x) is derivable on (1, 3) and $f'(x) = 3x^2 - 10x - 3$

Therefore $\exists$ at least one real $c \in (1,3)$,such that

$$f'(c) = \frac{f(3)-f(1)}{3-1} = \frac{-27+7}{2} = -\frac{20}{2} = -10$$

$$\Rightarrow 3c^2 - 10c - 3 = -10 \Rightarrow 3c^2 - 10c + 7 = 0$$

$$\Rightarrow (c-1)(3c-7) = 0 \Rightarrow c = 1 \notin (1,3) \text{ or } c = \frac{7}{3} \in (1,3).$$

Hence the mean theorem satisfied for given function in the given interval.

41. Given, $2y + x^2 = 3$. Differentiating w.r.t. x.

$$2\frac{dy}{dx} + 2x = 0 \Rightarrow \frac{dy}{dx} = \frac{-2x}{2} = -x$$

$\therefore$ Slope of the tangent $m = \left.\frac{dy}{dx}\right|_{(1,1).} = -(1) = -1$ and Slope of normal $m' = 1$.

$\Rightarrow$ Slope of normal $m' = 1$ and point is $(1, 1)$.

$\therefore$ Equation of normal, $y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 1) \Rightarrow x - y = 0.$

42. Given integrand is an improper fraction.

$$x^2 - 5x + 6)\ x^2 + 1\ (1$$
$$\underline{x^2 - 5x + 6}$$
$$5x - 5$$

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6}$$

Let $\frac{5x-5}{x^2-5x+6} = \frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$ $\Rightarrow 5x - 5 = A(x-3) + B(x-2)$

Put $x = 2$, we get $A = -5$, Put $x = 3$, we get $B = 10$

$$\therefore I = \int \frac{x^2+1}{x^2-5x+6}\,dx = \int\left(1 - \frac{5}{x-2} + \frac{10}{x-3}\right)dx = x - 5\log|x-2| + 10\log|x-3| + c$$

43. Let $\frac{x}{(x+1)\,(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

Put $x = -1$, in equation (1), we get $-1 = A(-1+2)\ \therefore\ A = -1$

Put $x = -2$, in equation (1), we get $-2 = B(-2+1)\ \therefore\ B = 2$

$$\therefore I = \int \frac{x}{(x+1)\,(x+2)}\,dx = \int\left(\frac{-1}{x+1} + \frac{2}{x+2}\right)dx = -\log|x+1| + 2\log|x+2| + C$$

$$= \log\frac{(x+2)^2}{|x+1|} + C$$

44. $I = \int x\tan^{-1}x\,dx = \int(\tan^{-1}x)x\,dx = (\tan^{-1}x)\frac{x^2}{2} - \int\frac{1}{1+x^2}\frac{x^2}{2}\,dx$

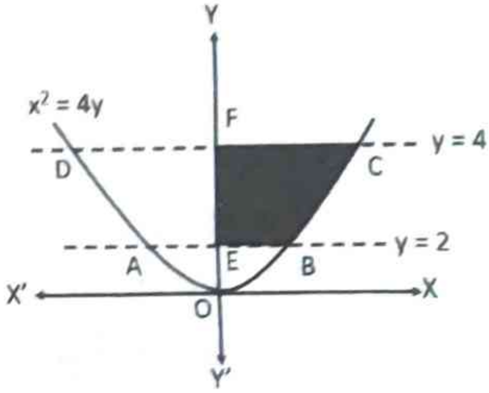
$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\frac{1+x^2-1}{1+x^2}\,dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left(x - \tan^{-1}x\right) + C$$

45. Equation of curve (parabola) is $x^2 = 4y \quad \Rightarrow x = 2\sqrt{y}$

Required area

$$= \left|\int_2^4 x\,dy\right| = \left|\int_2^4 2\sqrt{y}\,dy\right| = 2\left|\int_2^4 y^{\frac{1}{2}}dy\right|$$



$$= 2\left|\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4\right| \qquad = 2\left|\left[\left(\frac{4^{\frac{3}{2}}}{\frac{3}{2}}\right) - \left(\frac{2^{\frac{3}{2}}}{\frac{3}{2}}\right)\right]\right|$$

$$= 2\frac{2}{3}\left|\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]\right|$$

$$= \frac{4}{3}\left|\left[(2)^3 - (8)^{\frac{1}{2}}\right]\right| = \frac{4}{3}\left|\left[8 - 2\sqrt{2}\right]\right| = \frac{8}{3}\left(4 - \sqrt{2}\right) \text{ sq units.}$$

46. We have $y = a\sin(x+b)$ Diff.w.r.t.x

$\frac{dy}{dx} = a\cos(x+b)$ and $\frac{d^2y}{dx^2} = -a\sin(x+b)$

$\frac{d^2y}{dx^2} = -y$ or $\frac{d^2y}{dx^2} + y = 0.$

47. Given DE is $\frac{dy}{dx} + y = 1 \Rightarrow \frac{dy}{dx} = 1 - y \Rightarrow \frac{dy}{1-y} = \frac{dx}{1}$

Integrating w r t x on both sides we get $\int \frac{dy}{1-y} = \int dx$

$-\log|1-y| = x + C$ or $\log|1-y| + x = C.$

48. $LHS = [\vec{a}+\vec{b}\;\; \vec{b}+\vec{c}\;\; \vec{c}+\vec{a}] = (\vec{a}+\vec{b})\cdot\left((\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\right)$

$= (\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{c} + \vec{c}\times\vec{a}) = (\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{a}\cdot(\vec{b}\times\vec{a}) + \vec{a}\cdot(\vec{c}\times\vec{a}) + \vec{b}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{b}\times\vec{a}) + \vec{b}\cdot(\vec{c}\times\vec{a})$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{c}\times\vec{a}) = [\vec{a}\vec{b}\vec{c}] + [\vec{b}\vec{c}\vec{a}] = 2[\vec{a}\vec{b}\vec{c}] = RHS.$

49. Let O be the origin.

Then $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$

Let P be a point on AB such that

$\frac{AP}{PB} = \frac{m}{n}$. Then $nAP = mPB \Rightarrow n\overrightarrow{AP} = m\overrightarrow{PB}$

$\Rightarrow n(\overrightarrow{OP} - \overrightarrow{OA}) = m(\overrightarrow{OB} - \overrightarrow{OP}) \quad \therefore (m+n)\overrightarrow{OP} = m\overrightarrow{OB} + n\overrightarrow{OA}$

$\Rightarrow \vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$.

50. Let $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ ...(1)

$\vec{r} = (3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ ...(2)

(1) & (2) is a parallel lines with $\vec{a_1} = (1, 2, -4)$, $\vec{a_2} = (3, 3, -5)$ and

$\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k} = (2, 3, 6)$

$|\vec{b}| = = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

and $(\vec{a_2} - \vec{a_1}) = (3, 3, -5) - (1, 2, -4) = (2, 1, -1)$

$$\vec{b} \times (\vec{a_2} - \vec{a_1}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix} = \hat{\imath}(-3 - 6) - \hat{\jmath}(-2 - 12) + \hat{k}(2 - 6)$$

$$= -9\hat{\imath} + 14\hat{\jmath} - 4\hat{k} = (-9, 14, -4)$$

and $|\vec{b} \times (\vec{a_2} - \vec{a_1})| = \sqrt{(-9)^2 + 14^2 + (-4)^2} = \sqrt{81 + 196 + 16} = \sqrt{293}$

$\therefore$ shortest distance d = $\left|\frac{|\vec{b} \times (\vec{a_2} - \vec{a_1})|}{|\vec{b}|}\right| = \left|\frac{\sqrt{293}}{7}\right| = \frac{\sqrt{293}}{7}$

51. Let E_1 = man speaks the truth and E_1 = man lies

A= six on the die

We need to find the probability that it is actually a six, if the man reports that it a six i.e $P(E_1/A)$

$P(E_1) = \frac{3}{4}$ *and* $P(A/E_1) = \frac{1}{6}$ and $P(E_2) = \frac{1}{4}$ *and* $P(A/E_2) = \frac{5}{6}$

$$\therefore P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)} = \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}} = \frac{3}{8}.$$

PART – D

52. Let $f : R \to R$ defined by $f(x) = 3 - 4x \ \forall x \in R$

Let $x_1, x_2 \in R, f(x_1) = f(x_2)$

$\Rightarrow 3 - 4x_1 = 3 - 4x_2 \ \Rightarrow -4x_1 = -4x_2$

$\Rightarrow x_1 = x_2 \ \therefore f$ is one-one.

Let any $y \in R$ (co domain) and $f(x) = y$

$\Rightarrow 3 - 4x = y \Rightarrow 4x = 3 - y \Rightarrow x = \dfrac{3-y}{4}$

$\Rightarrow$ corresponding to every $y \in R$ there exists $x = \dfrac{3-y}{4} \in R$ such that $f(x) = y$.

$\therefore f$ is onto.

Hence f is a bijective function.

53. Given $f : \mathrm{N} \to \mathrm{Y}$ as $f(n) = n^2 = \mathbf{y}$, for some $n \in \mathrm{N}$.

$$\Rightarrow n^2 = y . \Rightarrow \mathrm{n} = \sqrt{y}.$$

$\therefore \ \exists$ *a new* function $g : \mathrm{Y} \to \mathrm{N}$, defined by $g(y) = \sqrt{y} = \mathrm{n}$.

Now, $gof(n) = g(f(n)) = g(n^2) = \sqrt{n^2} = n = I_N$

and $fog(y) = f(g(y)) = f(\sqrt{y}) = (\sqrt{y})^2 = y = I_Y$,

$\Rightarrow gof = I_N$ and $fog = I_Y$.

Hence, f is invertible with $f^{-1} = g$. $\therefore f^{-1}(y) = g(y) = \sqrt{y}$.

54. Let $A + B = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix}$

$$(A+B)C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0-14+24 \\ -10+0+30 \\ 16+12+0 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$

$$\text{and } AC=\begin{bmatrix}0&6&7\\-6&0&8\\7&-8&0\end{bmatrix}\begin{bmatrix}2\\-2\\3\end{bmatrix}=\begin{bmatrix}0-12+21\\-12+0+24\\14+16+0\end{bmatrix}=\begin{bmatrix}9\\12\\30\end{bmatrix}$$

$$BC=\begin{bmatrix}0&1&1\\1&0&2\\1&2&0\end{bmatrix}\begin{bmatrix}2\\-2\\3\end{bmatrix}=\begin{bmatrix}0-2+3\\2+0+6\\2-4+0\end{bmatrix}=\begin{bmatrix}1\\8\\-2\end{bmatrix}$$

$$AC+BC=\begin{bmatrix}9\\12\\30\end{bmatrix}+\begin{bmatrix}1\\8\\-2\end{bmatrix}=\begin{bmatrix}10\\20\\28\end{bmatrix}$$

Clearly (A+B)C = AC + BC.

55. The given system of equations can be written in form AX=B, where

$$A=\begin{bmatrix}1&-1&2\\3&4&-5\\2&-1&3\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix}\text{ and } B=\begin{bmatrix}7\\-5\\12\end{bmatrix}$$

$$|A|=\begin{vmatrix}1&-1&2\\3&4&-5\\2&-1&3\end{vmatrix}=1(12\text{-}5)\text{-}(\text{-}1)(9+10)+2(\text{-}3\text{-}8)=7+19\text{-}22=4\neq 0$$

and $X=A^{-1}B=\left(\frac{\text{adj } A}{|A|}\right)B$

$$\therefore \text{adj } A=\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}$$

$$X=A^{-1}B==\frac{1}{4}\begin{bmatrix}7&1&-3\\-19&-1&11\\-11&-1&7\end{bmatrix}\begin{bmatrix}7\\-5\\12\end{bmatrix}=\frac{1}{4}\begin{bmatrix}49-5-36\\-133+5+132\\-77+5+84\end{bmatrix}=\frac{1}{4}\begin{bmatrix}8\\4\\12\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}\Rightarrow x=2,\quad y=1,\quad z=3.$$

56. $y=3\cos(logx)+4\sin(logx)$

$$y_1=-\frac{3\sin(logx)}{x}+\frac{4\cos(logx)}{x}$$

$$\Rightarrow xy_1=-3\sin(logx)+4\cos(logx)$$

Again differentiating on both sides we get

$\Rightarrow xy_2 + (1)y_1 = -\frac{3\cos(logx)}{x} - \frac{4sin(logx)}{x}$

$\Rightarrow x^2y_2 + xy_1 = -[3\cos(logx) + 4\sin(logx)]$

$\therefore x^2y_2 + xy_1 = -y \Rightarrow x^2y_2 + xy_1 + y = 0.$

57. Given $\frac{dx}{dt} = -3$ cm/min $\frac{dy}{dt} = 2$ cm/min

$x = 10cmy = 6cm$

(i) The perimeter of the rectangle, $P = 2x + 2y$

$\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$

$= 2(-3) + 2(2)$

$= -6 + 4 = -2$ cm/min

$\therefore$ the perimeter is decreasing at the rate 2cm/min

(ii) The area of the rectangle $A = xy$

$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} = 10(2) + 6(-3)$

$= 20 - 18 = 2cm^2/min$

$\therefore$ Area of rectangle increasing at the rate of $2cm^2/min$.

58. Put $x = a\sec\theta$, then $dx = a\sec\theta\tan\theta\, d\theta$

$\therefore I = \int \frac{dx}{\sqrt{x^2-a^2}} = \int \frac{a sec\theta tan\theta d\theta}{\sqrt{a^2sec^2\theta - a^2}} = \int sec\theta d\theta = \log|sec\theta + tan\theta| + C_1$

$= \log\left|\frac{x}{a} + \sqrt{\frac{x^2}{a^2} - 1}\right| + C_1 = \log|x + \sqrt{x^2 - a^2}| - \log|a| + C_1$

$= \log|x + \sqrt{x^2 - a^2}| + C, where\ C = C_1 - \log|a|$

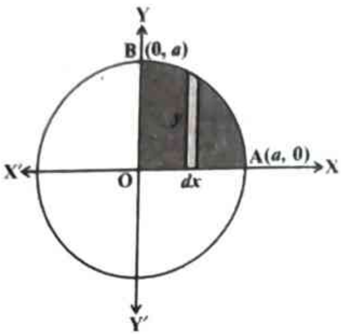
$\int \frac{dx}{\sqrt{x^2+6x+3^2-3^2-7}} = \int \frac{dx}{\sqrt{(x+3)^2-16}}$

$= log\left|(x+3) + \sqrt{(x+3)^2 - 16}\right| + C.$

59. The whole area enclosed by the given circle

= 4 (area of the region AOBA bounded by the curve, x-axis and the ordinates x = 0 and x = a)

$x^2 + y^2 = a^2 \Rightarrow y = \sqrt{a^2 - x^2}$



Required area $= 4\int_0^a y dx = 4\int_0^a \sqrt{a^2 - x^2} dx$

$$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left[\left(\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}1\right) - 0\right]$$

$$= 4\left(\frac{a^2}{2}\right)\left(\frac{\pi}{2}\right) = \pi a^2$$

60. Given $\frac{dy}{dx}$ + secx y = tanx

P = Secx Q = tanx

IF = $e^{\int secxdx} = e^{\log(secx+tanx)}$ = Secx + tanx

Solution is given by

y (secx+tanx) = ∫ tanx(secx + tanx)dx

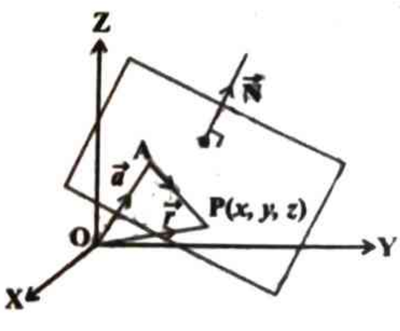
$\Rightarrow y(secx + tanx) = \int (secxtanx + tan^2x)\, dx$

$\Rightarrow y(secx + tanx) = \int (secxtanx + sec^2x - 1)\, d$

$\Rightarrow y(secx + tanx) = secx + tanx - x + C$

$\Rightarrow y = 1 - x + \frac{C}{secx + tanx}$

61. Let a plane passing through a point $A(x_1, y_1, z_1)$ with the position vector $\vec{a}$ and is perpendicular to $\vec{N}$ *with drs* A, B and C. and $\vec{r}$ be the position vector of any point P(x, y, z) on the plane.



$\therefore \overrightarrow{AP} \perp \vec{N} \Rightarrow \overrightarrow{AP} \cdot \vec{N} = 0$

$\Rightarrow (\overrightarrow{OP} - \overrightarrow{OA}) \cdot \vec{N} = 0$

$(\vec{r} - \vec{a}).\vec{N} = 0$ is the required vector equation of the plane.

Cartesian :Let $\therefore \overrightarrow{OA} = \vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{N} = A\hat{\imath} + B\hat{\jmath} + C\hat{k}$

Let $\vec{r} = \overrightarrow{OP} = x\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$

Equation of the place is,

$$((x - x_1)\,\hat{\imath} + (y - y_1)\,\hat{\jmath} + (z - z_1)\,\hat{\jmath}\,)\cdot(A\hat{\imath} + B\hat{\jmath} + C\hat{k})$$

$$\Rightarrow A(x - x_1) + B(y - y_1) + C(z - z_1) = 0.$$

62. S = {(1,1)(1,2)............(1,6), (2,1)(2,2) ....(2,6),.............(6,6)}

A = odd number on the first throw'

= {(1,1),(1,2)....(1,6),(3,1),(3,2)....(5,1),(5,2)...(5,6)}

$P(A) = \frac{18}{36} = \frac{1}{2}$

B = odd number on the second throw'

= {(1,1),(1,3),(1,5),(2,1),(2,3).....(6,5)}

$P(B) = \frac{18}{36} = \frac{1}{2}$.

$\therefore A \cap B = \{(1,1), (1,3) \ldots . (5,5)\}$

$P(A \cap B) = \frac{9}{36} = \frac{1}{4}$,

Now $P(A)P(B) = \frac{1}{2} . \frac{1}{2} = \frac{1}{4} = P(A \cap B)$

$\therefore$ *A and B* are independent of events.

63. Here $n = 5$

$$p = P(spade\,cards) = \frac{13}{52} = \frac{1}{4}; q = 1 - p = \frac{3}{4}$$

Where $P(X = x) = {}^nC_x q^{n-x} p^x = {}^5C_x \left(\frac{3}{4}\right)^{5-x} \left(\frac{1}{4}\right)^x = {}^5C_x \frac{(3)^{5-x}}{(4)^5}$

i) $P(X = 5) = {}^5C_5 \frac{(3)^0}{(4)^5} = \frac{1}{(4)^5} = \frac{1}{1024}$

ii) $P(X=3)={}^5C_3\dfrac{(3)^3}{(4)^5}=10\times\dfrac{(3)^2}{(4)^5}=\dfrac{90}{1024}$

iii) $P(X=0)={}^5C_0\dfrac{(3)^5}{(4)^5}=\dfrac{(3)^5}{(4)^5}=\dfrac{243}{1024}$.

PART - E

64(a). Region represented by $x+3y\le 60$.Consider the equation $x+3y=60$

Put $x=0\Rightarrow y=20$ $\therefore A(0,20)$ and Put $y=0\Rightarrow x=60$ $\therefore B(60,0)$

Putting $x=0$ and $y=0$, we get $0+3(0)\le 60$, is true.

$\therefore$ O (0, 0) lies in the region $x+3y\le 60\cdot$ So, the region containing the origin.

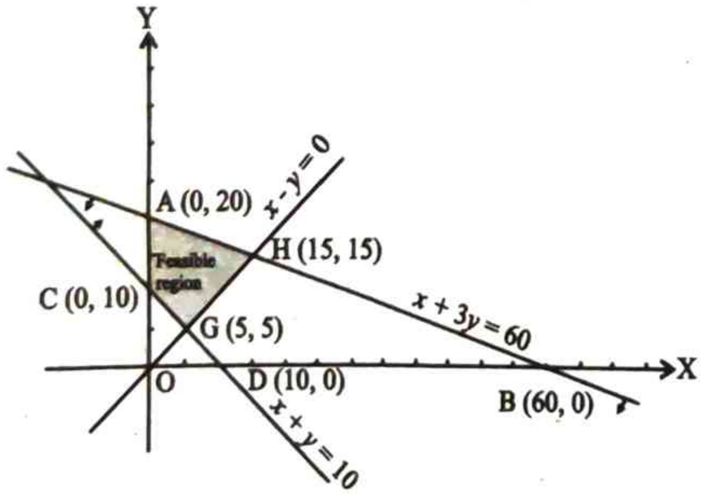
Region represented by $x+y\ge 10\cdot$Consider the equation $x+y=10$.

Put $x=0\Rightarrow y=10$ $\therefore C(0,10)$ and Put $y=0\Rightarrow x=10$ $\therefore D(10,0)$

Now, $x=0,\ y=0\Rightarrow 0+0\ge 10$, is not true. $\therefore$ O (0, 0) does not lie in the region .

Region represented by $x\le y$ i.e., $x-y\le 0$ .Consider $x=y$ i.e., $x-y=0$

Clearly P (0, 1) satisfies $x-y\le 0\cdot\therefore$ (0, 1) lies in the region $x-y\le 0\cdot$



Thus, the feasible region is ACGH, as shown in the figure.

Sl. No.	Corner points	Corresponding value of $z = 3x + 9y$
	C (0, 10)	90
	G (5, 5)	60 ← Minimum
	H (15, 15)	180 Maximum
	A (0, 20)	180 Maximum

∴ The minimum value of Z is 60 at (5,5) and maximum value of Z is 180.

At all points on the line segment joining the points (15, 15) and (0, 20).

64(b). LHS = $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + bc & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix}$

Taking out a from C_1, b fromC_2 and c from C_3

$= abc \begin{vmatrix} a & c & a + c \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$ Applying $C_1 \to C_1 + C_2 - C_3$

$= abc \begin{vmatrix} 0 & c & a + c \\ 2b & b & a \\ 2b & b + c & c \end{vmatrix}$ Applying $R_2 \to R_2 - R_3$

$= abc \begin{vmatrix} 0 & c & a + c \\ 0 & -c & a - c \\ 2b & b + c & c \end{vmatrix}$ = abc [2b (ac-c^2+ac+c^2)]

= abc[2b(2ac)] = $4a^2b^2c^2$ = RHS.

65(a). We have $I = \int_0^{2a} f(x)dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$ ...(1)

Consider $\int_a^{2a} f(x)\,dx$ Put $2a - x = t \Rightarrow dx = -dt$,

When $x = a,\ t = a$ When $x = 2a,\ t = 0$

$$\therefore \int_a^{2a} f(x)\,dx = \int_a^0 f(2a-t)\,(-dt) = \int_0^a f(2a-t)\,dt = \int_0^a f(2a-x)\,dx$$

$\therefore$ (1) reduces to I = $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

If $f(2a-x) = f(x)$ then I = $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(x)\,dx = 2\int_0^a f(x)\,dx$

If $f(2a-x) = -f(x)$ then I = $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(x)\,dx = 0$

$\int_0^{2\pi} \mathbf{cos^5 x dx} = 2\int_0^{\pi} cos^5 x\,dx$ (*since* $cos^5(2\pi - x) = cos^5 x$)

$\therefore 2\int_0^{\pi} cos^5 x\,dx = 2(0) = 0$ (since $cos^5(\pi - x) = -cos^5 x$)

65b). Given f(x) is continuous at x = 2

$\Rightarrow \lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$

$\Rightarrow \lim\limits_{x\to 2^-} 5 = \lim\limits_{x\to 2^+} (ax+b)$

$\Rightarrow 5 = 2a + b \Rightarrow 2a + b = 5$ ....(1)

Given f(x) is continuous at x = 10

$\Rightarrow \lim\limits_{x\to 10^-} f(x) = \lim\limits_{x\to 10^+} f(x)$

$\Rightarrow \lim\limits_{x\to 10^-} (ax+b) = \lim\limits_{x\to 10^+} 21$

$\Rightarrow 10a + b = 21$ ....(2)

eq(1) –eq(2)

$\Rightarrow 2a + b - (10a + b) = 5 - 21$

$\Rightarrow -8a = -16 \Rightarrow a = \frac{16}{8} \Rightarrow a = 2$

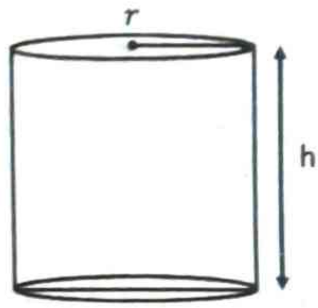
From eq(1) $\Rightarrow 2\,a + b = 5 \Rightarrow 2(2) + b = 5 \Rightarrow b = 5 - 4$

$\Rightarrow$ b = 1.

66(a). Let r be the base radius and h be the height of the right circular cylinder.

Given that surface area is fixed $A = 2\pi r^2 + 2\pi rh \Rightarrow h = \frac{A-2\pi r^2}{2\pi r}$

Volume of the cylinder $V = \pi r^2 h = \pi r^2\left(\frac{A-2\pi r^2}{2\pi r}\right)$



$V = \frac{r}{2}(A - 2\pi r^2) \Rightarrow \frac{dV}{dr} = \frac{1}{2}(A - 6\pi r^2)$

If $\frac{dV}{dr} = 0 \Rightarrow A = 6\pi r^2$

And $\frac{d^2V}{dr^2} = -6\pi r < 0$

$\Rightarrow$ volume of the cylinder is maximum when $A = 6\pi r^2$

$$\therefore h = \frac{6\pi r^2 - 2\pi r^2}{2\pi r} = \frac{6\pi r^2}{2\pi r} = 2r.$$

66(b). We have $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

LHS $= A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O =$ RHS

Now $A^2 - 4A + I = O \Rightarrow AA(A^{-1}) - 4AA^{-1} + IA^{-1} = O.A^{-1}$

$\Rightarrow A(AA^{-1}) - 4I + A^{-1} = O \quad \Rightarrow AI - 4I + A^{-1} = O$

$\Rightarrow A^{-1} = 4I - A \Rightarrow A^{-1} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

* * *

MODEL QUESTION PAPER-2

MATHEMATICS (35)

TIME: 3 Hours 15 Minutes **MAX. MARKS: 100**

Instructions:

i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii) Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer any TEN questions: **$10 \times 1 = 10$**

1. Find the number of all one-one functions from set A = {1, 2, 3} to itself.

2. Let * be the binary operation on N given by a * b = L.C.M. of a and b. Find 5 *7.

3. Write the principle branch value of $\cot^{-1} x$

4. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

5. Find the number of all possible matrices of order 3×3 with each entry 0 or 1.

6. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then find values of x.

7. Find $\frac{dy}{dx}$, if $y = e^{-x}$.

8. Find the derivative of the function $\cos(\sqrt{x})$ with respect to x.

9. Find: $\int \sec x(\sec x + \tan x)dx$

10. Find: $\int (e^x - x^e + e^e)\, dx$

11. Define collinear vectors.

12. For what value of λ, the vectors $\vec{a} = 2\hat{\imath} - 3\lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ are perpendicular to each other ?

13. Find the direction ratio's of the line $\frac{x-1}{2} = 3y = \frac{2z+3}{4}$

14. Define a feasible region of a LPP.

15. If P(A)= 3/5 and P(B)= 1/5 find P(A/B) if A and B are independent vents.

PART – B

Answer any TEN questions: **$10 \times 2=20$**

16. If $f: R \rightarrow R$ be given $f(x)=(3-x^3)^{\frac{1}{3}}$. Find fof(x).

17. Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

18. Prove that $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$.

19. Find Y, if $X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X-Y=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

20. Find equation of line joining (1, 2) and (3, 6) using determinants.

21. Differentiate sin^2x with respect to e^{cosx}.

22. If $y + siny = cosx$ then find $\frac{dy}{dx}$.

23. Differentiate $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ with respect to x.

24. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ isstrictly increasing and decreasing.

25. Find $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$

26. Find $\int \frac{x-1}{\sqrt{x^2-1}}dx$

27. Evaluate $\int_0^{2/3} \frac{dx}{4+9x^2}$

28. Find the order and degree of the differential equation $\frac{d^3y}{dx^3} - \sin\left(\frac{d^4y}{dx^4}\right) = 0$.

29. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

30. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

31. Find the angle between the planes whose vector equations are $\vec{r}.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r}.(3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$.

32. Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$.

33. The random variable X has probability distribution $P(X)$ of the following form.

$$P(X) = \begin{cases} k & if \quad x = 0 \\ 2k & if \quad x = 1 \\ 3k & if \quad x = 2 \\ 0 & otherwise \end{cases}$$

a) Determine value of K and b) Find $P(X < 2)$.

PART – C

Answer any TEN questions: **$10 \times 3 = 30$**

34. Show that the relation R in the set A = $\{x \in Z : 0 \le x \le 12\}$, given by R = $\{(a, b) : |a - b|$ is a multiple of 4$\}$ is an equivalence relation.

35. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

36. Express $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as sum of symmetric and skew symmetric matrix.

37. Examine the consistency of the system of equations $x + 3y = 5$ and $2x + 6y = 8$.

38. If $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, $y = a\sin t$ prove that $\frac{dy}{dx} = \tan t$.

39. Find $\frac{dy}{dx}$ if $y^x = x^y$.

40. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4,2]$.

41. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

42. Find $\int (x^2+1)\log x dx$.

43. Find $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$.

44. Evaluate $\int_0^5 (x+1)dx$ as the limit of sum.

45. Find the area of the region bounded by $y^2 = 9x$ and the lines x = 2, x = 4 and the x-axis in the first quadrant.

46. Form the differential equation representing the family of curves $y = ae^{3x} + be^{-2x}$ by eliminating the arbitrary constants.

47. Find the general solution of the differential equation

$y \log y \, dx - x \, dy = 0$.

48. Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.

49. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{\imath}+\hat{\jmath}+\hat{k}$, $\vec{b} = \hat{\imath}+2\hat{\jmath}+3\hat{k}$.

50. Find the vector equation of the plane passing through the intersection of the planes 3x - y + 2z – 4 = 0 and x + y + z -3 = 0 and the point (2, 2, 1).

51. Bag I contains 4 Red and 4 Black balls , Bag II contains 2 Red and 6 Black balls .One bag is selected at random and a ball is drawn is found to be Red. What is the probability that bag I is selected?

PART - D

Answer any SIX questions: **6 × 5=30**

52. State whether the function $f: R \to R$ defined by $f(x) = 1 + x^2$ is one-one, onto or bijective. Justify your answer.

53. Let f: N →Y be a function defined as $f(x) = 4x + 3$, where, Y= $\{y \in N: y = 4x + 3$ for some $x \in N\}$. Show that f is invertible. Find the inverse of f.

54. If $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, verify that $(A+B)' = A' + B'$.

55. Solve the following system of linear equations by matrix method:

$x - y + z = 4$, $2x + y - 3z = 0$ and $x + y + z = 2$.

56. If $e^y(x + 1) = 1$, prove that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

57. The volume of a cube is increasing at the rate of $8 cm^3/s$. How fast is the surface area increasing when the length of an edge is 12cm?

58. Find $\int \frac{dx}{x^2+a^2}$ and hence evaluate $\int \frac{dx}{3+2x+x^2}$

59. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

60. Find general solution of $x\frac{dy}{dx} + 2y = x^2 \log x$.

61. Derive the equation of the line in space passing through two given points both in vector and Cartesian form.

62. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize,

a) atleast once b) exactly once c) atleast twice?

63. Let A and B be independent events with $P(A) = 0.3$, $P(B) = 0.4$. Find (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A|B)$ (iv) $P(B|A)$.

PART-E

Answer any ONE question: **$1 \times 10 = 10$**

64(a). Solve LPP graphically: Minimize $z = 200x + 500y$, subject to the constraints $x + 2y \geq 10,\ 3x + 4y \leq 24,\ x \geq 0,\ y \geq 0$.

64(b). Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

65(a). Prove that $\int_a^b f(x)dx = \int_a^b (a+b-x)\,dx$ and hence

Evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx$.

65(b). Find the value of k,

If f(x) $= \dfrac{k\cos x}{\pi - 2x}$ if $x \neq \frac{\pi}{2}$

3 if $x = \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$.

66(a). Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

66(b). If A $= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

* * *

ANSWERS TO MODEL QUESTION PAPER – 02

PART-A

1. Let n(A)= 3 ∴Number of one-one functions = 3!=6.

2. 5 ∗ 7 = L.C.M. of 5 and 7 is 35.

3. Principle branch value is $(0, \pi)$.

4. $\frac{\pi}{6}$.

5. All possible matrices of order 3×3 with each entry 0 or 1 are $2^9 = 512$.

6. $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix} \Rightarrow x^2 - 36 = 36 - 36$

$$\Rightarrow x^2 = 36 \Rightarrow x = \pm 6$$

7. Given $y = e^{-x} \Rightarrow \frac{dy}{dx} = e^{-x}(-1) = -e^{-x}$.

8. $y = \cos(\sqrt{x}) \Rightarrow \frac{dy}{dx} = -\sin(\sqrt{x})\frac{d}{dx}(\sqrt{x}) = \frac{-\sin(\sqrt{x})}{2\sqrt{x}}$

9. $\int(sec^2x + secxtanx)dx$ = tanx + secx + C.

10. $\int(e^x - x^e + e^e)\, dx = e^x - \frac{x^{e+1}}{e+1} + e^e x + C$.

11. Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.

12. We know that two vectors are perpendicular $\Rightarrow \vec{a} \cdot \vec{b} = 0$

$\Rightarrow 2 \times 1 + (-3\lambda) \times 1 + 1 \times (-2) = 0 \Rightarrow 2 - 3\lambda - 2 = 0 \Rightarrow \lambda = 0$.

13. Let $\frac{x-1}{2} = 3y = \frac{2z+3}{4} \Rightarrow \frac{x-1}{2} = \frac{y}{\left(\frac{1}{3}\right)} = \frac{2(z-\left(\frac{-3}{2}\right)}{4} \Rightarrow \frac{x-1}{2} = \frac{y}{\left(\frac{1}{3}\right)} = \frac{(z-\left(\frac{-3}{2}\right)}{2}$

∴ direction ratio's are 2, $\frac{1}{3}$, 2.

14. The common region determined by all the constraints including non-negative constraints $x \geq 0$, $y \geq 0$ of an LPP is called the feasible region for the problem.

15. If A and B are independent events P(A/B)= P(A)= 3/5.

PART – B

16. $fof(x) = f(f(x)) = f\left((3-x^3)^{\frac{1}{3}}\right) = \left[3-\left((3-x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$

$= [3-3+x^3]^{\frac{1}{3}} = (x^3)^{\frac{1}{3}} = x.$

17. $\frac{\pi}{3} - \left[\frac{2\pi}{3}\right] = -\frac{\pi}{3}$

18. Substitute $\sin^{-1}(x) = \theta \Rightarrow x = \sin\theta$

$\Rightarrow x = \cos\left(\frac{\pi}{2} - \theta\right) \Rightarrow \cos^{-1}(x) = \frac{\pi}{2} - \theta \Rightarrow \cos^{-1}(x) + \theta = \frac{\pi}{2}$

$\therefore \sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}.$

19. $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ...(1) and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ...(2)

Subtracting (2) from (1) we get $2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}.$

20. $\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0 \Rightarrow 1(6-y) - 2(3-x) + 1(3y-6x) = 0$

$\Rightarrow 6-y-6+2x+3y-6 = 0$

$\Rightarrow -4x+2y = 0 \Rightarrow 2x-y=0.$

21. Let $u = sin^2x$, Differentiate with respect to x

$\Rightarrow \frac{du}{dx} = 2\sin x \cos x$

Let $v = e^{cosx}$, differentiate with respect to x

$$\Rightarrow \frac{dv}{dx} = e^{\cos x}(-\sin x) = -e^{\cos x}\sin x$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2\sin x\cos x}{-e^{\cos x}\sin x} = -\frac{2\cos x}{e^{\cos x}}$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2\sin x\cos x}{-e^{\cos x}\sin x} = -\frac{2\cos x}{e^{\cos x}}$$

22. Given, $y + \sin y = \cos x$,

Differeniate with respect to x.

$$\Rightarrow \frac{dy}{dx} + \cos y\frac{dy}{dx} = -\sin x \quad \Rightarrow \frac{dy}{dx}(1+\cos y) = -\sin x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin x}{1+\cos y}$$

23. Let $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

$$= \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2}$$

$$y = \frac{x}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}.$$

24. Given $f(x) = x^2 - 4x + 6$ $\quad \therefore f'(x) = 2x - 4$.

If $f(x)$ is strictly increasing $\Rightarrow f'(x) > 0 \Rightarrow 2x - 4 > 0 \Rightarrow 2x > 4$

$\Rightarrow x > 2$ i.e. $x \in (2, \infty)$.

And if f(x) is strictly decreasing.

$\Rightarrow f'(x) < 0 \Rightarrow 2x - 4 < 0 \Rightarrow 2x < 4$

$\Rightarrow x < 2$ i.e. $x \in (-\infty, 2)$.

25. $\int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2}dx = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}dx$

$= \log|\cos x + \sin x| + C.$

26. $\int \frac{x\,dx}{\sqrt{x^2-1}} - \int \frac{dx}{\sqrt{x^2-1}} = \frac{1}{2}\int \frac{2x\,dx}{\sqrt{x^2-1}} - \int \frac{dx}{\sqrt{x^2-1}}$

$$= \frac{1}{2}2\sqrt{x^2-1} - \log|x + \sqrt{x^2-1}| + C$$

27. $I = \frac{1}{2} \cdot \frac{1}{3} \cdot \tan^{-1} \frac{3x}{2} \Big]_0^{2/3} = \frac{1}{6}(\tan^{-1} 1 - \tan^{-1} 0)$

$= \frac{1}{6}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{24}$.

28. Order is 4 and degree is not defined.

29. The area of a parallelogram with $\vec{a}$ and $\vec{b}$ as its adjacent sides is given by $|\vec{a} \times \vec{b}|$.

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$= (1 - (-4))\hat{\imath} - (3 - 4)\hat{\jmath} + (-3 - 1)\hat{k} = 5\hat{\imath} + \hat{\jmath} - 4\hat{k}$.

$\therefore$ area of parallelogram

$= |\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + (-4)^2} = \sqrt{25 + 1 + 16} = \sqrt{42}$ sq units.

30. The projection of $\vec{a}$ on $\vec{b}$ is $= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$= \frac{(2\hat{\imath}+3\hat{\jmath}+2\hat{k}) \cdot (\hat{\imath}+2\hat{\jmath}+\hat{k})}{\sqrt{1^2+2^2+1^2}} = \frac{2+6+2}{\sqrt{6}} = \frac{10}{\sqrt{6}}$.

31. Let $\vec{r}.(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 5$ … (1). Here $\overrightarrow{n_1} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k} = (2, 2, -3)$

And $\vec{r}.(3\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) = 3$ … (2). $\overrightarrow{n_2} = 3\hat{\imath} - 3\hat{\jmath} + 5\hat{k} = (3, -3, 5)$

Now $\overrightarrow{n_1} . \overrightarrow{n_2} = 2(3) + 2(-3) + (-3)(5) = 6 - 6 - 15 = -15$

$|\overrightarrow{n_1}| = \sqrt{2^2 + 2^2 + (-3)^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$

$|\overrightarrow{n_2}| = \sqrt{3^2 + (-3)^2 + (5)^2} = \sqrt{9 + 9 + 25} = \sqrt{43}$

$\therefore \cos\theta = \left|\frac{\overrightarrow{n_1} . \overrightarrow{n_2}}{|\overrightarrow{n_1}|\,|\overrightarrow{n_2}|}\right| = \left|\frac{-15}{\sqrt{17}\sqrt{43}}\right| = \frac{15}{\sqrt{731}}$

$\therefore \boldsymbol{\theta = \cos^{-1}\left(\frac{15}{\sqrt{731}}\right)}$

32. Let $\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 4$

This can be written as $6x - 3y + 2z = 4$ and $(x_1, y_1, z_1) = (2, 5, -3)$

$$PQ = \left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right| = \left|\frac{6(2)+(-3)(5)+2(-3)-4}{\sqrt{2^2+(-1)^2+2^2}}\right|$$

$$= \left|\frac{12-15-6-4}{\sqrt{4+4+1}}\right| = \left|\frac{-13}{3}\right| = \frac{13}{3}.$$

33. a) $\sum P(X)=1 \quad \Rightarrow k+2k+3k+0=1 \quad \Rightarrow 6k=1 \Rightarrow k=\frac{1}{6}$

b) $P(X<2)=P(X=0)+P(X=1)=k+2k$

$$= \frac{1}{6}+2\frac{1}{6} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$$

PART – C

34. Here $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$R = \{(a, b) : |a-b|$ is a multiple of $4\}$

R is reflexive: $\forall\, a \in A,\ a-a=0$ and 0 is multiple of 4

$|a-a| = 0$ is multiple of 4 $\Rightarrow (a,a) \in R \, \forall a \in A$ $\therefore$ R is reflexive

R is symmetric: $(a,b) \in R \Rightarrow |a-b|$ is a multiple of 4

$\Rightarrow |-(b-a)|$ is a multiple of 4 $\quad \Rightarrow |b-a|$ is a multiple of 4

$\Rightarrow (b,a) \in R$ $\therefore$ R is symmetric

R is transitive: $(a, b) \in R$ and $(b,c) \in R$ $\quad \Rightarrow |a-b|$ and $|b-c|$ are multiples of 4

$\Rightarrow (a-b)$ and $(b-c)$ are multiples of 4

$\Rightarrow (a-b)+(b-c)$ is a multiple of 4 $\Rightarrow a-c$ is a multiple of 4

$\Rightarrow |a-c|$ is a multiple of 4 $\Rightarrow (a, c) \in R$ $\therefore$ R is transitive.

$\therefore$ R is an equivalence relation.

35. $\text{LHS} = 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left[\frac{2.\frac{1}{2}}{1-\frac{1}{4}}\right] + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left[\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\frac{1}{7}}\right] = \tan^{-1}\left[\frac{\frac{28+3}{21}}{1-\frac{4}{21}}\right]$$

$$= tan^{-1}\frac{31/21}{17/21} = tan^{-1}\frac{31}{17} = RHS.$$

36. Here $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Let $P = \frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 1+1 & 5-1 \\ -1+5 & 2+2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

Now $P = P'$ Thus $P = \frac{1}{2}(A+A')$ is a symmetric matrix.

$Q = \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 1-1 & 5+1 \\ -1-5 & 2-2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

Since $Q' = -Q$ Thus $Q = \frac{1}{2}(A-A')$ is a skew symmetric matrix.

$P+Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A$

37. The given system of equations can be written in form AX=B,

where $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ *and* $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Here $|A| = 6 - 6 = 0$

$\therefore$ A is singular

$\therefore$ adj A = $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$ and (adj A)B = $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$

$\therefore$ solution does not exist and the system of equations is inconsistent.

38. Now, $x = a\left(\cos t + log tan\frac{t}{2}\right) \Rightarrow \frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\left(\frac{t}{2}\right)}(sec^2 t/2)\frac{1}{2}\right)$

$$\Rightarrow \frac{dx}{dt} = a\left(-\sin t + \frac{1}{\frac{\sin\left(\frac{t}{2}\right)}{\cos\left(\frac{t}{2}\right)}}\frac{1}{\cos^2\left(\frac{t}{2}\right)}\frac{1}{2}\right) = a\left(-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right)$$

$$\Rightarrow \frac{dx}{dt} = a\left(-\sin t + \frac{1}{\sin t}\right) = a\left(\frac{-\sin^2 t + 1}{\sin t}\right) = a\left(\frac{\cos^2 t}{\sin t}\right)$$

Now, $y = a\sin t \quad \Rightarrow \frac{dy}{dt} = a\cos t$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)} = \cos t\frac{\sin t}{\cos^2 t} = \frac{\sin t}{\cos t} = \tan t$$

39. Let $y^x = x^y$, Taking logarithm on both sides

$logy^x = logx^y \Rightarrow \; xlogy = ylogx$

Differentiating with respect to x, on both sides, we get

$$x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x) = y\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(y)$$

$$\frac{x}{y}\frac{dy}{dx} + \log y = \frac{y}{x} + \log x\frac{dy}{dx} \Rightarrow \left(\frac{x-ylogx}{y}\right)\frac{dy}{dx} = \frac{y-xlogy}{x}$$

$$\therefore \; \frac{dy}{dx} = \frac{y(y-xlogy)}{x(x-ylogx)}.$$

40. Given $f(x) = x^2 + 2x - 8, x \in [-4,2]$.

Since a polynomial function is continuous and derivable on R.

(1) f(x) is continuous on $[-4,2]$

(2) f(x) is derivable on $[-4,2]$, and $f'(x) = 2x + 2$

Also $f(-4) = (-4)^2 + 2(-4) - 8 = 0$ and

$f(2) = 2^2 + 2 \times 2 - 8 = 0 \Rightarrow f(-4) = f(2)$

Therefore there exist at least one real number c $\in(-4,2)$ such that $f'(c) = 0$.

$\Rightarrow f'(c) = 0 \Rightarrow 2c + 2 = 0 \Rightarrow c = -1 \in (-4,2)$

$\therefore$ Rolle's theorem is verified.

41. Given radius of the sphere $r = 9cm$ and Error in radius

$\Delta r = 0.03cm$

Volume of the sphere, $V = \frac{4}{3}\pi r^3$ ∴

$\frac{dV}{dr} = \frac{4}{3}\pi \times 3r^2 = 4\pi r^2 = 4\pi(9)^2 = 4\pi \times 81 = 324\pi$

Approximate error in calculating its volume is

$$\Delta V = \frac{dV}{dr}\Delta r = 324\pi \times 0.03 = 9.72\pi \text{cm}^3.$$

42. $I = \int (x^2+1)\log x\, dx = \int (\log x)(x^2+1)dx$

$$= (\log x)\left(\frac{x^3}{3}+x\right) - \int \frac{1}{x}\left(\frac{x^3}{3}+x\right)dx = \left(\frac{x^3}{3}+x\right)\log x - \int\left(\frac{x^2}{3}+1\right)dx$$

$$= \left(\frac{x^3}{3}+x\right)\log x - \frac{x^3}{9} - x + C$$

43. $I = \int e^x \left(\dfrac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx = \int e^x\left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right)dx \; = e^x \tan\frac{x}{2} + c$

44. $I = \int_0^5 (x+1)dx = \int_0^5 f(x)\, dx$, where $f(x) = x+1$

$= \lim_{h\to 0}[f(0) + f(0+h) + f(0+2h) + \cdots\ldots + f\{0+(n-1)h\}]$

where $nh = (5 - 0) = 5$.

$\therefore I = \lim_{h\to 0} h\{1+h+1+2h+1+\cdots+(n-1)h+1\} = \lim_{h\to 0} h\{n + h(1+2+\cdots+(n-1))\}$

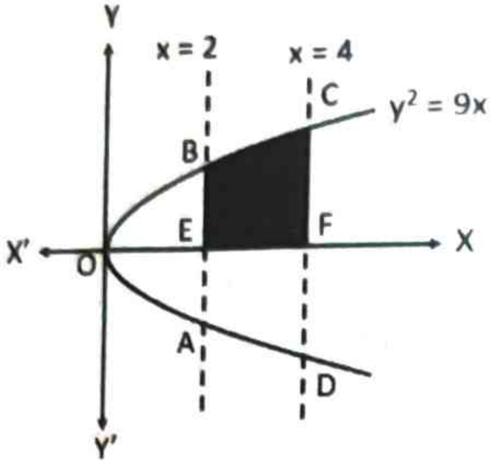
$$= \lim_{h\to 0}\left\{nh + \frac{h^2(n-1)n}{2}\right\} = \lim_{h\to 0}\left\{nh + \frac{nh(nh-h)}{2}\right\}$$

$$= \lim_{h\to 0}\left\{5 + \frac{5(5-h)}{2}\right\} = 5 + \frac{25}{2} = \frac{35}{2}$$

45. Equation of the curve (rightward parabola)

is $y^2 = 9x \Rightarrow y = 3\sqrt{x}$

∴ Required area



$$=\left|\int_2^4 y\,dx\right| = \left|\int_2^4 3\sqrt{x}\,dx\right| = 3\left|\int_2^4 x^{\frac{1}{2}}dx\right|$$

$$= 3\left|\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4\right| = 3\left|\left[\left(\frac{4^{\frac{3}{2}}}{\frac{3}{2}}\right)-\left(\frac{2^{\frac{3}{2}}}{\frac{3}{2}}\right)\right]\right|$$

$$=\frac{3.2}{3}\left|\left[(4)^{\frac{3}{2}}-(2)^{\frac{3}{2}}\right]\right|$$

$$=2\left|\left[(2)^3-(8)^{\frac{1}{2}}\right]\right| = 2\left|\left[8-2\sqrt{2}\right]\right| = 4\left(4-\sqrt{2}\right) \text{ squnits}$$

46. We have $y = ae^{3x} + be^{-2x}$ Diff.w.r.t.x

$\frac{dy}{dx} = 3ae^{3x} - 2be^{-2x}$ Diff .again w.r.t.x we get

$$\frac{d^2y}{dx^2} = 9ae^{3x} + 4be^{-2x}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 9ae^{3x} + 4be^{-2x} - 3ae^{3x} + 2be^{-2x} = 6ae^{3x} + 6be^{-2x} = 6y$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

47. Given DE is $y\log y\,dx - x\,dy = 0 \Rightarrow \frac{dx}{x} = \frac{dy}{y\log y}$

Integrating on both sides we get $\int\frac{dx}{x} = \int\frac{dy}{y\log y}$

$\log|x| = \log|\log y| + \log C.$

48. Given position vectors are $\overline{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$, $\overline{OB} = -\hat{j} - \hat{k}$, $\overline{OC} = 3\hat{i} + 9\hat{j} + 4\hat{k}$ and

$\overline{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$

$\overline{AB} = \overline{OB} - \overline{OA} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = -\hat{i} + 4\hat{j} + 3\hat{k}$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = -8\hat{i} - \hat{j} + 3\hat{k}$$

$$\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$$= -4(12+3) + 6(-3+24) - 2(1+32)$$

$$= -60 + 126 - 66 = 0$$

$\therefore$ $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar. Hence, given points are coplanar.

49. Let $\vec{a} + \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{a} - \vec{b} = -\hat{\jmath} - 2\hat{k}$.

Now, $\vec{c} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$.

$\therefore |\vec{c}| = \sqrt{(-2)^2 + 4^2 + (-2)^2} = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}$.

Therefore, the required unit vector is $\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{6}}\left(-\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$.

50. Let $\pi_1 = 3x - y + 2z - 4 = 0$, $\pi_2 = x + y + z - 3 = 0$) and the pt A = (2, 2, 1)

Now required equation of the plane will be of the form $\pi_1 + \lambda\pi_2 = 0$

i.e $\pi = (3x - y + 2z - 4) + \lambda(x + y + z - 3) = 0$,the plane π passes through the point (2, 2, 1),

$\Rightarrow$ $(3(2) - 2 + 2(1) - 4) + \lambda(2 + 2 + 2 - 3) = 0 \Rightarrow \lambda = \frac{-2}{3}$.

$\therefore \pi = (3x - y + 2z - 4) \frac{-2}{3} (x + y + z - 3) = 0$

$3(3x - y + 2z - 4) - 2(x + y + z - 3) = 0 \Rightarrow 7x - 5y + 4z - 6 = 0$.

51. Let A : Bag I and B ; Bag II

$$\therefore P(A) = \frac{1}{2}, P(B) = \frac{1}{2}$$

Let E:'drawn ball is red'

$$\therefore P(E \mid A) = \frac{4}{8} = \frac{1}{2}, P(E \mid B) = \frac{2}{8} = \frac{1}{4}$$

Required probability = $P(A|E) = \dfrac{P(A).P(E|A)}{P(A).P(E|A)+P(B)P(E|B)}$

$$= \frac{\frac{1}{2}.\frac{1}{2}}{\frac{1}{2}.\frac{1}{2}+\frac{1}{2}.\frac{1}{4}} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$$

PART – D

52. Let $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 1 + x^2$

Here $f(1) = 1+1 = 2$. Also $f(-1) = 1+1 = 2$

$\therefore f(1) = f(-1) \Rightarrow 1 \neq -1$

i.e., $f(x_1) = f(x_2) \Rightarrow x_1 \neq x_2$ $\therefore f$ is not one-one

Range of (Range of f contains only those real numbers greater than or equal to 1

i.e., Range of f is $[1, \infty)$ codomain of f is R

$\therefore$Range of $f \neq$ codomain of f. $\therefore f$ is not onto.

Thus, f is neither one-one nor onto.

53. Given $f : \mathbf{N} \to Y$ as $f(x) = 4x + 3 = y$.

$\Rightarrow 4x + 3 = y$. $\Rightarrow x = \frac{y-3}{4}$

$\therefore$ $\exists$ *a new* function $g : Y \to N$, defined by $g(y) = \frac{y-3}{4} = x$.

Consider $gof(x) = g(f(x))$

$$= g(4x+3) = \frac{4x+3-3}{4} = \frac{4x}{4} = x = \mathbb{I}_x.$$

Consider $(fog)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3$

$$= y - 3 + 3 = y = \mathbb{I}_y.$$

$\Rightarrow gof = \mathbb{I}_x$ and $fog = I_Y$.

Hence, f is invertible with $f^{-1} = g$. $\therefore f^{-1}(y) = g(y) = \frac{y-3}{4}$.

54. $A+B=\begin{bmatrix}3 & \sqrt{3} & 2\\4 & 2 & 0\end{bmatrix}+\begin{bmatrix}2 & -1 & 2\\1 & 2 & 4\end{bmatrix}=\begin{bmatrix}3+2 & \sqrt{3}-1 & 2+2\\4+1 & 2+2 & 0+4\end{bmatrix}=\begin{bmatrix}5 & \sqrt{3}-1 & 4\\5 & 4 & 4\end{bmatrix}$

$$(A+B)'=\begin{bmatrix}5 & 5\\\sqrt{3}-1 & 4\\4 & 4\end{bmatrix}$$

$$A'+B'=\begin{bmatrix}3 & 4\\\sqrt{3} & 2\\2 & 0\end{bmatrix}+\begin{bmatrix}2 & 1\\-1 & 2\\2 & 4\end{bmatrix}=\begin{bmatrix}3+2 & 4+1\\\sqrt{3}-1 & 2+2\\2+2 & 0+4\end{bmatrix}=\begin{bmatrix}5 & 5\\\sqrt{3}-1 & 4\\4 & 4\end{bmatrix}$$

Clearly,

$$(A+B)'=A'+B'$$

55. The given system of equations can be written in form AX=B, where

$$A=\begin{bmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\0\\2\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -1 & 1\\2 & 1 & -3\\1 & 1 & 1\end{vmatrix}=1(1+3)-(-1)(2+3)+1(2-1)=4+5+1=10\neq 0$$

and $X=A^{-1}B=\left(\frac{\text{adj }A}{|A|}\right)B$

$$\therefore\quad \text{adj }A=\begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}$$

$$\therefore X=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\-5 & 0 & 5\\1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\0\\2\end{bmatrix}=\frac{1}{10}\begin{bmatrix}16+0+4\\-20+0+10\\4+0+6\end{bmatrix}=\frac{1}{10}\begin{bmatrix}20\\-10\\10\end{bmatrix}=\begin{bmatrix}2\\-1\\1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-1\\1\end{bmatrix}\Rightarrow x=2,\quad y=-1,\quad z=1$$

56. Let $e^y(x+1)=1$. Differentiate w.r.t x on both sides

$$e^y+(x+1)e^y\frac{dy}{dx}=0\Rightarrow e^y+\frac{dy}{dx}=0$$

$\frac{dy}{dx} = -e^y$

Again differentiate w.r.t x, we get

$\frac{d^2y}{dx^2} = -e^y\frac{dy}{dx}$

$\therefore \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2.$

57. Given $\frac{dV}{dt} = 8cm^3/s \; \frac{dA}{dt} = ?\; x = 12\text{cm}$

Where V is the volume ,x is the length of edge and A is the surface area of the cube

Volume of the cube, $V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$\Rightarrow 8 = 3(12)^2\frac{dx}{dt}$

$\Rightarrow 8 = 3 \times 144 \times \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = \frac{8}{3\times144} = \frac{1}{54}\text{cm/s}$

Surface area of the cube, $A = 6x^2$

$\frac{dA}{dt} = 6 \times 2x\frac{dx}{dt} = 12 \times 12 \times \frac{1}{54} = \frac{8}{3}cm^2/s$

$\therefore$ surface area is increasing at the rate of $\frac{8}{3}cm^2/s$

58. Put $x = a\tan\theta$ then $dx = a\sec^2\theta d\theta$

$\therefore I = \int\frac{dx}{x^2+a^2} = \int\frac{a\sec^2\theta d\theta}{a^2\tan^2\theta+a^2} = \frac{1}{a}\int d\theta$

$= \frac{1}{a}\theta + C \quad = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

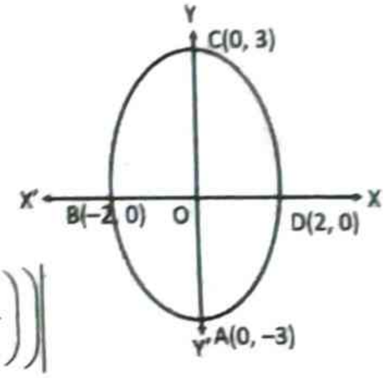
$\int\frac{dx}{x^2+2x+1+2} = \int\frac{dx}{(x+1)^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + C.$

59. Equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{9}=1 \Rightarrow \frac{y^2}{9} = 1-\frac{x^2}{4} = \frac{4-x^2}{4} \Rightarrow y^2 = \frac{9}{4}(4-x^2)$

$$\Rightarrow y=\sqrt{\frac{9}{4}\left(4-x^2\right)} \Rightarrow y=\frac{3}{2}\sqrt{\left(4-x^2\right)}$$

Required area = 4 x Area OCD of ellipse in first quadrant

$$=4\left|\int_0^2 y dx\right| = 4\left|\int_0^2 \frac{3}{2}\sqrt{\left(4-x^2\right)}dx\right| = 6\left|\int_0^2 \sqrt{\left(4-x^2\right)}dx\right|$$



$$=6\left|\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right|_0^2$$

$$=6\left|\left(\frac{2}{2}\sqrt{4-2^2}+\frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right)\right)-\left(\frac{0}{2}\sqrt{4-0^2}+\frac{4}{2}\sin^{-1}\left(\frac{0}{2}\right)\right)\right|$$

$$=6\left|\left(2\frac{\pi}{2}\right)\right| = 6\pi \text{ Sq units}$$

60. Given $\frac{dy}{dx}+\frac{2}{x}y=\frac{x^2\log x}{x}$

$P=\frac{2}{x}$ Q= xlogx

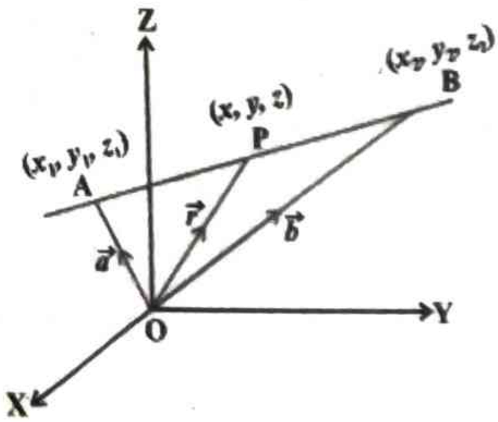
IF $= e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

Solution is given by $y(x^2) = \int x^2 x\log x\, dx + C$

$$\Rightarrow yx^2 = \int x^3 \log x\, dx + C \Rightarrow yx^2 = \log x\frac{x^4}{4} - \int \frac{1}{x}\frac{x^4}{4}\, dx + C$$

$$\Rightarrow yx^2 = \log x\frac{x^4}{4} - \frac{x^4}{16} + C.$$

61. Let $\vec{a}$ and $\vec{b}$ be the position vectors of two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, respectively .i.e $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$



Let $\vec{r}$ be the position vector of an arbitrary point P(x, y, z) ie $\overrightarrow{OP}=\vec{r}$.

Clearly A, P, B are collinear.

$\Rightarrow \overrightarrow{AP}$ and $\overrightarrow{AB}$ are collinear.

$\therefore \overrightarrow{AP} = \lambda \overrightarrow{AB}$, (where λ is any real)

$\Rightarrow \overrightarrow{OP} - \overrightarrow{OA} = \lambda(\overrightarrow{OB} - \overrightarrow{OA}) \Rightarrow \vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$.

Cartesian,

Let $\vec{r} = \overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{a} = \overrightarrow{OA} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

and $\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Equation of the line

$\Rightarrow \quad x\hat{i} + y\hat{j} + z\hat{k}$

$= \quad (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda[(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})]$

$= \quad (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda[(x_2 - x_1)\hat{i} + (y_2 - y_1\hat{j} + z_2 - z_1)\hat{k})]$

$\Rightarrow \quad x = x_1 + \lambda(x_2 - x_1)$, $y = y_1 + \lambda(y_2 - y_1)$, $z = z_1 + \lambda(z_2 - z_1)$

By eleminating λ, we get $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

62. Here $n = 50, p = \frac{1}{100}, q = \frac{99}{100}$

$$P(X = x) = {}^nC_x (q)^{n-x} (p)^x = {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^x$$

Required

a) $P(X \geq 1) = 1 - P(0) = 1 - \left(\frac{99}{100}\right)^{50}$

b) $P(X = 1) = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$

c) $P(X \geq 2) = 1 - [P(0) + P(1)]$

$$= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + 50 \times \frac{1}{100}\right] = 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$$

63. i) $P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.4 = 0.12$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= P(A) + P(B) - P(A) \cdot P(B)$$

$$= 0 \cdot 3 + 0 \cdot 4 - 0 \cdot 12 = 0 \cdot 7 - 0 \cdot 12 = 0 \cdot 58$$

iii) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot P(B)}{P(B)} = P(A) = 0 \cdot 3$

iv) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(A) \cdot P(B)}{P(A)} = P(B) = 0 \cdot 4$.

PART – E

64(a). Region represented by $x + 2y \geq 10$. Consider $x + 2y = 10$

Put $x = 0 \Rightarrow y = 5 \quad \therefore A(0, 5)$

Put $y = 0 \Rightarrow x = 10 \therefore B(10,0)$

Put $x = 0,\ y = 0 \Rightarrow 0 + 0 \geq 10$ which is not true.

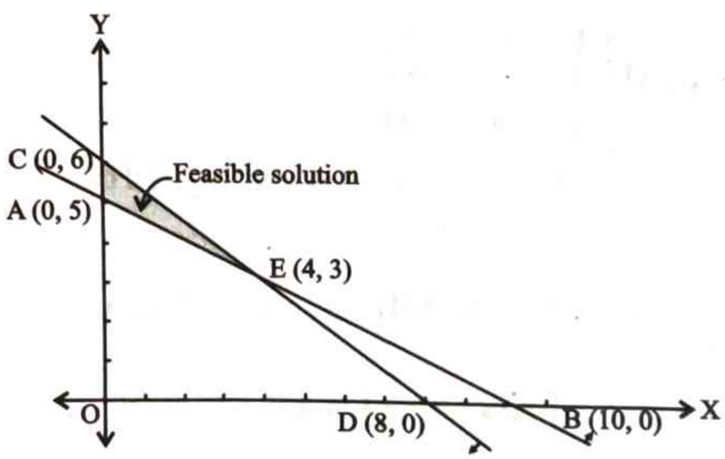
$\therefore$ Solution region does not contain the origin.

Region represented by $3x + 4y \leq 24$. Consider $3x + 4y = 24$ ...(2)

Put $x = 0 \Rightarrow y = 6 \quad \therefore C(0, 6)$. Put $y = 0 \Rightarrow x = 8 \quad \therefore D(8,0)$

Put $x = 0, y = 0 \Rightarrow 0 + 0 \leq 24$ is true.

$\therefore$ Solution region contains the origin.



Solution lies in the region ACE.

Sl. No.	Corner points	Corresponding value of $z = 200x + 500y$
	A (0, 5)	2500
	C (0, 6)	3000
	E (4, 3)	2300 ← Minimum

Hence, minimum value of z is 2300 at the point (4, 3).

64(b). LHS = $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$$= (a+b+c)^3 \begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

$$= (a+b+c)^3[1(1-0)-0+0] = (a+b+c)^3 = \text{RHS}.$$

65(a). Put $a+b-x=t \Rightarrow dx=-dt$. When $x=a, t=b$ When $x=b,\ t=a$

$$\therefore \int_a^b f(a+b-x)\,dx = \int_b^a f(t)(-dt) = \int_a^b f(t)\,dt = \int_a^b f(x)\,dx$$

$$\therefore \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx \quad I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad \ldots(1)$$

$$\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}\,dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \qquad \ldots(2)$$

(1) + (2) gives $2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \int_{\pi/6}^{\pi/3} dx = \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{6}$

65(b). Given f(x) is continuous at $x = \frac{\pi}{2}$

$\Rightarrow \lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x\to\frac{\pi}{2}} \left(\frac{k\cos x}{\pi-2x}\right) = 3$, taking $\left(\frac{\pi}{2}-\theta\right) = x \Rightarrow if\ x \to \frac{\pi}{2}$, then $\theta \to 0$

$\Rightarrow \lim_{\theta\to 0} \left(\frac{k\cos\left(\frac{\pi}{2}-\theta\right)}{\pi-\pi+2x}\right) = 3 \ \Rightarrow \frac{k}{2}\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)(1) = 3 \Rightarrow \frac{k}{2} = 3$

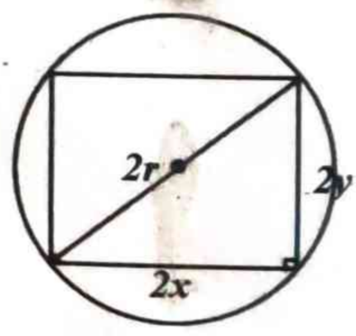
$\Rightarrow k = 6$

66(a). Let $2x$ and $2y$ be length and breadth of the rectangle inscribed in the circle of radius r.

From the fig, $(2x)^2 + (2y)^2 = (2r)^2$

$\Rightarrow x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2$

$\Rightarrow y = \sqrt{r^2 - x^2} \quad \ldots(1)$



Area of rectangle,

$A = 2x.2y = 4xy = 4x\sqrt{r^2 - x^2}$ *maximum*

$\Rightarrow A^2 = 16x^2(r^2 - x^2) = Z$ *maximum*

$\therefore \frac{dZ}{dx} = 32r^2x - 64x^3 = 32x(r^2 - 2x^2)$

If $\frac{dZ}{dx} = 0$; $x = 0$ or $r^2 - 2x^2 = 0 \Rightarrow r^2 = 2x^2 \Rightarrow x = \frac{r}{\sqrt{2}}$.

And $\frac{d^2Z}{dx^2} = 32r^2 - 192x^2 < 0 \text{ if } r^2 = 2x^2$

$\Rightarrow$ *A has* maximum area when $x = \frac{r}{\sqrt{2}}$.

If $x = \frac{r}{\sqrt{2}}$, *then* $y = \frac{r}{\sqrt{2}} \Rightarrow x = y = \frac{r}{\sqrt{2}} \Rightarrow 2x = 2y = \sqrt{2}r.$

Rectangle is a square.

66(b). We have $A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

LHS $= A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= O =$ RHS.

Now, $A^2 - 5A + 7I = O. \Rightarrow A(AA^{-1}) - 5AA^{-1} + 7A^{-1} = O.A^{-1}$

$$\Rightarrow AI - 5I + 7A^{-1} = O$$

$$\Rightarrow A^{-1} = -\frac{1}{7}(A - 5I)$$

$$= -\frac{1}{7}\left(\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right) = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$$

* * *

MODEL QUESTION PAPER-3

MATHEMATICS (35)

TIME: 3 Hours 15 Minutes **MAX. MARKS: 100**

Instructions:

i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii) Use the graph sheet for the question on Linear programming in PART E.

PART-A

Answer any TEN questions: **$10 \times 1 = 10$**

1. Let $*$ be a binary operation on N given by $a * b = L.C.M.of\ a\ and\ b$. Find $5 * 7$.

2. Define a empty relation.

3. Write the range of the function $y = \sec^{-1} x$

4. Find the principal value of $\operatorname{cosec}^{-1}(2)$.

5. If a matrix has 5 elements, what are the possible orders it can have?

6. Find the values of x for which $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.

7. If $y = \tan\sqrt{x}$, find $\frac{dy}{dx}$.

8. Find $\frac{dy}{dx}$, if $y = e^{x^3}$

9. Find $\int (2x^2 + e^x)\,dx$.

10. Find $\int_a^b x\,dx$.

11. Define a negative vector.

12. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

13. If a line makes angles 90°, 135° 45° with the x, y and z-axis respectively, find its direction cosines.

14. Define Optimal solution in linear programming problem.

15. If $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$ find $P(A\cap B)$ if A and B are independent events.

PART-B

Answer any TEN questions: **$10\times 2=20$**

16. If $f:R\to R$ and $g:R\to R$ are given by $f(x)=\cos x$ and $g(x)=3x^2$. Find $g\circ f$ *and* $f\circ g$.

17. Prove that $\cot^{-1}(-x)=\pi-\cot^{-1}x,\ x\forall R.$

18. Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

19. Let $A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$ *and* $B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$. Find $A+B$ and $A-B$

20. Find the area of the triangle whose vertices are $(-2,-3),(3,2)$ and $(-1,-8)$ using determinants method.

21. Find $\frac{dy}{dx}$, if $\sin^2 x+\cos^2 y=1$.

22. If $y=x^x$, find $\frac{dy}{dx}$.

23. Find $\frac{dy}{dx}$, if $y=\cos(\log x+e^x),\ x>0$

24. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly decreasing.

25. Find $\int \cot x\log(\sin x)\,dx$.

26. Find $\int x\sec^2 x\,dx$

27. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$.

28. Find the order and degree (if define) of the differential equation,
$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$$

29. Find the projection of the vector $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ on the vector $\vec{b}=7\hat{i}-\hat{j}+8\hat{k}$.

30. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

31. Find the equation of the plane with intercepts 2, 3 and 4 on *X*, *Y* and *Z* axes respectively.

32. Find the vector equation of the line passing through the points (-1,0,2) and (3,4,6).

33. A random variable X has the following probability distribution.

X	0	1	2	3	4
$P(X)$	0.1	k	$2k$	$2k$	k

. Find k.

PART-C

Answer any TEN questions: **10 × 3=30**

34. Show that the relation R defined in the set A of all triangles as $R=\{(T_1,T_2):T_1\ is\ similar\ to T_2\}$, is equivalence relation.

35. **Prove that** $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$ **Proof: we have** $2\tan^{-1}\frac{1}{2}$

36. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, then show that $F(x)F(y)=F(x+y)$.

37. Without expanding, prove that $\Delta=\begin{vmatrix}a & b & c\\ a+2x & b+2y & c+2z\\ x & y & z\end{vmatrix}=0$.

38. Differentiate $\sin(\cos(x^2))$ with respect to x.

39. If $x=2at^2,\ y=at^4$, then find $\frac{dy}{dx}$

40. Verify Mean Value theorem, if $f(x)=x^2-4x-3,\ x\in(1,4)$

41. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

42. Find $\int \frac{(x-3)e^x}{(x-1)^3}\,dx$.

43. Evaluate: $\int \frac{x}{(x+1)(x+2)}\,dx$.

44. Evaluate: $\int_0^{\pi/2} \cos^2 x\,dx$

45. Find the area of the region bounded by $x^2=4y,\ y=2, y=4$ and the y-axis in the first quadrant.

46. Find the equation of the curve passing through the point $(-2, 3)$, given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

47. Form the differential equation representing family of curves $\frac{x}{a}+\frac{y}{b}=1$ where a and b arbitrary constants.

48. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

49. Find x, such that the four points
A (3, 2, 1), B (4, x, 5), C (4, 2, −2) and D (6, 5, −1) are coplanar.

50. Find the equation of the plane through the intersection of the planes $3x-y+2z-4=0,\ x+y+z-2=0$ and the point (2, 2, 1).

51. Bag I contains 3 red and 4 black balls. While another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.

PART D

Answer any SIX questions: **$6\times 5=30$**

52. Show that the function $f: R\to R$ defined by $f(x)=3x$ is one-one onto. Justify your answer.

53. Prove that the function $f: R \to R$ defined by $f(x) = 4x + 3$ is invertible. Find the inverse of f.

54. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then

compute $A + B$ and $B - C$. Also, verify $A + (B - C) = (A + B) - C$.

55. Solve the system of equations by matrix method:
$2x + 3y + 3z = 5$, $x - 2y + z = -4$, $3x - y - 2z = 3$.

56. If $y = (\tan^{-1} x)^2$, show that $(1 + x^2)^2 y_2 + 2x(1 + x^2) y_1 = 2$.

57. A ladder $5m$ long is leaning against a wall. The bottom of the ladder is Pulled along the ground, away from the wall at the rate of $2m/sec$. How fast is its height on the wall decreasing when the foot of the ladder is $4m$ away from the wall?

58. Find the integral of $\frac{1}{x^2 + a^2}$ w.r.to x and hence evaluate $\int \frac{dx}{x^2 + 2x + 2}$.

59. Using the method of integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

60. Find the general solution of the differential equation
$x\frac{dy}{dx} + 2y = x^2 \log x$

61. Derive the equation of the line in space passing through a given point and parallel to a vector both in vector and Cartesian form.

62. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize (a) exactly once? (b) at least once?

63. In a girl's hostel 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

a) Find the probability that she reads neither Hindi nor English newspaper.

b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

c) If she reads English newspaper, find the probability she read Hindi newspaper.

PART-E

Answer any ONE question: **1 × 10=10**

64. (a) Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx, & \text{if } f(x) \text{ is even function} \\ 0, & \text{if } f(x) \text{ is odd function} \end{cases}$

and hence evaluate $\int_{-1}^{1} sin^5 x\, cos^4 x\, dx$.

(b) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

65. (a) Miximise $z = 4x + y$ subject to constraints $x + y \leq 50,\ 3x + y \leq 90,$ $x \geq 0,\ y \geq 0$ by graphical method.

(b) Find the value of k if $f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$

is continuous at $x = \pi$.

66. (a) A sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand form a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is $4cm$?

(b) Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

* * *

ANSWERS TO MODEL QUESTION PAPER – 3

PART-A

1. We have $5*7 = L.C.M. of\ 5 and\ 7 = 35$.

2. A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \phi \subset A \times A$.

3. Range of $y = \sec^{-1} x$ is $[0,\pi] - \left\{\frac{\pi}{2}\right\}$.

4. Let $\operatorname{cosec}^{-1}(2) = y$. Then $\operatorname{cosec} y = 2 = \operatorname{cosec}\frac{\pi}{6}$.

   We know that the range of the principal value branch of
   $$\operatorname{cosec}^{-1} x = \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$
   Therefore principal value of $\operatorname{cosec}^{-1}(2)\ is\ \frac{\pi}{6}$.

5. Matrix containing 5 elements can have any one of the following order; $1\times 5\ or\ 5\times 1$.

6. $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

   $\Rightarrow x^2 - 36 = 36 - 36 \Rightarrow x^2 = 36 \qquad \therefore x = \pm 6$

7. Let $y = \tan\sqrt{x}$

   Differentiate with respect to x,

   we get $\frac{dy}{dx} = \sec^2\sqrt{x} \times \frac{1}{2\sqrt{x}}$.

8. $\frac{dy}{dx} = e^{x^3}\frac{d}{dx}x^3 = e^{x^3} \times 3x^2$.

9. $= 2\int (x^2)\,dx + \int (e^x)\,dx$

   $= 2\frac{x^{2+1}}{2+1} + e^x + C \ = 2\frac{x^3}{3} + e^x + C$.

10. $I = \left[\frac{x^2}{2}\right]_a^b = \frac{1}{2}\left(b^2 - a^2\right).$

11. A vector having the same magnitude as that of a given vector $\vec{a}$ and the direction opposite to that of $\vec{a}$ is called the negative of $\vec{a}$ and denoted by $-\vec{a}$.

12. The unit vector in the direction of $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{1+1+4}} = \frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$

13. The direction cosines of given lines are

$\cos 90°, \cos 135°, \cos 45°$ i.e., $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

14. A feasible solution which leads to the optimal value of an objective function is known as an optimal solution of the system of inequations.

15. Since A and B are independent events the

$$P(A \cap B) = P(A) \cdot P(B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}.$$

PART-B

16. We have

$gof(x) = g\left(f(x)\right) = g(\cos x) = 3(\cos x)^2 = 3\cos^2 x$ and

$fog(x) = f\left(g(x)\right) = f(3x^2) = \cos(3x^2)$.

17. Let $\cot^{-1}(-x) = y \Rightarrow -x = \cot y$

$\Rightarrow x = -\cot y = \cot(\pi - y)$

$\Rightarrow \cot^{-1} x = \pi - y$

$\Rightarrow y = \pi - \cot^{-1} x$

$\therefore \cot^{-1}(x) = \pi - \cot^{-1} x$.

18. Let $\tan^{-1}\sqrt{3} = y$.

Then $\tan y = \sqrt{3} = \tan\left(\frac{\pi}{3}\right) \Rightarrow y = \frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Let $\sec^{-1}(-2) = \theta$.

Then $\sec\theta = -2 = -\sec\frac{\pi}{3} = \sec\left(\pi - \frac{\pi}{3}\right) \Rightarrow \theta = \frac{2\pi}{3} \in (0, \pi)$

Now G.E., $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.

19. $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3+(-2) & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

20. Area of the triangle is given by, $\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$

$$\Delta = \frac{1}{2}\{-2[2+8] + 3[3+1] + 1[-24+2]\} = \frac{1}{2}[-20+12-22] = \left|\frac{-30}{2}\right| = 15.$$

21. We differentiate the relationship directly with respect to x,

i.e., $\frac{d}{dx}\sin^2 x + \frac{d}{dx}\cos^2 y = \frac{d}{dx}1$

$\Rightarrow\ 2\sin x \cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0$

$\Rightarrow (2\sin y \cos y)\frac{dy}{dx} = 2\sin x \cos x$

$\therefore \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$.

22. Let $y = x^x$

Then $\log y = \log x^x = x \log x$

Differentiating with respect to x, $\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log x \cdot 1$

$\frac{dy}{dx} = x^x(1 + \log x)$.

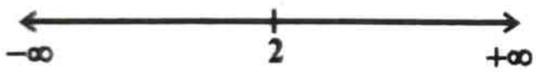
23. $\frac{dy}{dx} = -\sin(\log x + e^x)\frac{d}{dx}(\log x + e^x) = -\sin(\log x + e^x)\left[\frac{1}{x} + e^x\right]$.

24. We have $f(x) = x^2 - 4x + 6$

$f'(x) = 2x - 4$

Therefore, $f'(x) = 0$ gives $x = 2$.

Now the point $x = 2$ divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$



In the interval $(-\infty, 2)$, $f'(x) = 2x - 4 < 0$.

Therefore, f is strictly decreasing in this interval.

25. $I = \int \log(\sin x) \cot x \, dx$

Put $\log(\sin x) = t$

$\Rightarrow \frac{1}{\sin x} \cdot \cos x dx = dt \Rightarrow \cot x \, dx = dt$

$= \int t \, dt = \frac{t^2}{2} + C$

$= \frac{[\log(\sin x)]^2}{2} + C$.

26. $\int x \sec^2 x \, dx = x \int \sec^2 x \, dx - \int 1 \cdot \tan x \, dx$

$= x \tan x - \log|\sec x| + C$

27. $I=\left[\tan^{-1}x\right]_0^1=\tan^{-1}(1)-\tan^{-1}(0)=\frac{\pi}{4}-0=\frac{\pi}{4}$.

28. Order is 2 and Degree is not defining.

29. $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ and $\vec{b}=7\hat{i}-\hat{j}+8\hat{k}$

The projection of $\vec{a}$ on $\vec{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$=\frac{1\times7+3(-1)+7\times8}{\sqrt{7^2+(-1)^2+8^2}}=\frac{7-3+56}{\sqrt{49+1+64}}=\frac{60}{\sqrt{114}}.$$

30. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&3\\2&-7&1\end{vmatrix}=20\hat{i}-5\hat{j}-5\hat{k}=5(4\hat{i}-\hat{j}-\hat{k})$

The area of the parallelogram $=|\vec{a}\times\vec{b}|$

$=5\sqrt{16+1+1}=5\sqrt{18}$

$=15\sqrt{2}$ Square unit.

31. Required equation of a plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$,

where $a=2\,,b=3$ *and* $c=4$ $\therefore\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$ OR $6x+4y+3z=12$.

32. Let $\vec{a}=(-1,0,2)=-\hat{i}+2\hat{k}$ and $\vec{b}=(3,4,6)=3\hat{i}+4\hat{j}+6\hat{k}$

$\vec{b}-\vec{a}=4\hat{i}+4\hat{j}+4\hat{k}$

Vector equation of the line $\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)=-\hat{i}+2\hat{k}+\lambda\left(4\hat{i}+4\hat{j}+4\hat{k}\right)$.

33. The probability distribution of X is

$$\sum_{i=1}^{n}p_i=1\Rightarrow0.1+k+2k+2k+k=1$$

$$\Rightarrow6k=1-0.1=0.99\qquad\therefore k=\frac{0.99}{6}=0.15.$$

PART-C

34. Let A be the set of all triangles in a plane. The relation is $R = \{(T_1, T_2) : T_1 \text{ is similar to} T_2\}$

Reflexive: Every triangle is similar to itself; so R is reflexive.

Symmetric: If the triangle $T_1 \text{ is similar to} T_2$, then clearly $T_2 \text{ is similar to} T_1$. Therefore R is symmetric.

Transitive: Let $T_1 \text{ is similar to} T_2$ and $T_2 \text{ is similar to} T_3$. Then the triangle $T_1 \text{ is similar to} T_3$. Therefore R is transitive.

Hence R is an equivalence relation.

35. Proof: we have $2\tan^{-1}\frac{1}{2} = \tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right) = \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$$LHS = \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\left(\frac{4}{3}\right)\times\left(\frac{1}{7}\right)}\right) = \tan^{-1}\left(\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right)$$

$$= \tan^{-1}\frac{31}{17} = \text{RHS}.$$

36. Given $F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y).$$

37. we have $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix} = 0+0=0$.

38. Let $y=\sin(\cos(x^2))$ Differentiate with respect to x,

we get $\frac{dy}{dx}=\frac{d}{dx}\sin(\cos(x^2))$

$$=\cos(\cos x^2)\frac{d}{dx}(\cos x^2)$$

$$=\cos(\cos x^2)(-\sin x^2)\frac{d}{dx}(x^2)$$

$$=\cos(\cos x^2)(-\sin x^2)(2x)$$

$$=-2x\sin x^2\cos(\cos x^2).$$

39. $\frac{dx}{dt}=2a\times 2t=4at$ *and* $\frac{dy}{dt}=a\times 4t^3=4at^3$

$$\therefore \frac{dy}{dx}=\frac{4at^3}{4at}=t^2.$$

40. The function $f(x)=x^2-4x-3$, $x\in[1,4]$ which is polynomial function, so it is continuous in [1,4] and differentiable in $(1,4)$. Hence, there is at least one real number, $c\in(1,4)$ such that

$$f'(c)=\frac{f(4)-f(1)}{4-1}$$

$$\Rightarrow 2c-4=\frac{\left(4^2-4\times 4-3\right)-\left(1^2-4\times 1-3\right)}{4-1}$$

$$=\frac{-3+6}{3}=1 \quad (\because f'(x)=2x-4\Rightarrow f'(c)=2c-4)$$

$$\Rightarrow 2c-4=1\Rightarrow c=\frac{5}{2}\in(1,4)$$

Therefore MVT is verified for $f(x)$ in [1.4].

41. Let one of the numbers be x. Then the other number is $(15-x)$.

Let $S(x)$ denote the sum of the squares of these numbers.

Then $S(x) = x^2 + (15-x)^2 = 2x^2 - 30x + 225$

$$\begin{cases} S'(x) = 4x - 30 \\ S''(x) = 4 \end{cases}$$

Now $S'(x) = 0$ gives $x = \frac{15}{2}$.

Also $S''\left(\frac{15}{2}\right) = 4 > 0$. Therefore, by second derivative test, $x = \frac{15}{2}$ is the point of local minima of S. Hence the sum of squares of numbers is minimum when the numbers are $x = \frac{15}{2}$ and $15 - \frac{15}{2} = \frac{15}{2}$.

42. $$I = \int e^x \frac{(x-1-2)}{(x-1)^3} dx$$

$$= \int e^x \left[\frac{(x-1)-2}{(x-1)^3}\right] dx$$

$$= \int e^x \left[\frac{(x-1)}{(x-1)^3} + \left(\frac{-2}{(x-1)^3}\right)\right] dx$$

$$= \int e^x \left[\frac{1}{(x-1)^2} + \left(\frac{-2}{(x-1)^3}\right)\right] dx$$

$f(x) = \frac{1}{(x-1)^2}$, *then* $f'(x) = \frac{-2}{(x-1)^3}$

$$= e^x \frac{1}{(x-1)^2} + C.$$

43. Let $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

Put $x = -1$, in equation (1), we get

$$-1 = A(-1+2) \therefore A = -1$$

Put $x=-2$, in equation (1), we get

$$-2 = B(-2+1) \therefore B = 2$$

$$\therefore I = \int \frac{x}{(x+1)(x+2)}\,dx = \int \left(\frac{-1}{x+1} + \frac{2}{x+2} \right) dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log \frac{(x+2)^2}{|x+1|} + C.$$

44. Let $I = \int_0^{\pi/2} \cos^2 x\,dx \quad \ldots (1)$

$$\therefore\ I = \int_0^{\pi/2} \cos^2\left(\frac{\pi}{2} - x\right) dx$$

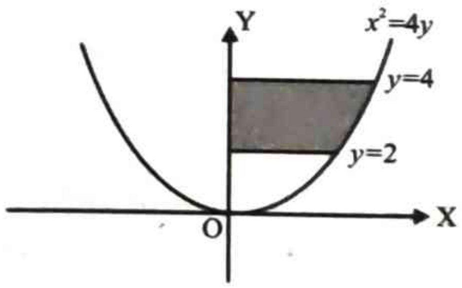
$$= \int_0^{\pi/2} \sin^2 x\,dx \quad \ldots (2)$$

(1) + (2) gives

$$2I = \int_0^{\pi/2} (\cos^2 x + \sin^2 x)\,dx = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{4}$$

45. The required area of the region bounded by $x^2 = 4y$, $y=2$, $y=4$ and the y-axis in the first quadrant.



$$A = \int_2^4 x\,dy$$

$$= \int_2^4 2\sqrt{y}\,dy$$

$$= 2 \cdot \frac{2}{3} y^{3/2} \Big]_2^4$$

$$= \frac{4}{3}\left(4^{3/2} - 2^{3/2}\right) = \frac{4}{3}\left(8 - 2\sqrt{2}\right) \text{ square unit.}$$

46. We know that, the slope of the tangent to the curve at any point (x, y) is $\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{2x}{y^2}$

By variable separable, $y^2 dy = 2x dx$

$$\Rightarrow \int y^2 dy = \int 2x dx + C$$

$$\frac{y^3}{3} = x^2 + C$$

It passes through $(-2, 3)$

$\therefore 9 = 4 + C \Rightarrow C = 5$

$\therefore$ Required equation of curve is $\frac{y^3}{3} = x^2 + 5$.

47. Given $\frac{x}{a} + \frac{y}{b} = 1$

Differentiating w.r.to x, we get, $\frac{1}{a} + \frac{1}{b}\frac{dy}{dx} = 0$

Differentiating w.r.to x, we get, $0 + \frac{1}{b}\frac{d^2y}{dx^2} = 0$

$\therefore \frac{d^2y}{dx^2} = 0$ is the required differential equation.

48. Solution: $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ *and* $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ $\quad \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j} + 0\hat{k}$ and $\vec{a} - \vec{b} = 2\hat{i} + 0\hat{j} + 4\hat{k}$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$$\left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right| = \sqrt{16^2 + (-16)^2 + (-8)^2} = 24$$

The unit vector perpendicular to $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is

$$\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{\left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|}=\pm\frac{16\hat{i}-16\hat{j}-8\hat{k}}{24}=\pm\frac{1}{3}(2\hat{i}-2\hat{j}-\hat{k}).$$

49. $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}\equiv(1,\ x-2,\ 4)$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}\equiv(1,0,-3)$

$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}\equiv(3,\ 3,\ -2)$

Since given points are coplanar, therefore $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right]=0$

$$\therefore\begin{vmatrix}1 & x-2 & 4\\ 1 & 0 & -3\\ 3 & 3 & -2\end{vmatrix}=0$$

$$1(0+9)-(x-2)(-2+9)+4(3-0)=0$$

$$9-7x+14+12=0$$

$$x=5.$$

50. Equation of the plane through the intersection of the given planes is of the form.

$$(3x-y+2z-4)+\lambda(x+y+z-2)=0$$

It passes through (2, 2, 1)

$$(6-2+2-4)+\lambda(2+2+1-2)=0$$

$$2+3\lambda=0 \quad\Rightarrow\quad \lambda=\frac{-2}{3}$$

Required plane is, $\quad (3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$

$$\Rightarrow 7x-5y+4z-8=0.$$

51. Let E_1 be the event of choosing the bag I, E_2 the event of choosing the bag II and A be the event of drawing a red ball.

Then $P(E_1)=P(E_1)=\frac{1}{2}$

Also, $P(A\,|\,E_1)=P$ (drawing a red ball from Bag I) $=\frac{3}{7}$

Now, the probability of drawing a ball from Bag II, being given that it is red,is

By using **Bayes' theorem**, we have

$$P(E_2 \mid A) = \frac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$$

$$P(E_2 \mid A) = \frac{\frac{1}{2} \times \frac{5}{11}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{5}{11}} = \frac{35}{68}.$$

PART-D

52. Let $f(x) = 3x$

Let $x_1, x_2 \in R, f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2 \ \therefore f$ is one – one

Let any $y \in R$ and $f(x) = y$ then

$$f(x) = y \Rightarrow 3x = y \Rightarrow x = \frac{y}{3} \in R$$

$\therefore$ Corresponding to each $y \in R$ (codomain), there exists $\left(\frac{y}{3}\right) \in R$ (domain) such that.

$f\left(\frac{y}{3}\right) = y \ \therefore \ f$ is onto.

$\therefore \ f$ is one-one onto.

Hence f is a bijective function.

53. Let us define $g : R \to R$ by $g(x) = \frac{x-3}{4}$

Now $(gof)(x) = g\left[f(x)\right] = g(4x+3)$

$$= \frac{(4x+3)-3}{4} = \frac{4x}{4} = x$$

And $(fog)(x) = f[g(x)] = f\left(\frac{y-3}{4}\right)$

$$= 4\left(\frac{y-3}{4}\right) + 3 = y - 3 + 3 = y$$

Therefore, $gof = fog = I_R$

Hence, f is invertible and the inverse of f is given by $f^{-1}(y) = \frac{y-3}{4}$.

OR

Proving f is both one one and onto.

Hence $f^{-1}(x)$ exists

Finding $f^{-1}(x) = \frac{x-3}{4}$.

54. Let $A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$

$$B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

LHS = A + (B − C)

$$= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \quad \ldots(1)$$

RHS = (A + B) − C

$$= \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \quad \ldots(2)$$

From (1) and (2), $A + (B - C) = (A + B) - C$.

55. Let $AX=B \Rightarrow X=A^{-1}B$, where $A=\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and

$$B=\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}=2[4+1]-3[-2-3]+3[-1+6]$$

$$=10+15+15=40 \neq 0$$

$$\text{adjA}=\begin{bmatrix} +(4+1) & -(-6+3) & +(3+6) \\ -(-2-3) & +(-4-9) & -(2-3) \\ +(-1+6) & -(-2-9) & +(-4-3) \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}=\frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x=1$, $y=2$ *and* $z=-1$.

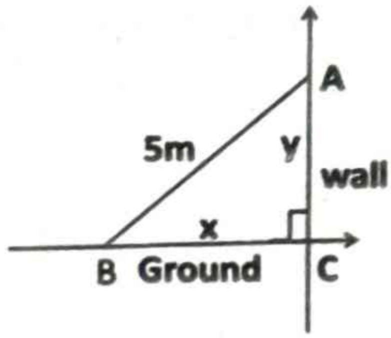
56. $y_1=2\tan^{-1}x \cdot \dfrac{1}{1+x^2} \Rightarrow (1+x^2)y_1=2\tan^{-1}x$

Again differentiating w. r. to x, we get,

$$(1+x^2)y_2+y_1(0+2x)=2\times\frac{1}{1+x^2}$$

$$(1+x^2)^2 y_2+2x(1+x^2)y_1=2.$$

57. Let $AB = 5m$ be the ladder and y be the height of the wall at which the ladder touches. Also, let the foot of the ladder be at B whose distance from the wall is x.



Given that the bottom of the ladder is pulled along theground at 2 m/s,

So, $\dfrac{dx}{dt} = 2m \text{ per sec}$

As we know that triangle is right angled, so Pythagoras theorem, we have

$x^2 + y^2 = 25 \ldots\ldots (1)$,

when x = 4 then

$y^2 = 25 - 16$

$\Rightarrow y = \sqrt{25-16} \quad \therefore y = 3$

Differentiating equation (1) with respect to t on both sides

We get, $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0 \Rightarrow 2\times4\times2 + 2\times3\dfrac{dy}{dt} = 0 \qquad \therefore \dfrac{dy}{dt} = -\dfrac{8}{3}$.

The rate of fall of height on the wall $\dfrac{dy}{dx} = -\dfrac{8}{3} m \text{ per sec}$. negative sign shows that height on the wall is decreasing at the rate of $\dfrac{8}{3} m \text{ per sec}$.

58. Put $x = a\tan\theta \Rightarrow dx = a\sec^2\theta\, d\theta$

$$\therefore I = \int \frac{1}{x^2+a^2}dx = \int \frac{1}{a^2\tan^2\theta + a^2} a\sec^2\theta\, d\theta = \int \frac{a\sec^2\theta}{a^2(\tan^2\theta+1)} d\theta$$

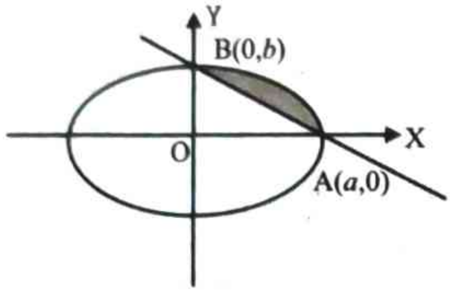
$$= \frac{1}{a}\int d\theta = \frac{1}{a}\theta + C = \frac{1}{a}\tan^{-1}\frac{x}{a} + C.$$

Consider $I = \int \dfrac{dx}{x^2+2x+2} = \int \dfrac{dx}{(x+1)^2+1} \quad = \tan^{-1}(x+1) + C$.

59. The ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ ...(1)

and the line $\frac{x}{3}+\frac{y}{2}=1$ ...(2)

Required area is shown shaded



$$=\int_0^3 \frac{2}{3}\sqrt{9-x^2}\,dx-\int_0^3 \frac{2}{3}(3-x)\,dx$$

$$=\frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3-\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

$$=\frac{2}{3}\left[0+\frac{9}{2}\frac{\pi}{2}-0-0\right]-\frac{2}{3}\left[9-\frac{9}{2}-0+0\right]$$

$$=\frac{3\pi}{2}-3 \text{ square unit.}$$

60. Dividing by x on both side, we get,

$$\frac{dy}{dx}+\frac{2y}{x}=\frac{x^2\log x}{x}\Rightarrow\frac{dy}{dx}+\frac{2}{x}\times y=x\log x$$

This is of the form, $\frac{dy}{dx}+Py=Q$

Where $P=\frac{2}{x}$ *and* $Q=x\log x$

Integrating factor, $I.F.=e^{\int p\,dx}=e^{2\int\frac{1}{x}dx}=e^{2\log x}=e^{\log x^2}=x^2$

$$\therefore\ y(I.F.)=\int Q(I.F.)\,dx+C$$

$$y\cdot x^2=\int x\log x\cdot x^2dx+C=\int \underbrace{\log x}_{I}\ \underbrace{x^3}_{II}\,dx+C$$

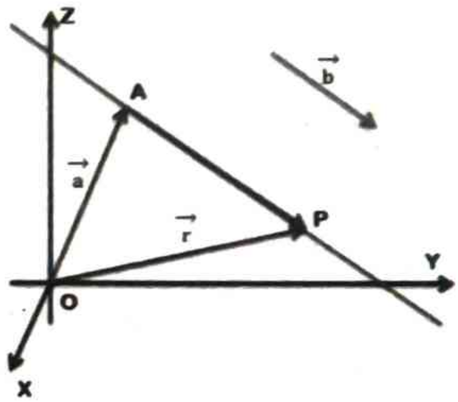
$$y\cdot x^2=\left[\log x\,\frac{x^4}{4}-\int\frac{1}{x}\,\frac{x^4}{4}dx\right]+C$$

$$=\frac{x^4\log x}{4}-\frac{1}{4}\int x^3dx+C$$

$$y\cdot x^2=\frac{x^4\log x}{4}-\frac{1}{4}\left(\frac{x^4}{4}\right)+C.$$

61. Let A be a given point whose position vector $\vec{a}$ and $\vec{b}$ be given vector. Let 'l' be the line passing through the point A and is parallel to a given vector $\vec{b}$.

Let P be any point on the line 'l', whose position vector $\vec{r}$.



Then $\overrightarrow{AP}$ is parallel to the vector $\vec{b}$.

i.e., $\overrightarrow{AP}=\lambda\vec{b}$,where λ is a real number.

But $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}$ i.e., $\lambda\vec{b}=\vec{r}-\vec{a}$

$\vec{r}=\vec{a}+\lambda\vec{b}$. Which is the required vector equation of the line.

Cartesian form

Let $A(x_1,y_1,z_1)$ be the co-ordinates of the point A. Let a, b, c be the direction ratios of the given line $\vec{b}$. Let $P(x,y,z)$ be a point on the required line.

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$; $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{b}=a\hat{i}+b\hat{j}+c\hat{k}$

Substitute these values in $\vec{r}=\vec{a}+\lambda\vec{b}$ and equating the coefficients of $\hat{i},\hat{j}$ *and* $\hat{k}$. We get $x=x_1+\lambda \mathrm{a} \Rightarrow x-x_1=\lambda \mathrm{a}$

$y=y_1+\lambda \mathrm{b} \Rightarrow y-y_1=\lambda \mathrm{b}$ and $z=z_1+\lambda \mathrm{c} \Rightarrow z-z_1=\lambda \mathrm{c}$

Eliminating the parameter λ, we get $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$

This is the required equation of given line.

62. Let X denotes the number of times that the person wins the prize.

Here $n=50$, $p=\dfrac{1}{100}$, $q=\dfrac{99}{100}$

Therefore $\mathrm{P}(\mathrm{X}=\mathrm{r})={}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}\ \mathrm{q}^{\mathrm{n-r}}\ \mathrm{p}^{\mathrm{r}}$

$$= {}^{50}C_r \left(\frac{99}{100}\right)^{50-r} \left(\frac{1}{100}\right)^r$$

(a) P(he wins a prize exactly once)

$$= {}^{50}C_1 \ q^{50-1} \ p^1$$

$$= 50 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 = \frac{1}{2} \times \left(\frac{99}{100}\right)^{49}.$$

(b) P (he wins a prize at least once)

$$= 1 - P(0) = 1 - {}^{50}C_0 \, q^{50} \, p^0 = 1 - \left(\frac{99}{100}\right)^{50}$$

63. Let H and E be the events of a student reading Hindi and English news paper respectively.

$$\therefore P(E) = \frac{40}{100} = \frac{2}{5}, \quad P(H) = \frac{60}{100} = \frac{3}{5}$$

$$P(E \cap H) = \frac{20}{100} = \frac{1}{5}$$

(a) P (she reads neither Hindi nor English newspaper)

$$= P(E' \cap H') = P((E \cup H)') = 1 - P(E \cup H) = 1 - [P(E) + P(H) - P(E \cap H)]$$

$$= 1 - \left[\frac{2}{5} + \frac{3}{5} - \frac{1}{5}\right] = 1 - \frac{4}{5} = \frac{1}{5}.$$

(b) Required $= P(E|H) = \dfrac{P(E \cap H)}{P(H)} = \dfrac{1/5}{3/5} = \dfrac{1}{3}.$

(c) Required $= P(H|E) = \dfrac{P(E \cap H)}{P(E)} = \dfrac{1/5}{2/5} = \dfrac{1}{2}.$

PART-E

64. (a) **Proof:** Let $\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$

Putting $x=-t$ in the first integral on the RHS, then $dx=-dt$

Also, $x=-a \Rightarrow t=a$ and $x=0 \Rightarrow t=0$

$$\Rightarrow \int_{-a}^{0} f(x)dx = \int_{a}^{0} f(-t)(-dt) = -\int_{a}^{0} f(-t)dt = \int_{0}^{a} f(-x)dx$$

$$\therefore \int_{-a}^{a} f(x)dx = \int_{0}^{a} f(-x)dx + \int_{0}^{a} f(x)dx$$

Case(1): If $f(x)$ is even, then $f(-x)=f(x)$

$$\therefore \int_{-a}^{a} f(x)dx = \int_{0}^{a} f(x)dx + \int_{0}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

Case(2): If $f(x)$ is odd, then $f(-x)=-f(x)$

$$\therefore \int_{-a}^{a} f(x)dx = -\int_{0}^{a} f(x)dx + \int_{0}^{a} f(x)dx = 0.$$

Hence the proof.

Consider $I = \int_{-1}^{1} sin^5 x\, cos^4 x\, dx$

Since $sin^5 x cos^4 x$ is an odd function, by the above property, $I = \int_{-1}^{1} sin^5 x\, cos^4 x\, dx = 0$.

(b) $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

Hence $A^2 + (-5)A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Now $A^2 - 5A + 7I = O$

Therefore $7IA^{-1} = -A^2A^{-1} + 5AA^{-1}$

(post multiply by A^{-1} because $|A| \neq 0$)

Or $7IA^{-1} = (-A)(AA^{-1}) + 5(AA^{-1})$

Or $7A^{-1} = (-A)I + 5I = 5I - A$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Or $7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Therefore $A^{-1} = \left(\frac{1}{7}\right)\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.

65. **(a)** Region represented by $x + y \leq 50$.

Consider $x + y = 50$ ...(1)

Put $x = 0 \Rightarrow y = 50$ $\therefore A(0, 50)$

Put $y = 0 \Rightarrow x = 50$ $\therefore B(50,0)$

Plot the points A (0, 50) and B (50, 0). Join AB and produce it both ways.

Clearly O (0, 0) satisfies $x + y \leq 50$.

$\therefore$ Solution region is on origin side.

Region represented by $3x + y \leq 90$

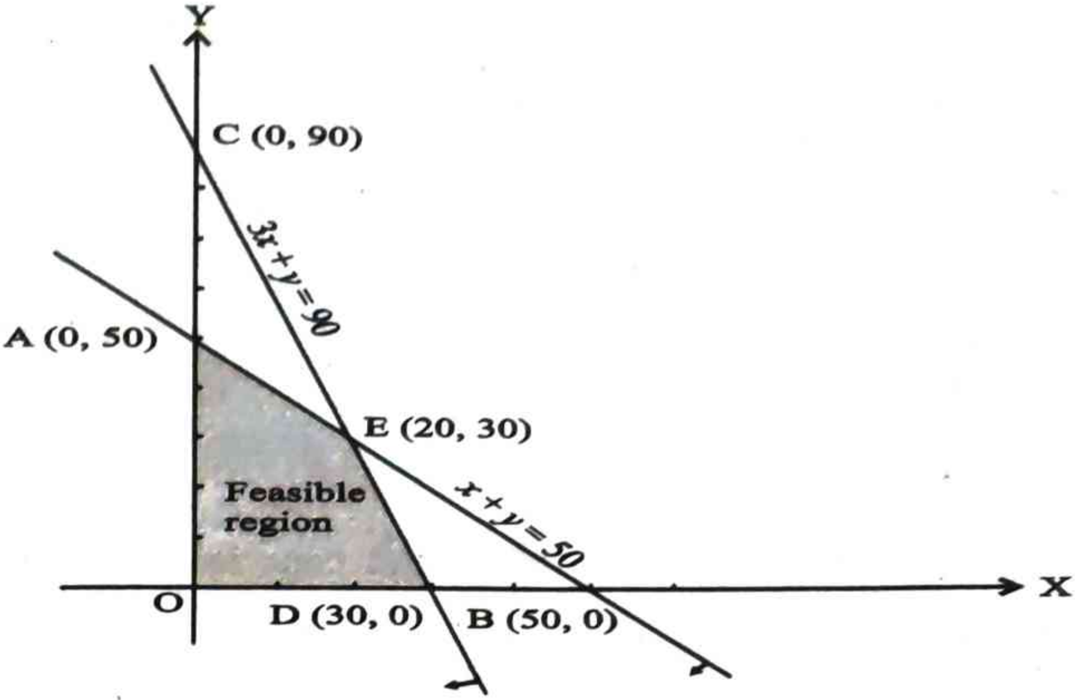
Consider $3x + y = 90$ ...(2)

If $x = 0$ then $y = 90$ $\therefore C(0,90)$

If $y = 0$ then $x = 30$ $\therefore D(30,0)$

Put $x = 0, y = 0$ then $0 + 0 \leq 90$ which is true. Therefore solution region contains the origin.

Solve (1) and (2), we get $x = 20, y = 30$ $\therefore E(20,30)$



$\therefore$ Solution lies in the region OAED.

Sl. No.	Corner points	Corresponding value of $z = 4x + y$
	O (0, 0)	0
	A (0, 50)	50
	E (20, 30)	110
	D (30, 0)	120 ← Maximum

Hence, maximum value of Z is 120 at the point (30, 0).

(b) Given function is continuous at $x = \pi$

Therefore $f(\pi) = LHL = RHL$

Here $f(\pi) = \pi k + 1$

$$RHL = \lim_{x \to \pi^+} f(x) = \lim_{x \to \pi^+} (\cos x) = \cos \pi = -1$$

$$\Rightarrow \pi k + 1 = -1 \Rightarrow \pi k = -2 \qquad \therefore k = \frac{-2}{\pi}.$$

66. (a) Let r be the radius, h be the height and V be the volume of the sand cone at any time t.

Given $h = 4\ \frac{dv}{dt} = 12\,cm^3/s$ and $h = \frac{1}{6}r \Rightarrow r = 6h$

$\therefore V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6h)^2 h = 12\pi h^3$

Differentiating with respect to t, we get

$$\frac{dv}{dt} = (12\pi)3h^2\frac{dh}{dt} = 36\pi h^2\frac{dh}{dt} \Rightarrow 12 = 36\pi(4)^2\frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{12}{36\pi 16} = \frac{1}{48\pi}cm/s.$$

Hence, when the height of the sand cone is $4\,cm$, its heights is increasing at the rate of $\frac{1}{48\pi}cm/s$.

(b) $LHS = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Applying $R_1 \to R_1 - R_2 - R_3$, we get $= \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Expanding along first column

$$= 0 - (-2c)\left[b(a+b) - bc\right] + (-2b)\left[bc - c(c+a)\right]$$

$$= 2c\left[ab + b^2 - bc\right] - 2b\left[bc - c^2 - ca\right]$$

$$= 2abc + 2b^2c - 2bc^2 - 2b^2c + 2bc^2 + 2abc = 2abc + 2abc = 4abc.$$

* * *

MODEL QUESTION PAPER-4

MATHEMATICS (35)

TIME: 3 Hours 15 Minutes **MAX. MARKS: 100**

Instructions:

i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii) Use the graph sheet for the question on Linear programming in PART E.

PART-A

I. Answer any TEN questions: **$10 \times 1 = 10$**

1. Examine whether the operation $* : Z^+ \to Z^+$ defined by $a * b = |a - b|$, where Z^+ is the set of all positive integers, is a binary operation or not.

2. The relation R in the set {1, 2, 3} given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is not transitive. Why?

3. Write the domain of $f(x) = \sin^{-1} x$.

4. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

5. If a matrix has 5 elements, what are the possible orders it can have?

6. If A is a square matrix and $\text{adj}(A) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then find $|A|$.

7. If $y = \log(\sin x)$ find $\frac{dy}{dx}$.

8. Find $\frac{dy}{dx}$, if $y = \cos(1 - x)$.

9. Find $\int \sec^2(7 - 4x)\, dx$.

10. Evaluate $\int_{1}^{e} \frac{1}{x} dx$.

11. Find a value of x for which $x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector.

12. Write two different vectors having same direction.

13. If a line makes angle 90^0, 60^0 and 30^0 with the positive direction of x, y and z axis respectively, find its direction cosines.

14. In a linear programming problem, define optimal solution.

15. If $P(A)=0.6$, $P(B)=0.3$ and $P(E \cap F)=0.2$ find $P(F \mid E)$.

PART-B

Answer any TEN questions: **10 × 2=20**

16. Show that the function $f: N \rightarrow N$, given by $f(x)=2x$, is one-one but not onto.

17. Prove that $\sin^{-1} x+\cos^{-1} x=\frac{\pi}{2}$, $x \in[-1,1]$.

18. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $|x|>1$ in the simplest form.

19. Prove that $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}+\sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}=I$

20. Find equation of line joining $(3, 1)$ and $(9, 3)$ using determinants.

21. If $\sqrt{x}+\sqrt{y}=\sqrt{10}$, show that $\frac{dy}{dx}+\sqrt{\frac{y}{x}}=0$.

22. Find $\frac{dy}{dx}$, if $y=(\log x)^{\cos x}$.

23. Find $\frac{dy}{dx}$, if $y=\log_7(\log x)$.

24. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

25. Find $\int \sin 2x \cos 3x \, dx$.

26. Find $\int \frac{1}{\sin^2 x \cdot \cos^2 x} dx$

27. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$.

28. Find the order and degree (if define) of the differential equation,

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0.$$

29. Let the $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $|\vec{a} \times \vec{b}| = 1$. Find the angle between $\vec{a}$ and $\vec{b}$.

30. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, prove that $\vec{a}$ and $\vec{b}$, are perpendicular.

31. Find the angle between the pair of lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

32. Find the vector equations of the plane which passes through the point (5, 2, –4) and perpendicular to the line with direction ratios 2, 3, –1.

33. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even', then prove that E and F are independent events.

PART-C

Answer any TEN questions: **10 × 3=30**

34. Determine whether the relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$ is reflexive, symmetric and transitive.

35. Solve for x, $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, $x > 0$.

36. By using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

37. Without expanding, prove that $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$

38. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.

39. If $x = a(\theta + \sin\theta)$ *and* $y = a(1 - \cos\theta)$. Prove that $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$.

40. Verify Rolle's Theorem for the function $y = x^2 + 2x - 8,\ x \in [-4, 2]$.

41. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to $x-axis$.

42. Find $\int x \cdot \log x\, dx = \int \log x \cdot x\, dx$.

43. Evaluate $\int \frac{1}{(x+1)(x+2)} dx$

44. Evaluate $\int \frac{1}{1 - \tan x} dx$

45. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

46. Form the differential equation representing family of curves $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are arbitrary constants.

47. Find the equation of the curve passing through the point $(1,1)$, given that the slope of the tangent to the curve at any point is $\frac{x}{y}$.

48. Show that the position vector of the point P, which divides the line joining the points A and B having the position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m : n$ is $\frac{m\vec{b} + n\vec{a}}{m + n}$.

49. Find λ, such that the four points A (3, 2, 1), B (4, λ, 5), C (4, 2, –2) and D (6, 5, –1) are coplanar.

50. Find the shortest distance between the lines $r = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

51. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of accidents are 0.01, 0.03 and 0.15 respectively. One of the insured meets with an accident. What is the probability that he is a scooter driver?

PART-D

Answer any SIX questions: **$6 \times 5 = 30$**

52. Verify whether the function $f: N \to N$ defined by $f(x) = x^2$ is one-one, onto and bijective.

53. Let R_+ be the set of all non-negative real numbers. Show that the function $f: R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$ is invertible and write the inverse of f.

54. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$.

55. Solve the system of equations by matrix method: $4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$.

56. If $y = \sin^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

57. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the $y-$coordinate is changing 8 times as fast as the $x-$coordinate.

58. Find the integral of $\int \frac{dx}{\sqrt{a^2 - x^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{9 - 25x^2}} dx$.

59. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

60. Find the general solution of the differential equation $(x + 3y^2)\frac{dy}{dx} = y$ $(y > 0)$

61. Derive the equation of a line passing through two given points.

62. If a fair coin is tossed 10 times. Find the probability of

i) at least six heads and

ii) at most six heads

iii) exactly six heads.

63. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

i) both balls are red

ii) first ball is black and second is red.

iii) one of them is black and other is red.

PART-E

Answer any ONE question: **$1 \times 10 = 10$**

64. (a) Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

and hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$.

(b) Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$. Where I is 2×2 identity matrix and O is 2×2 zero matrix. Using this equation, find A^{-1}.

65. (a) Maximise Z=250x + 75y *subject to constraints*

$5x + y \leq 100$; $x + y \leq 60$; $x \geq 0; y \geq 0$

by graphical method.

(b) Find the value of k if

$f(x) = \begin{cases} kx+1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$.

66. (a) Find the two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

(b) Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} (5x+4)(4-x)^2$.

* * *

ANSWERS TO MODEL QUESTION PAPER-04

PART-A

1. For all $a,b \in Z^+$, there is a unique element $|a-b| in Z^+$. So * is a binary operation

2. $R=\{(1,\ 1),\ (2,\ 2),\ (3,\ 3),\ (1,\ 2),\ (2,\ 3)\}$

R is not transitive:

Q $(1,\ 2) \in R$ and $(2,\ 3) \in R$ but $(1,3) \notin R$.

3. The domain of $f(x) = \sin^{-1} x$ is $-1 \le x \le 1$.

4. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$ Q $\frac{\pi}{6} \in [0,\ \pi]$

5. Matrix containing 5 elements can have any one of the following order; 1×5 *or* 5×1 .

6. adjA $| = |A| = \begin{vmatrix} 5 & 0 \\ 0 & 5 \end{vmatrix} = 25$.[using $|\text{adjA}| = |\text{A}|^{\text{n-1}}$

7. $\therefore \frac{dy}{dx} = \frac{1}{\sin x}\cos x = \cot x$.

8. $\frac{dy}{dx} = -\sin(1-x)[0-1] = \sin(1-x)$.

9. $I = -\frac{1}{4}\tan(7-4x)$.

10. $I = \left[\log|x|\right]_1^e = \log|e| - \log|1| \ = 1 - 0 = 1$.

11. $\left(x\hat{i} + x\hat{j} + x\hat{k}\right) = 1 \Rightarrow \sqrt{3x^2} = 1$

$$\Rightarrow \pm\sqrt{3}x = 1 \Rightarrow x = \pm\frac{1}{\sqrt{3}}.$$

12. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $2\vec{a} = 2\hat{i} + 2\hat{j} + 2\hat{k}$ Then $\vec{a}$ and $2\vec{a}$ having same direction.

13. The direction cosines of given line are cos 90°, cos 60°, cos 30°, i.e., $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$.

14. Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

15. $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{0.2}{0.6} = \frac{2}{6} = \frac{1}{3}$.

PART-B

16. Let $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$.

Therefore f is one one

Further, f is not onto, as for $1 \in N$, there does not exist any x in N such that $f(x) = 2x = 1$.

17. **Proof:** Let $\sin^{-1} x = y$.

Then $x = \sin y = \cos\left(\frac{\pi}{2} - y\right)$

Therefore $\cos^{-1} x = \frac{\pi}{2} - y = \frac{\pi}{2} - \sin^{-1} x$

Hence $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

18. Let $x = sec\theta$, then $\theta = \sec^{-1} x$

Now $\sqrt{x^2 - 1} = \sqrt{sec^2\theta - 1} = \tan\theta$

Therefore $\cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) = \cot^{-1}\left(\frac{1}{tan\theta}\right)$

$$= \cot^{-1}(\cot\theta) = \theta = \sec^{-1}x .$$

19. $LHS = \cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = RHS .$$

20. Let $P(x,y)$ be any point on the line joining $A(3,1)$ and $B(9,3)$.

Therefore points A, P and B are collinear.

Area of the triangle $APB = 0$

$$\Delta = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0 \Rightarrow x[1-3] - y[3-9] + 1[9-9] = 0$$

$-2x + 6y = 0 \quad or \quad 3y = x$. This is the required equation of the line AB.

21. Differentiating with respect to x,

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \frac{1}{\sqrt{y}}\frac{dy}{dx} = -\frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\therefore \frac{dy}{dx} + \frac{\sqrt{y}}{\sqrt{x}} = 0 .$$

22. Let $y = (\log x)^{\cos x} \Rightarrow \log y = \cos x \cdot \log(\log x)$

Now, differentiating both sides with respect to x, we get,

$$\frac{1}{y}\frac{dy}{dx} = \cos x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(\cos x)$$

$$\frac{dy}{dx}=y\left[\cos x\times\frac{1}{(\log x)}\times\frac{1}{x}+\log(\log x)\times(-\sin x)\right]$$

$$\therefore\frac{dy}{dx}=(\log x)^{\cos x}\left[\frac{\cos x}{x\log x}-(\sin x)\log(\log x)\right].$$

23. Let $y=log_7(log\,x)=\dfrac{log(log\,x)}{log\,7}$

Differentiating with respect to x,

$$\frac{dy}{dx}=\frac{1}{\log 7}\frac{d}{dx}\log(\log x)\quad=\frac{1}{\log 7}\times\frac{1}{\log x}\times\left[\frac{1}{x}\right].$$

24. Volume of cube = $V=x^3\Rightarrow\dfrac{dV}{dx}=3x^2$

where x is the length of edge of cube.

$$dV=\left(\frac{dV}{dx}\right)\Delta x=\left(3x^2\right)\Delta x=\left(3x^2\right)(0.02x)$$

$$=0.06x^3m^3\qquad(as\ 2\%\ of\ x\ is\ 0.02x\ )$$

Thus, the approximate change in the volume is $0.06\ x^3m^3$.

25. Using $\sin x\cos y=\dfrac{1}{2}[\sin(x+y)+\sin(x-y)]$

$$I=\frac{1}{2}\int[\sin(2x+3x)+\sin(2x-3x)]dx$$

$$=\frac{1}{2}\int[\sin(5x)+\sin(-x)]dx\quad=\frac{1}{2}\int[\sin(5x)-\sin(x)]dx$$

$$=\frac{1}{2}\left[-\frac{\cos(5x)}{5}+\cos(x)\right]+C.$$

26. $I=\displaystyle\int\frac{\sin^2 x+\cos^2 x}{\sin^2 x\cdot\cos^2 x}dx$

$$=\int\frac{\sin^2 x}{\sin^2 x\cdot\cos^2 x}dx+\int\frac{\cos^2 x}{\sin^2 x\cdot\cos^2 x}dx$$

$$=\int\frac{1}{\cos^2 x}dx+\int\frac{1}{\sin^2 x}dx$$

$$=\int \sec^2 x\,dx+\int \operatorname{cosec}^2 x\,dx \quad =\tan x-\cot x+C$$

27. $I=\left[\tan^{-1}x\right]_0^1=\tan^{-1}(1)-\tan^{-1}(0)=\frac{\pi}{4}-0=\frac{\pi}{4}$.

28. Order is 2 and Degree is not defining.

29. We have $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

$$1=3\cdot\frac{\sqrt{2}}{3}\sin\theta$$

$$\therefore \sin\theta=\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{\pi}{4}.$$

30. Let $\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}-\vec{b}\right|^2\Rightarrow\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\vec{a}\cdot\vec{b}=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\vec{a}\cdot\vec{b}$

$\Rightarrow 2\vec{a}\cdot\vec{b}=-2\vec{a}\cdot\vec{b}\Rightarrow 4\vec{a}\cdot\vec{b}=0$

$\therefore \vec{a}\cdot\vec{b}=0$, then $\vec{a}$ and $\vec{b}$, are perpendicular.

31. Here $\vec{b}_1=\hat{i}+2\hat{j}+2\hat{k}$ *and* $\vec{b}_2=3\hat{i}+2\hat{j}+6\hat{k}$

Then, $\cos\theta=\left|\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

$$=\left|\frac{(\hat{i}+2\hat{j}+2\hat{k})\cdot(3\hat{i}+2\hat{j}+6\hat{k})}{\sqrt{1+4+4}\sqrt{9+4+36}}\right|=\left|\frac{3+4+12}{3\times 7}\right|=\frac{19}{21}$$

Hence $\theta=\cos^{-1}\left(\frac{19}{21}\right)$.

32. Let A (5, 2, –4) be given point.

$\therefore\ \vec{a}=5\hat{i}+2\hat{j}-4\hat{k}$

The direction ratios of perpendicular Vector are 2, 3, –1.

$\therefore\ \ \vec{N}=2\hat{i}+3\hat{j}-\hat{k}$

Let $P(x,y,z)$ be any point on the plane.

$\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Therefore, the vector equation of the plane is $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$[\vec{r} - (5\hat{i} + 2\hat{j} - 4\hat{k})] \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 0$.

33. We know that the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

Now $E = \{3, 6\}$, $F = \{2, 4, 6\}$ and $E \cap F = \{6\}$

Then $P(E) = \frac{2}{6} = \frac{1}{3}$, $P(F) = \frac{3}{6} = \frac{1}{2}$ *and* $P(E \cap F) = \frac{1}{6}$

Clearly $P(E \cap F) = P(E) \cdot P(F)$

Hence E and F are independent events.

PART-C

34. Here $y = 3x \Rightarrow R = \{(1,3),(2,6),(3,9),(4,12)\}$

Reflexive: As $1 \in R$ but $(1,1) \notin R$

$\therefore$ R is not reflexive.

Symmetric: As $(1,3) \in R$ but $(3,1) \notin R$

$\therefore$ R is not symmetric.

Transitive: As $(1,3) \in R$ and $(3,9) \in R$ but $(1,9) \notin R$ $\therefore$ R is not transitive.

35. Given: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ or $\tan^{-1}\left(\frac{2x + 3x}{1 - 2x \cdot 3x}\right) = \frac{\pi}{4}$

Therefore $\frac{5x}{1 - 6x^2} = \tan\frac{\pi}{4} = 1$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow 6x^2 + 6x - x - 1 = 0$$

$$\Rightarrow 6x(x+1) - 1(x+1) = 0$$

$$\Rightarrow (6x-1)(x+1) = 0$$

$$\Rightarrow x=-1 \, or \, x=\frac{1}{6}.$$

The given equation is not satisfied by $x=-1$. $\therefore x=-1$ cannot be the solution as the LHS of the equation becomes negative,

Thus $x=\frac{1}{6}$ is the only solution.

36. In order to use elementary row operations we may write $A=IA$

Or $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$

Applying $R_2 \to R_2 - 2R_1$, we get $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}A$

Applying $R_2 \to -\frac{1}{5}R_2$, then $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}A$

Applying $R_1 \to R_1 - 2R_2$, we get $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}A$.

Thus $A^{-1}=\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$.

37. Applying $R_1 \to R_1 + R_2$ to Δ, we get $\Delta=\begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

$$=(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}=(x+y+z)\times 0=0.$$

38. Let $u(x)=\sin^2 x$ and $v(x)=e^{\cos x}$

$$\frac{du}{dx}=2\sin x\cos x \qquad \frac{dv}{dx}=e^{\cos x}(-\sin x)$$

$\therefore \frac{du}{dv} = \frac{2\sin x \cos x}{e^{\cos x}(-\sin x)} = -\frac{2\cos x}{e^{\cos x}}.$

39. Let $x = a(\theta + \sin\theta)$

Therefore, $\frac{dx}{d\theta} = a(1+\cos\theta)$

Let $y = a(1-\cos\theta)$

Therefore, $\frac{dy}{d\theta} = a(0-[-\sin\theta]) = a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}.$

40. The function $y = x^2 + 2x - 8$ is continuous in $[-4,2]$ and differentiable in $(-4,2)$. Also $f(-4) = (-4)^2 + 2(-4) - 8 = 0$ and $f(2) = 2^2 + 2\times 2 - 8 = 0 \Rightarrow f(-4) = f(2) = 0$ hence the value of $f(x)$ *at* -4 *and* 2 coincide. Rolle's theorem states that there is a point $c \in (-4,2)$, where $f'(c) = 0$.

Since $f'(x) = 2x + 2$, we get $c = -1$.

Thus at $c = -1$, we have $f'(-1) = 0$ and $c = -1 \in (-4,2)$.

41. Differentiating $\frac{x^2}{4} + \frac{y^2}{25} = 1$, w.r.to x

We get $\frac{2x}{4} + \frac{2y}{25}\frac{dy}{dx} = 0 \Rightarrow \frac{2y}{25}\frac{dy}{dx} = -\frac{x}{2} \therefore m = \frac{dy}{dx} = -\frac{25x}{4y}$

Now, the tangent is parallel to the $x-axis$ if the slope of the tangent is zero which gives $-\frac{25x}{4y} = 0$.

This is possible if $x = 0$.

Then $\frac{x^2}{4} + \frac{y^2}{25} = 1$, for $x = 0$ gives $y^2 = 25$, *i.e.*, $y = \pm 5$

Thus, the points at which the tangents are parallel to the $x-axis$

are $(0,5)$ and $(0,-5)$.

42. $= \log x \int x\,dx - \int\left[\frac{d}{dx}(\log x)\int x\,dx\right]dx$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx = \frac{x^2}{2}\log x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

43. Let $\frac{1}{(x+1)\,(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$$\Rightarrow 1 = A(x+2) + B(x+1) \qquad \ldots(1)$$

Put $x=-1$ in equation (1), we get

$1 = A(-1+2) \therefore \boxed{A=1}$

Put $x=-2$, in equation (1), we get

$1 = B(-2+1) \;\therefore \boxed{B=-1}$

$$\therefore \int \frac{1}{(x+1)(x+2)}dx = \int\left[\frac{1}{x+1} - \frac{1}{x+2}\right]dx$$

$$= \log|x+1| - \log|x+2| + C$$

$$= \log\left|\frac{x+1}{x+2}\right| + C$$

44. Let $I = \int \frac{1}{1-\tan x}\,dx = \int \frac{\cos x}{\cos x - \sin x}dx$

$$= \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{2(\cos x - \sin x)}dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \frac{-\cos x - \sin x}{\cos x - \sin x}dx$$

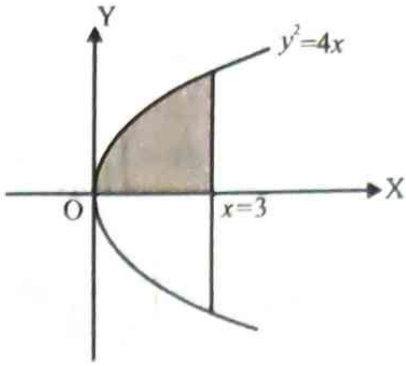
$$= \frac{1}{2}\int dx - \frac{1}{2}\int \frac{1}{t}dt,$$

where $t = \cos x - \sin x$

$\Rightarrow \; dt = (-\sin x - \cos x)\,dx$

$I = \frac{1}{2}x - \frac{1}{2}\log|t| + C \qquad = \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$

45. The required area of the region bounded by $y^2 = 4x$ and $x = 3$.



$$A = 2\int_0^3 y\,dx$$

$$= 2\int_0^3 2\sqrt{x}\,dx = 4\cdot\frac{2}{3}x^{3/2}\Big]_0^3$$

$$= \frac{8}{3}(3^{3/2}) = 8\sqrt{3} \text{ square unit.}$$

46. Given $\frac{x}{a} + \frac{y}{b} = 1$

Differentiating w.r.to x, we get, $\frac{1}{a} + \frac{1}{b}\frac{dy}{dx} = 0$

Differentiating w.r.to x, we get, $0 + \frac{1}{b}\frac{d^2y}{dx^2} = 0$

$\therefore \frac{d^2y}{dx^2} = 0$ is the required differential equation.

47. We know that slope of the tangent to the curve at any point is $\frac{dy}{dx}$

So, $\frac{dy}{dx} = \frac{x}{y} \Rightarrow y\,dy = x\,dx$ and integrating

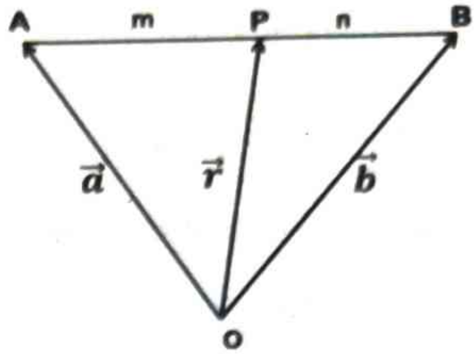
$$\int y\,dy = \int x\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C \;.....(1)$$

Equation (1) passes through the point

$(1,1)$, we get $C = 0$ in (1), we get of the required curve as

$$\frac{y^2}{2} = \frac{x^2}{2} + 0 \Rightarrow \frac{x^2}{2} - \frac{y^2}{2} = 0.$$

48. Let O be the origin.



Then $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$

Let P be a point on AB such that

$\frac{AP}{PB} = \frac{m}{n}$. Then $nAP = mPB$

$\Rightarrow n\overrightarrow{AP} = m\overrightarrow{PB}$

$\Rightarrow n(\overrightarrow{OP} - \overrightarrow{OA}) = m(\overrightarrow{OB} - \overrightarrow{OP})$

$\therefore\ (m+n)\,\overrightarrow{OP} = m\overrightarrow{OB} + n\overrightarrow{OA}$

$\Rightarrow \vec{r} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$.

49. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \equiv (1, \lambda - 2, 4)$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} \equiv (1, 0, -3)$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} \equiv (3, 3, -2)$

Since given points are coplanar, therefore, $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$

$$\therefore \begin{vmatrix} 1 & \lambda-2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$1(0+9) - (\lambda-2)(-2+9) + 4(3-0) = 0$

$9 - 7\lambda + 14 + 12 = 0$

$\lambda = 5$.

50. Here $\vec{a}_1 = \hat{i} + \hat{j},\ \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k},\ \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$\therefore \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9+1+49} = \sqrt{59}.$$

$\therefore$ The shortest distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{|3-0+7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}.$$

51. Let E_1 : 'Insured person is a scooter driver'.

E_2 : 'Insured person is a car driver'.

E_3 : 'Insured person is a truck driver'.

$$\therefore P(E_1) = \frac{2000}{2000+4000+6000} = \frac{2}{12} = \frac{1}{6};$$

$$P(E_2) = \frac{4}{12} = \frac{1}{3} \text{ and } P(E_3) = \frac{6}{12} = \frac{1}{2}.$$

Let E: 'a person meets with an accident'.

$\therefore P(E \mid E_1) = 0 \cdot 01$; $P(E \mid E_2) = 0 \cdot 03$ and $P(E \mid E_3) = 0 \cdot 15$

Required probability $= P(E_1 \mid E)$

$$= \frac{P(E_1) \cdot P(E \mid E_1)}{P(E_1)P(E \mid E_1) + P(E_2)P(E \mid E_2) + P(E_2) \cdot P(E \mid E_3)}$$

$$= \frac{\frac{1}{6} \times 0 \cdot 01}{\frac{1}{6} \times 0 \cdot 01 + \frac{1}{3} \times 0 \cdot 03 + \frac{1}{2} \times 0 \cdot 15}$$

$$= \frac{0 \cdot 01}{0 \cdot 01 + 0 \cdot 06 + 0 \cdot 45} = \frac{0 \cdot 01}{0 \cdot 52} = \frac{1}{52}.$$

PART-D

52. Let $f: R \to R$ is given by $f(x) = x^2$

It is seen that for $x, y \in N, f(x) = f(y)$

$\Rightarrow x^2 = y^2 \Rightarrow x = y$

Therefore, f is injective.

Now, $2 \in N$ but there does not exists any x in N such that $f(x) = x^2 = 2$

It means there is some elements in co-domain in which do not have any images. Therefore, f is not surjective.

Hence, function f is injective but not surjective.

53. Let $y = f(x)$. Then $y = x^2 + 4$.

Since $x^2 \geq 0$, we get, $x^2 + 4 \geq 4$. Thus $y \geq 4$. Thus $y \in [4, \infty)$.

Now $y = x^2 + 4 \Rightarrow x^2 = y - 4 \Rightarrow x = \pm\sqrt{y-4}$

But $x \in R_+$ .Hence, $y = \sqrt{y-4}$.

Define, $g: [4, \infty) \to R_+$, $g(y) = \sqrt{y-4}$

Now $(gof)(x) = g(f(x)) = g(x^2 + 4) = \sqrt{x^2 + 4 - 4} = x$

Also, $(fog)(y) = f(g(y)) = f(\sqrt{y-4})$

$$= \left(\sqrt{y-4}\right)^2 + 4 = y - 4 + 4 = y$$

Hence, $gof = I_{R_+}$ and $fog = I_{[4, \infty)}$

$\therefore f$ is invertible with the inverse given by $f^{-1} = g \Rightarrow f^{-1}(y) = \sqrt{y-4}$.

OR

Proving f is both one one and onto

Hence $f^{-1}(x)$ exists

Finding $f^{-1}(x)=\sqrt{x-4}$.

54. We have $A^2 = A\times A = \begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}$

$$=\begin{bmatrix}1+6+12 & 2-4+6 & 3+2+3\\ 3-6+4 & 6+4+2 & 9-2+1\\ 4+6+4 & 8-4+2 & 12+2+1\end{bmatrix}=\begin{bmatrix}19 & 4 & 8\\ 1 & 12 & 8\\ 14 & 6 & 15\end{bmatrix}$$

$$A^3 = AA^2 = \begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}\begin{bmatrix}19 & 4 & 8\\ 1 & 12 & 8\\ 14 & 6 & 15\end{bmatrix}$$

$$=\begin{bmatrix}63 & 46 & 69\\ 69 & -6 & 23\\ 92 & 46 & 63\end{bmatrix}$$

$$A^3-23\text{A}-40\text{I} = \begin{bmatrix}63 & 46 & 69\\ 69 & -6 & 23\\ 92 & 46 & 63\end{bmatrix}-23\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}-40\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}63-23-40 & 46-46+0 & 69-69+0\\ 69-69+0 & -6+46-40 & 23-23+0\\ 92-92+0 & 46-46+0 & 63-23-40\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=\text{O}.$$

55. Let $AX=B \Rightarrow X=A^{-1}B$, where $A=\begin{bmatrix}4&3&2\\2&4&6\\6&2&3\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}60\\90\\70\end{bmatrix}$

$$|A|=\begin{vmatrix}4&3&2\\2&4&6\\6&2&3\end{vmatrix}=4[12-12]-3[6-36]+2[4-24]=90-40=50\neq 0$$

$$\text{adjA}=\begin{bmatrix}+(12-12)&-(9-4)&+(18-8)\\-(6-36)&+(12-12)&-(24-4)\\+(4-24)&-(8-18)&+(16-6)\end{bmatrix}=\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}$$

$$\therefore\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{50}\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}\begin{bmatrix}60\\90\\70\end{bmatrix}=\frac{1}{50}\begin{bmatrix}0-450+700\\1800-0-1400\\-1200+900+700\end{bmatrix}=\frac{1}{50}\begin{bmatrix}250\\400\\400\end{bmatrix}=\begin{bmatrix}5\\8\\8\end{bmatrix}$$

Hence, $x=5,\ y=8$ *and* $z=8$.

56. Given $y=\sin^{-1}x$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}} \quad \Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx}=1$$

Squaring on both sides, we get,

$$\left(1-x^2\right)\left(\frac{dy}{dx}\right)^2=1$$

Differentiate w. r. to x

$$\left(1-x^2\right)\cdot 2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2=0$$

Divide by $2\cdot\frac{dy}{dx}$, we get,

$$\left(1-x^2\right)\cdot\frac{d^2y}{dx^2}-x\cdot\frac{dy}{dx}=0.$$

57. Given $6y = x^3 + 2$ .....(1) and $\frac{dy}{dt} = 8\frac{dx}{dt}$. Differentiating (1) with respect to t, we get $6\frac{dy}{dt} = 3x^2\frac{dx}{dt} + 0 \Rightarrow 6\times 8\frac{dx}{dt} = 3x^2\frac{dx}{dt}$

$\Rightarrow 3x^2 = 48 \Rightarrow x^2 = 16 \therefore x = \pm 4$

When $x = 4$ *then* $y = 11$ *and* $x = -4$ *then* $y = \frac{-62}{6} = -\frac{31}{3}$

Hence, the required points on the curve are $(4,11)$ and $\left(-4, \frac{-31}{3}\right)$.

58. Put $x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$

$$\therefore I = \int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{a\cos\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\, d\theta$$

$$= \int \frac{a\cos\theta}{a\sqrt{1 - \sin^2\theta}}\, d\theta$$

$$= \int 1\cdot d\theta = \theta + C = \sin^{-1}\frac{x}{a} + C$$

Let $I = \int \frac{1}{\sqrt{9 - 25x^2}}\, dx = \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$.

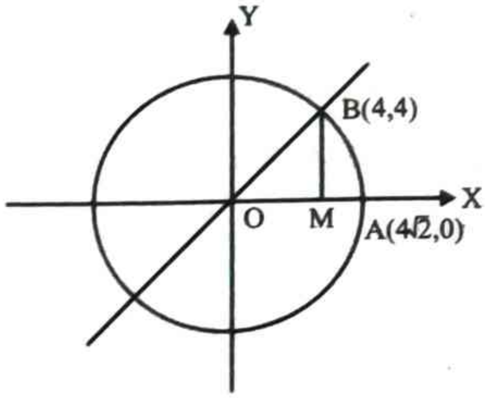
59. The given circle: $x^2 + y^2 = 32$ ...(1)

The given line is $y = x$ ...(2)

Solve (1) and (2) we get $x = \pm 4,\ y = \pm 4$

$\therefore$ B (4, 4) is the point of intersection.

Draw $BM \perp^r$ to x-axis.



$\therefore$ Required area

$= \text{(Area OMB)} + \text{(Area MBAM)}$

$$= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32-x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32-x^2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= (8-0) + \left[0 + 16\left(\frac{\pi}{2}\right) - \frac{4}{2}\sqrt{16} - 16\sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= 8 + 8\pi - 8 - 16\left(\frac{\pi}{4}\right) = 4\pi \text{ square unit.}$$

60. Given differential equation is $\frac{x+3y^2}{y} = \frac{dx}{dy}$

$$\therefore \frac{dx}{dy} - \frac{1}{y}x = 3y$$

Here $P = -\frac{1}{y}, Q = 3y$

$$I \cdot F \cdot = e^{\int pdy} = e^{\int \frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$$

$\therefore$ Required solution is

$$x\frac{1}{y} = \int \frac{1}{y} 3y dy + C = 3y + C$$

$$\therefore x = 3y^2 + Cy$$

61. Let $\vec{a}$ and $\vec{b}$ be the position vector of two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively that are lying in a line.

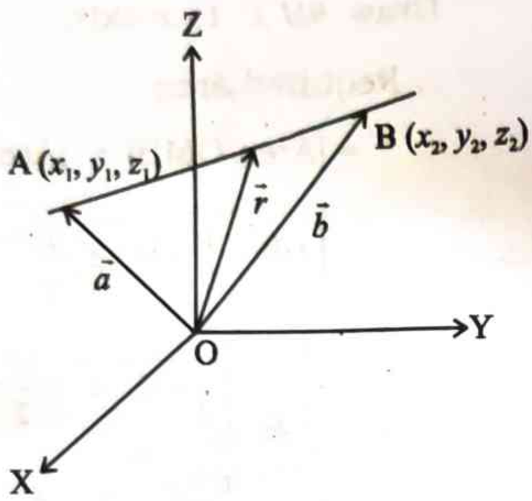
Let $\vec{r}$ be the position vector of an arbitrary point $P(x, y, z)$

Now P lies on AB iff $\overrightarrow{AP} = \vec{r} - \vec{a}$ and $\overrightarrow{AB} = \vec{b} - \vec{a}$ are collinear vectors.

$$\Rightarrow \vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$$

$$\therefore \ \vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

is required equation of the line.



Cartesian form:

We have $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

Equation of a line in vector form is

$$x\hat{i} + y\hat{j} + z\hat{k} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} +$$

$$\lambda[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}]$$

Equating the co-efficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ we get,

$$x = x_1 + \lambda(x_2 - x_1)$$

$$y = y_1 + \lambda(y_2 - y_1)$$

$$z = z_1 + \lambda(z_2 - z_1)$$

On eliminating λ, we get

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

is the Cartesian form of the equation of a line.

62. The trials are Bernoulli trials.

Let X denote the number of heads in an experiment of 10 trials.

Then X has the binomial distribution.

Here n=10 , p=$\frac{1}{2}$ (*probality of getting head in single trial*); q=1-$\frac{1}{2}$ = $\frac{1}{2}$

Now P(X=x)=$^{n}C_x q^{n-x} p^x$, $x = 0,1,2,\ldots\ldots n$

$$\therefore P(X=x) = {}^{10}c_x\left(\frac{1}{2}\right)^{10-x}.\left(\frac{1}{2}\right)^{x} = {}^{10}c_x\left(\frac{1}{2}\right)^{10}$$

i) Probality of atleast six heads=P(X≥6)

$= P(X=6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$

$$= {}^{10}c_6\left(\frac{1}{2}\right)^{10} + {}^{10}c_7\left(\frac{1}{2}\right)^{10} + {}^{10}c_8\left(\frac{1}{2}\right)^{10} + {}^{10}c_9\left(\frac{1}{2}\right)^{10} + {}^{10}c_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}[{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}]$$

$$= \left(\frac{1}{1024}\right)[{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + 1]$$

$$= \frac{1}{1024}[{}^{10}C_4 + {}^{10}C_3 + {}^{10}C_2 + {}^{10}C_1 + 1]$$

$$= \frac{1}{1024}[210 + 120 + 45 + 10 + 1] = \frac{1}{1024}.386 = \frac{193}{512}$$

ii) $P(X \leq 6)$

$= 1 - [\ P(X=7)+P(X=8)+P(X=9)+P(X=10)]$

$$= 1 - \frac{1}{1024}[210 + 45 + 10 + 1] = 1 - \frac{176}{1024} = \frac{848}{1024} = \frac{53}{64}$$

iii) $P(X = 6) = {}^{10}c_6\left(\frac{1}{2}\right)^{10} = \frac{10.9.8.7}{24}.\frac{1}{1024} = \frac{105}{512}$

63. Let B = black ball, R = red ball.

Total number of balls = 18

Number of black balls $= 10 \Rightarrow P(B) = \frac{10}{18}$

Number of red balls $= 8 \Rightarrow P(R) = \frac{8}{18}$

(i) P(both are red balls)

$$= P(RR) = P(R)P(R) = \frac{8}{18} \times \frac{8}{18} = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}.$$

(ii) P(first ball is black and second is red)

$$= P(BR) = P(B) \cdot P(R) = \frac{10}{18} \times \frac{8}{18} = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$$

(iii) P(one of them is black and other is red)

$$= P(BR) + P(RB)$$

$$= P(B) \cdot P(R) + P(R) \cdot P(B)$$

$$= \frac{10}{18} \times \frac{8}{18} + \frac{10}{18} \times \frac{8}{18} = \frac{40}{81}.$$

PART-E

64. (a) Proof: Consider $I = \int_0^a f(x)\,dx$

Putting $x = a - t$, then $dx = -dt$.

Also $x = 0$, *then* $0 = a - t \Rightarrow t = a$ and

$x = a$, *then* $a = a - t \Rightarrow t = 0$

$$\therefore I = \int_a^0 f(a-t)(-dt)$$

$$= -\int_a^0 f(a-t)\,dt \text{ by property}$$

$$= \int_0^a f(a-t)\,dt \text{ by property}$$

$$= \int_0^a f(a-x)\,dx.$$

Consider $I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(1)

Using the above property, we have

$$I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$$I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \qquad ...(2)$$

Adding (1) and (2) we get, $2I=\int_0^{\frac{\pi}{2}}1\,dx=[x]_0^{\frac{\pi}{2}}$

$$2I=\frac{\pi}{2} \qquad \therefore I=\frac{\pi}{4}.$$

(b) We have $A^2=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}$

$$LHS=A^2+(-4)A+I=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}+\begin{bmatrix}-8 & -12\\ -4 & -8\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O=RHS$$

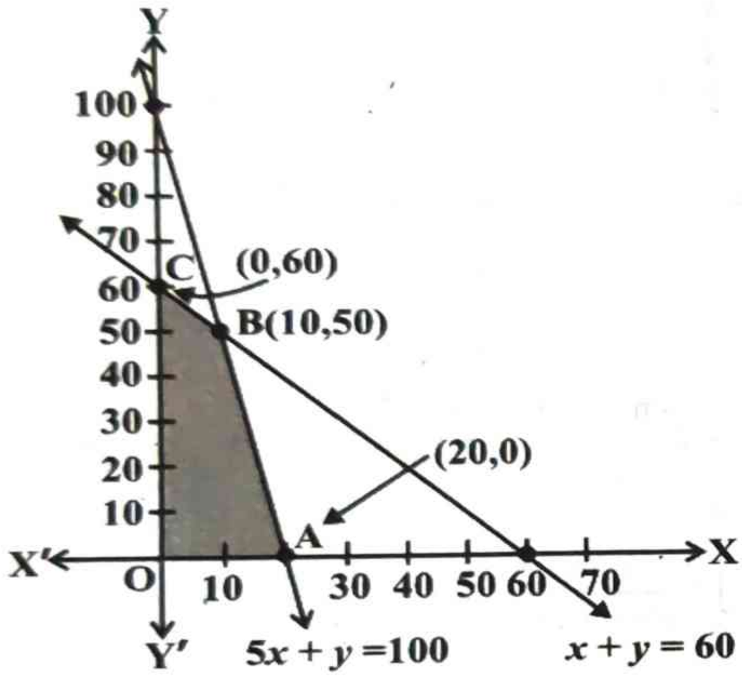
Now $A^2-4A+I=O\Rightarrow I=-A^2+4A$

Therefore $IA^{-1}=-A^2A^{-1}+4AA^{-1}$ (post multiply by A^{-1} because $|A|\neq 0$)

Or $IA^{-1}=(-A)(AA^{-1})+4(AA^{-1})$

Or $A^{-1}=(-A)I+4I=\begin{bmatrix}-2 & -3\\ -1 & -2\end{bmatrix}+\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}=\begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}.$

65. (a): Drawing graph of the system of linear inequalities :



Equation	x	y	Conclusion
5x+y=100	20	0	The line cuts X axis at (20,0) and Y axis at (0,100)
	0	100	
X+y=60	60	0	The line cuts X axis at (60,0) and Y axis at (0,60)

Verification of feasible region

INEQUATION	TESTING CO ORDINATES	VERIFICATION	INFERENCE
$5x + y \leq 100$	(0,0)	0+0≤100 true (put x=0, y=0 in the inequation)	The solution region contains origin
$x + y \leq 60$	(0,0)	0+0≤100 true (put x=0, y=0 in the inequation)	The solution region contains origin

The shaded region represents feasible region and corner points are (0,0), (20,0) , (10,50), (0,60).

Evalauting corresponding value of objective function,

$Z=250x+75y$ at each corner point.

Vertex of the Feasible Region	Corresponding value of Z (in Rs)
O (0,0)	0
C (0,60)	4500
B (10,50)	**6250**←Maximum
A (20,0)	5000

(b) Given function is continuous at $x=\pi$

Therefore $f(\pi)=LHL=RHL$

Here $f(\pi)=\pi k+1$

$$RHL=\lim_{x\to\pi^+} f(x)=\lim_{x\to\pi^+}(\cos x)=\cos\pi=-1$$

Therefore $\Rightarrow \pi k+1=-1\Rightarrow \pi k=-2 \quad \therefore k=\dfrac{-2}{\pi}$.

66.(a) Given $x+y=35$ and $x>0,\ y>0$

$\Rightarrow x=35-y$ ... (1)

Let $P=x^2y^5 \qquad \therefore P=(35-y)^2y^5$

We have,

$$\frac{dp}{dy}=(35-y)^2(5y^4)+y^5\cdot 2(35-y)(-1)$$

$$=y^4(35-y)\left[(35-y)\,5-2y\right]=y^4(35-y)\,(175-7y)$$

$$=y^4(6125-420y+7y^2)\ =6125\cdot y^4-420y^5+7y^6$$

and $\dfrac{d^2p}{dy^2}=24500y^3-2100y^4+42y^5$

For maxima, $\frac{dp}{dy}=0$

$$\Rightarrow y^4(35-y)(175-7y)=0$$

$$\Rightarrow y=0,\ 35-y=0, 175-7y=0$$

$$\Rightarrow 175-7y=0 \text{ Q } y>0 \text{ and } y<35$$

$$\Rightarrow y=\frac{175}{7}=25$$

Also $\left(\frac{d^2p}{dy^2}\right)_{y=25}=\left(y^2(24500-2100y+42y^2)\right)_{y=25}$

$$=-(25)^3(1750)<0$$

$\therefore$ By second derivative test, $y=25$ is a point of local maxima of p.

Now, from (1), $x=35-25=10$.

Hence, P is maximum when the numbers are 10 and 25.

$\therefore$ Required numbers are x = 10 and y = 25.

66.(b) LHS = $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$$\begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix} \text{Q } c_1 \to c_1+c_2+c_3$$

Takeout $(5x + 4)$ from c_1

$$=(5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix} \quad =(5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & 4-x & 0 \\ 0 & 0 & 4^{-x} \end{vmatrix}$$

Q $R_2 \to R_2-R_1$ and $R_3 \to R_3-R_1$

Expanding along R_1, we get

$$\text{LHS}=(5x+4)[1(4-x)(4-x)-0+0]$$

$$(5x+4)(4-x)^2=\text{RHS}.$$

* * *

DEPARTMENTAL MODEL QUESTION PAPER-2022

MATHEMATICS (35)

TIME: 3 Hours 15 Minutes **MAX. MARKS: 100**

Instructions:

i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii) Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer any TEN questions: **10 × 1=10**

1. Give an example of a relation which is symmetric and transitive but not reflexive.

2. Define a binary operation.

3. Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$.

4. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.

5. Define a row matrix.

6. Find the value of x if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.

7. If $y = e^{cosx}$, $find\ \frac{dy}{dx}$.

8. If $y = sin(x^2+5)$, $find\ \frac{dy}{dx}$.

9. Find $\int(2x^2+e^x)dx$.

10. Evaluate $\int_2^3 \frac{1}{x}dx$.

11. Find the unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$.

12. Write two different vectors having same magnitude.

13. Write the direction cosines of x-axis.

14. Define feasible region of a linear programming problem.

15. Find P(A|B), if P(B) = 0.5 and P(A∩B) = 0.32.

PART- B

Answer any TEN questions. **10 × 2=20**

16. Show that the signum function f:R→R given by

$$f(x) = \begin{cases} 1 & if \;\; x > 0 \\ 0 & if \;\; x = 0 \\ -1 & if \;\; x < 0 \end{cases}$$ is neither one-one nor onto.

17. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

18. Write the domain and range of y =$\tan^{-1} x$.

19. Find the values of x, y and z from the equation $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$.

20. Find equation of line joining (1, 2) and (3, 6) using determinants.

21. If $x^2 + xy + y^2 = 100$, find $\frac{dy}{dx}$.

22. If $x = at^2$, $y = 2at,\ find\ \frac{dy}{dx}$.

23. Differentiate $\sin(cos(x^2))$ with respect to x.

24. Find the slope of tangent to curve $y = x^3 - x + 1$ at the point whose x-co-ordinate is 2.

25. Find $\int \frac{(logx)^2}{x}$dx.

26. Find $\int \frac{sin^2x - cos^2x}{sin^2x\ cos^2x}$ dx.

27. Evaluate $\int_0^{\frac{\pi}{2}} cos^2x\ dx$.

28. Find the order and degree of the differential equation $y^1 + y = e^x$.

29. Find the projection of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$.

30. Find the area of the parallelogram whose adjacent sides are given by the vectors: $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.

31. Find the intercepts cut-off by the plane 2x+y-z=5.

32. Find the distance of the point (-6,0,0) from the plane 2x – 3y + 6z -2 = 0

33. The random variable X has a probability distribution P(X) of the following form where k is some number. Find the value of k.

$$P(X) = \begin{cases} k, & if\ x = 0 \\ 2k, & if\ x = 1 \\ 3k, & if\ x = 2 \\ 0, & otherwise \end{cases}$$

PART – C

Answer any TEN questions: **10 × 3=30**

34. Show that the relation R in the set {1, 2, 3} given by

R = { (1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

35. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

36. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ using elementary operations.

37. Verify that the value of the determinant remains unchanged if its rows and columns are interchanged by considering third order determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

38. If $xy = e^{x-y}\ find\ \frac{dy}{dx}$.

39. If $x = a(cos\theta + \theta sin\theta),\ y = a(sin\theta - \theta cos\theta)$ find $\frac{dy}{dx}$.

40. Verify mean value theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a, b] where a=1 and b=4.

41. Find the intervals in which the function f given by

$f(x) = 2x^3 - 3x^2 - 36x + 7$ is a) increasing (b) decreasing.

42. Find $\int \frac{xe^x}{(1+x)^2}$dx.

43. Evaluate $\int \frac{1}{(x+1)(x+2)} dx$.

44. Evaluate $\int_0^2 e^x dx$ as the limit of a sum.

45. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x= 4$ and the x-axis in the first quadrant.

46. Form the deferential equation representing the family of curves $y=a.\sin(x+b)$, where a, b are arbitrary constants.

47. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

48. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+ \vec{b} + \vec{c} = \vec{o}$, find the value of $\vec{a} .\vec{b} + \vec{b} . \vec{c} + \vec{c}. \vec{a}$.

49. Find x such that the four points A(3,2,1), B(4,x,5), C (4,2, -2) and D(6,5, -1) are co-planar.

50. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

51. A man is known to speak truth 3 out of 4times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART –D

Answer any SIX questions: **6 × 5=30**

52. Let A = R – {3} and B = R–{1}. Consider the function f: A → B defined $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

53. Let f: N→R be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that f: N→S where, S is the range of f is invertible. Find the inverse of f.

54. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)^T= A^T+B^T$ (ii) $(A-B)^T= A^T-B^T$.

55. Solve system of linear equation, using matrix method.

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

56. If $y = 3\cos(logx) + 4\sin(logx)$ Show that $x^2y_2 + xy_1 + y = 0$.

57. The length x of a rectangle is decreasing at the rate of 5cm/minute and the width y is increasing at the rate of 4cm/minute. When x = 8 cm and y = 6cm find the rates of change of

a) the perimeter b) the area of the rectangle.

58. Find the integral of $\frac{1}{x^2+a^2}$ with respect to x and hence evaluate $\int \frac{3x^2}{x^6+1}$ dx.

59. Find the area enclosed by the circle $x^2 + y^2 = a^2$ using integration.

60. Find the general solution of the differential equation

$x\frac{dy}{dx} + 2y = x^2 \quad (x \neq 0)$.

61. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

62. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize i) at least once ii) exactly once

63. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

PART – E

Answer any ONE question: **1 × 10=10**

64. **(a)** Maximise Z = 3x + 2y subject to the constraints $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$

(b) If the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, when I is 2×2 identify matrix and O is 2×2 zero matrix. Using this equation find A^{-1}.

65. **(a)** Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx.$$

(b) Find the value of K so that the function

$$f(x) = \begin{cases} Kx + 1 & if\ x \leq 5 \\ 3x - 5 & if\ x > 5 \end{cases}$$ is continuous at x = 5.

66. **(a)** Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

b) By using properties of determinants

Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

* * *

ANSWERS TO DEPARTMENTAL MODEL QUESTION PAPER-2022

PART-A

1. For A = { 4, 6, 8}; R = { (4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6) }

2. A binary operation * on a set A is a function * : *A ′ A*'!*A*. We denote * (*a*, *b*) by *a* * *b*.

3. $$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \pi - \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

4. $$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$$

$$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}(1) \Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \cos^{-1}x + \sin^{-1}x$$

$$\Rightarrow \sin^{-1}\frac{1}{5} = \sin^{-1}x \Rightarrow x = \frac{1}{5}$$

5. A matrix is said to be a row matrix if it has only one row and many number of columns.

6. $$\begin{vmatrix} x & 2 \\ 18 & 2 \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$2x - 36 = 36 - 36$$

$$2x = 36$$

$$x = \frac{\cancel{36}^{18}}{\cancel{2}} = 18$$

7. $$y = \cos(1-x)$$

$$\frac{dy}{dx} = -\sin(1-x)(-1) = \sin(1-x)$$

8. $\frac{dy}{dx} = \cos(x^2+5) \times 2x = 2x\cos(x^2+5)$

9. Given integral $= 2\frac{x^3}{3} + e^x + C$

10. Given integral= $\log x]_2^3 = \log 3 - \log 2 = \log\frac{3}{2}$

11. $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\hat{a} = \frac{a}{|a|} = \frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}$

$|\vec{a}| = \sqrt{2^2+3^2+1^2}$

$= \sqrt{4+9+1}$

$= \sqrt{14}$

12. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$\therefore |\vec{a}| = |\vec{b}|$

13. The direction cosines of x-axis is (0, 1, 0).

14. The common region determined by all the constraints including non-negative constraints of a linear programming problems is called feasible region.

15. $P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{0\cdot 32}{0\cdot 5} = 0\cdot 64$

PART- B

16. Since $f(1) = 1 = f(2)$ Q $1>0,\ 2>0$

and $f(-2) = -1 = f(-3)$ Q $-2<0,\ -3>0$

Here $f(x_1) = f(x_2) \Rightarrow x_1 \neq x_2$

$\therefore f$ is not one-one.

Also, range of $f = \{1, 0, -1\}$

$\therefore$ Range of $f \neq R \Rightarrow$ f is not onto.

$\therefore$ f is neither one-one nor onto.

17. $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

$$= \tan^{-1}(\sqrt{3}) - \left[\pi - \sec^{-1}(2)\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \left[\pi - \sec^{-1}\left(\sec\frac{\pi}{3}\right)\right]$$

$$= \frac{\pi}{3} - \left[\pi - \frac{\pi}{3}\right] = \frac{\pi}{3} - \pi + \frac{\pi}{3} = -\frac{\pi}{3}$$

18. Domain $=$ R and Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

19. $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix} \Rightarrow 4 = y, 3 = z, x = 1$

20. The equation of the line joining (1, 2) and (3, 6) is $\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$

$$x(2-6) - y(1-3) + 1(6-6) = 0$$

$$-4x + 2y + 0 = 0$$

$$-4x + 2y = 0 \quad \text{OR} \quad 4x - 2y = 0$$

21. Given $x^2 + y^2 + xy = 100$

Differentiate w.r.t. 'x'.

$$2x + 2y \cdot \frac{dy}{dx} + \left[x \cdot \frac{dy}{dx} + y(1)\right] = 0$$

$$2x + 2y\frac{dy}{dx} + x \cdot \frac{dy}{dx} + y = 0$$

$$[2y + x]\frac{dy}{dx} = -(2x + y)$$

$$\frac{dy}{dx} = \frac{-(2x+y)}{2y+x}$$

22. Given that, $x = at^2, y = 2at$

$$\therefore \frac{dx}{dt} = 2at, \frac{dy}{dx} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

23. Let, $f(x) = \sin\left(\cos(x^2)\right)$

$$f'(x) = \cos(\cos x^2)\frac{d}{dx}\cos(x^2)$$

$$= \cos(\cos x^2)\left(-\sin(x^2)\right)\frac{d}{dx}x^2$$

$$= \cos(\cos x^2)(-\sin(x^2))\frac{d}{dx}x^2$$

$$= -\cos(\cos x^2)\sin(x^2)\cdot 2x$$

24. Given: $y = x^3 - x + 1$

$$\therefore \frac{dy}{dx} = 3x^2 - 1$$

$$\therefore \text{Required slope} = \left.\frac{dy}{dx}\right)_{x=2} = 3(2)^2 - 1 = 11$$

25. $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$$\therefore I = \int \frac{(\log x)^2}{x}dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C = \frac{1}{3}(\log x)^3 + C$$

26. $I = \int\left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}\right)dx = \int\left(\sec^2 x - \cos ec^2 x\right)dxs$

$$= \tan x - \cot x + C$$

27. $I = \int_0^{\pi/2} \frac{1+\cos 2x}{2}dx$

$$=\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi/2}=\frac{1}{2}\left[\left(\frac{\pi}{2}+0\right)-(0+0)\right]=\frac{\pi}{4}$$

28. Order = 1, degree 1.

29. $\vec{a} = 2\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$

The projection of $\vec{a}$ on $\vec{b}$ is $\dfrac{\vec{a}\,.\,\vec{b}}{|\vec{b}|}$

$\vec{a}\,.\vec{b} = 2+6+2=10$; $|\vec{b}| = \sqrt{1^2+2^2+1^1} = \sqrt{1+4+1} = \sqrt{6} = \dfrac{10}{\sqrt{6}}$ *units.*

30. $\vec{a} = \hat{i}-\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}-7\hat{j}+\hat{k}$

The area of the parallellogram is $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$$= \hat{i}(-1+21)-\hat{j}(1-6)+\hat{k}(-7+2) \Rightarrow 20\hat{i}+5\hat{j}-5\hat{k}$$

$$\Rightarrow \sqrt{(20)^2+(5)^2+(-5)^2} \Rightarrow \sqrt{400+75+25} = \sqrt{450} = 15\sqrt{2}$$

31. Given plane is $2x+y-z=5$

$$\Rightarrow \frac{x}{\frac{5}{2}}+\frac{y}{5}+\frac{z}{-5}=1$$

$\therefore$ x – intercept = $\dfrac{5}{2}$

y - intercept = 5

z – intercept = 5.

32. $p=\dfrac{|-12+0+0-2|}{\sqrt{4+9+36}}=\dfrac{14}{7}=2$

33.

$$\Sigma p(n) = 1$$

$$k+2k+3k+0 = 1$$

$$6k = 1$$

$$k = \frac{1}{6}$$

PART - C

34. R = { (1, 2), 92, 2), (3, 3), (1, 2), (2, 3) }

R is reflexive: Q (1, 1), (2, 2) and (3, 3) belong to R.

R is symmetric: Q $(1,2) \in R$ but $(2,1) \notin R$

R is not transitive: Q $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$

35. LHS = $= 2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$$= \tan^{-1}\left(\frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{\frac{3}{4}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\left(\frac{4}{3}\right)\left(\frac{1}{7}\right)}\right)$$

$$= \tan^{-1}\left(\frac{28+3}{21-4}\right) = \tan^{-1}\left(\frac{31}{17}\right) = \text{RHS}$$

36. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ $\qquad A = IA$

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \qquad R_2^1 = R_2 - 2R_1$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \qquad R_2^1 = \frac{R_2}{5}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{2}{5} & -\frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \qquad R_1^1 = R_1 + R_2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{2}{5} & -\frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \qquad \therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

37. $$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = -28$$

Interchaning rows R_2 and R_3 i.e. $R_2 \leftrightarrow R_3$, we have

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 5 & -7 \\ 6 & 0 & 4 \end{vmatrix}$$

Expanding the determinant Δ_1 along first row, we have

$$\begin{aligned} \Delta_1 &= 2\begin{vmatrix} 5 & -7 \\ 0 & 4 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -7 \\ 6 & 4 \end{vmatrix} + 5\begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix} \\ &= 2(20-0) + 3(4+42) + 5(0-30) \\ &= 40 + 138 - 150 \\ &= 28 \end{aligned}$$

Clearly $\Delta_1 = -\Delta$; Hence property is verified.

38. Given $xy = e^{(x-y)}$ ...(1)

Differentiate w.r.to x, we get, $x\frac{dy}{dx} + y = e^{(x-y)}\left[1 - \frac{dy}{dx}\right]$

$$\left[x + e^{x-y}\right]\frac{dy}{dx} = e^{x-y} - y$$

$$\frac{dy}{dx} = \frac{e^{x-y} - y}{x + e^{x-y}}$$

$$\text{or} \quad \frac{dy}{dx} = \frac{xy - y}{x + xy} = \frac{y(x-1)}{x(y+1)} \qquad \text{using (1)}$$

39. Given that $x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta)$

$$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta) = a\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

40. Given function $f(x) = x^2 - 4x - 3,\ x \in [1,4]$, which being a polynomial, is continuous and differentiable.

$\therefore$ $f(x)$ is continuous in $[1, 4]$ and $f(x)$ is differentiable in $(1, 4)$.

$f'(x) = 2x - 4 \quad ; \quad f(1) = 1 - 4 - 3 = -6 \quad ; \quad f(4) = 16 - 16 - 3 = -3$

Suppose $f'(c) = \dfrac{f(4) - f(1)}{4-1} \Rightarrow 2c - 4 = \dfrac{-3-(-6)}{4-1} = 1$

$$c = \frac{5}{2} \in [1,4]$$

$$\therefore \exists \frac{5}{2} \in (1,4) \ni f'\left(\frac{5}{2}\right) = \frac{f(4) - f(1)}{4-1}$$

Hence, mean value theorem is verified for $f(x)$ in $[1, 4]$

41. $f(x) = 2x^3 - 3x^2 - 36x + 7$

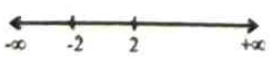
$\therefore \ f'(x) = 6x^2 - 6x - 36$

$\therefore f'(x) = 0 \Rightarrow 6(x^2 - x - 6) = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0 \qquad Q6 \neq 0$

$\Rightarrow (x-3)\,(x+2) = 0 \ \Rightarrow x = 3, -2$

$\therefore$ The points $x = 3$, $x = -2$ divides the real line into three disjoint intervals namely, $(-\infty, -2)$, $(-2, 3)$ and $(3, \infty)$



Interval	Sign of $f'(x)$	Nature of f
$(-\infty, -2)$	$(-)(-) > 0$	f is strictly increasing
$(-2, 3)$	$(-)(+) < 0$	f is strictly decreasing
$(3, \infty)$	$(+)(+) > 0$	f is strictly increasing

f is strictly increasing if $x \in (-8, -2) \cup (3, \infty)$ and strictly decreasing in $(-2, 3)$.

42. $I = \int \frac{x}{(1+x)^2} e^x dx = \int e^x \left[\frac{x+1-1}{(1+x)^2}\right] dx$

$$= \int e^x \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] dx = e^x \left(\frac{1}{x+1}\right) + C$$

43. Let $\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$\Rightarrow 1 = A(x+2) + B(x+1)$ . . . (1)

Put $x = -1$ in equation (1), we get

$1 = A(-1+2) \therefore \boxed{A = 1}$

Put $x = -2$, in equation (1), we get

$1 = B(-2+1) \therefore \boxed{B = -1}$

$$\therefore \int \frac{1}{(x+1)(x+2)} dx = \int \left[\frac{1}{x+1} - \frac{1}{x+2}\right] dx$$

$$= \log|x+1| - \log|x+2| + C$$

$$= \log\left|\frac{x+1}{x+2}\right| + C$$

44. Let $I = \int e^x \sin x \, dx$

Integrating by parts

$$I = ex(-\cos x) - \int (-\cos x) e^x dx$$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

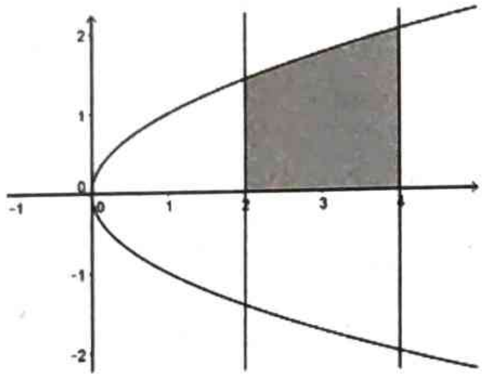
$$= -e^x \cos x + e^x \sin x - \int \sin x\, e^x dx$$

$$I = -e^x \cos x + e^x \sin x - I$$

$$2I = -e^x \cos x + e^x \sin x$$

$$I = \frac{e^x}{2}[\sin x - \cos x] + c$$

45. The required area of the region bounded by $y^2 = 9x,\ x = 2, x = 4$ and the x-axis,



$$A = \int_2^4 y\, dx$$

$$= \int_2^4 3\sqrt{x}\, dx$$

$$= 3 \cdot \frac{2}{3} x^{3/2} \Big]_2^4$$

$$= 2(4^{3/2} - 2^{3/2}) = 2(8 - 2\sqrt{2}) \text{ square unit.}$$

46. $y = a \sin (x + b)$ ... (1)

Differentiate w.r.t. 'x'. $\quad \dfrac{dy}{dx} = a \cos (x + b)$

Differentiate again w.r.t. 'x'

$$\frac{d^2y}{dx^2} = -a \sin (x + b) \ ; \quad \frac{d^2y}{dx^2} = -y \qquad \text{[From 1]}$$

then, $\dfrac{d^2y}{dx^2} + y = 0$ is the required differential equation.

47. Given equation is $\dfrac{dy}{dx} = \dfrac{1 + y^2}{1 + x^2}$ ... (1)

By variable separable,

$$\frac{dy}{1 + y^2} = \frac{dx}{1 + x^2}$$

$$\Rightarrow \int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2} + C$$

$\therefore \tan^{-1} y = \tan^{-1} + C$ which is the general solution of (1).

48. If $\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$ and $\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$

Let $c = (\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 16\hat{i} - 16\hat{j} - 8\hat{k} = 8(2\hat{i} - 2\hat{j} - \hat{k})$

$|\vec{c}| = 8\sqrt{4+4+1} = 8\sqrt{9} = 8 \times 3 = 24$

$\therefore$ Required unit vector perpendicular to both vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is

$$\frac{\vec{c}}{|\vec{c}|} = \frac{8(2\hat{i} - 2\hat{j} - \hat{k})}{24} = \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3}$$

49. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \equiv (1, x-2, 4)$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} \equiv (1, 0, -3)$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} \equiv (3, 3, -2)$

Since given points are coplanar, therefore $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$.

$$\therefore \begin{vmatrix} 1 & x-2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$1(0+9) - (x-2)(-2+9) + 4(3-0) = 0$$
$$9 - 7x + 14 + 12 = 0$$
$$x = 5$$

50. $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1}$ and $\dfrac{x-3}{1} = \dfrac{y-5}{-2} = \dfrac{z-7}{1}$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3-(-1) & 5-(-1) & 7-(-1) \\ 7 & -6 & -1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & -1 \\ 1 & -2 & 1 \end{vmatrix} = 4(-6-2) - 6(7+1) + 8\ (-14+6)$$

$$= -32 - 48 - 64 = -144$$

$$\sqrt{b_1c_2 - b_2c_1)^2 + (a_1c_2 - a_2c_1)^2 + (a_1b_2 - a_2b_1)^2} = \sqrt{(-8)^2 + 8^2 + 8^2}$$

$$= \sqrt{64 + 64 + 64} = 8\sqrt{3}$$

$$\therefore S.D. = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (a_2c_2 - a_2c_1)^2 + (a_2b_2 - a_2b_1)^2}}$$

$$= \left|\frac{-144}{8\sqrt{3}}\right| = \frac{144}{8\sqrt{3}} = \frac{18}{\sqrt{3}}$$

51. Let A : 'six occurs in the throw'. ; A^1 : 'six does not occurs.'

$$\therefore P(A) = \frac{1}{6}\ ;\ P(A^1) = 1 - \frac{1}{6} = \frac{5}{6}$$

Let E : 'man reports that six.'

$$\therefore P(E / A) = \frac{3}{4}\ ;\quad P(E / A') = 1 - \frac{3}{4} = \frac{1}{4}$$

Required probability =

$$P\left(\frac{A}{E}\right) = \frac{P(A)\ P\left(\frac{E}{A}\right)}{P(A)\ P\left(\frac{E}{A}\right) + P(A^1)\ P\left(\frac{E}{A^1}\right)} = \frac{\frac{1}{6}\cdot\frac{3}{4}}{\frac{1}{6}\cdot\frac{3}{4} + \frac{5}{6}\cdot\frac{1}{4}} = \frac{3}{3+5} = \frac{3}{8}$$

PART –D

52. $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in A = R - \{3\}$,

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$\Rightarrow \cancel{x_1x_2} - 3x_1 - 2x_2 + 6 = \cancel{x_1x_2} - 3x_2 - 2x_1 + 6$$

$$\Rightarrow 3x_2 - 2x_2 = 3x_1 - 2x_1$$

$$\Rightarrow x_2 = x_1 \;\; \Rightarrow x_1 = x_2$$

$\therefore$ f is one-one

Let $y \in B$ and let $f(x) = y$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x - 2 = xy - 3y \;\; \Rightarrow x - xy = 2 - 3y$$

$$\Rightarrow x(1-y) = 2 - 3y \;\; \Rightarrow x = \frac{2-3y}{1-y} \in A$$

$\therefore$ Corresponding to each $y \in B$ there exists $\left(\frac{2-3y}{1-y}\right) \in A$ such that $f\left(\frac{2-3y}{1-y}\right) = y$.

$\therefore$ f is onto.

53. Given f: N $\rightarrow$ R defined as $f(x) = 4x^2 + 12x + 15$

Let $y \in R_f$ (range of f)

then $y = 4x^2 + 12x + 15 = (2x + 3)^2 + 6 \Rightarrow (2x + 3)^2 = y - 6$

$$\Rightarrow x = \frac{\sqrt{y-6} - 3}{2}, \; \forall x \in N$$

$$\therefore y - 6 \geq 0 \Rightarrow y \geq 6 \Rightarrow y \in [6, \infty)$$

Define $g : S \to N$ by $g(y) = \dfrac{\sqrt{y-6}-3}{2}$, $\forall y \in R_f = S = [6, \infty)$

Now $g \circ f(x) = g\left(f(x)\right) = g(4x^2+12x+15)$

$$= g\left((2x+3)^2+6\right) = \frac{\sqrt{(2x+3)^2+6-6}-3}{2} = \frac{2x+3-3}{2} = x = I_N$$

and $f \circ g(y) = f\left(g(y)\right) = f\left(\dfrac{\sqrt{y-6}-3}{2}\right)$

$$= \left[2\frac{(\sqrt{y-6}-3)}{2}+3\right]^2 + 6 = \left[\sqrt{y-6}-3+3\right]^2+6 = y = I_S$$

i.e., $g \circ f = I_N$ and $f \circ g = I_S$

$\Rightarrow f$ is invertible with $f^{-1} = g$;

$$\therefore f^{-1} = g = \frac{\sqrt{y-6}-3}{2} \Rightarrow f^{-1}(x) = \frac{\sqrt{x-6}-3}{2}$$

54. We have,

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$$

and $$B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

(i) $$A+B = \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\therefore (A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \qquad \ldots(1)$$

$$\text{Now, } A'+B'=\begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix}=\begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \quad \ldots(2)$$

From (1) and (2), we get $(A+B)'=A'+B'$

$$\text{(ii) } A-B=\begin{bmatrix} -1+4 & 2-1 & 3+5 \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix}=\begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix} \quad \ldots(3)$$

$$\text{Now, } A'-B'=\begin{bmatrix} -1+4 & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3+5 & 9-0 & 1-1 \end{bmatrix}=\begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \quad \ldots(4)$$

From (3) and (4), we get $(A-B)'=A'-B'$.

55. Given: $2x+3y+3z=5$

$x-2y+z=-4$

$3x-y-2z=3$

The system of equations can be written as $AX=B$, where

$$A=\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, B=\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Now, $|A|=2(4+1)-3(-2-3)+3(-1+6)$

$=10+15+15=40\neq 0$

$\Rightarrow$ Given system has unique solution.

We have, $A^{-1}=\dfrac{adj\ A}{|A|}$

To find adj A:

$A_{11}=(4+1)=5,\ A_{12}=-(-2-3)$

$=5, A_{13}=(-1+6)=5$

$$A_{21} = -(-6+3) = 3,\ A_{22} = (-4-9)$$
$$= -13,\ A_{23} = -(-2-9) = 11$$

$$A_{31} = (3+6) = 9,\ A_{32} = -(2-3) = 1,$$
$$A_{33} = (-4-3) = -7$$

$$\therefore \text{adj A} = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Since $AX = B$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow x = 1, y = 2, z = -1$$

56. $y = 3\cos(\log x) + 4\sin(\log x)$

Differentiating w.r.t. 'x' $\quad y_1 = \dfrac{-3\sin(\log x)}{x} + \dfrac{4\cos(\log x)}{x}$

Multiplying both the sides by 'x' $\quad xy_1 = -3\sin(\log x) + 4\cos(\log x)$

Again differentiate w.r.t. 'x' $\quad xy_2 + y_1 = \dfrac{-3\cos(\log x)}{x} - \dfrac{4\sin(\log x)}{x}$

Multiplying each term by 'x' $\quad x^2y_2 + xy_1 = -[3\cos(\log x) + 4\sin(\log x)]$

$$x^2y_2 + xy_1 = -y$$

$$x^2y_2 + xy_1 + y = 0$$

57. Since the length x is decreasing and the width y is increasing with respect to time, then we have

$\dfrac{dx}{dt} = -5$ cm/min and $\dfrac{dy}{dt} = 4$ cm / min.

(a) The perimeter P of a rectangle is given by $P = 2(x+y)$

$$\therefore \frac{dp}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$= 2(-5+4) = -2 \text{ cm / min}$$

(b) The area A of the rectangle is given by $A = x \cdot y$

$$\therefore \frac{dA}{dt} = \frac{dx}{dt}y + \frac{dy}{dt}\cdot x$$

$$= (-5)(6) + (4)(8) \quad (\because x = 8 \text{ cm};\ y = 6 \text{ cm})$$

$$= -30 + 32 = 2 \text{ cm}^2\text{/min}$$

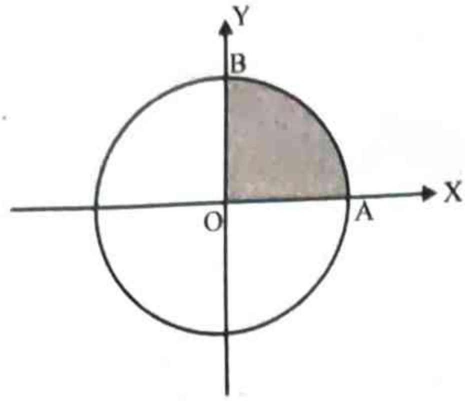
58. $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \int \frac{a\sec 2\theta\, d\theta}{a^2 + a^2\tan 2\theta} \Rightarrow \int \frac{a\sec 2\,\theta\, d\theta}{a^2(1+\tan^2\theta)}$

$$\Rightarrow \int \frac{d\theta}{a} = \frac{1}{a}\theta + \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

Put $x^3 = t$; $3x^2\,dx = dt$

Consider $\displaystyle\int \frac{3x^2}{x^6+1}\,dx = \int \frac{dt}{t^2+1} = \frac{1}{1}\tan^{-1}\left(\frac{t}{1}\right) + c = \tan^{-1}(x^3) + c$

59. Required area = 4 (area AOB)



$$= 4\int_0^a y\,dx = 4\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= 4\left\{\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a$$

$$= 4\left[\left(0 + \frac{a^2}{2}\sin^{-1}1\right) - (0+0)\right] \quad = 4\frac{a^2}{2}\cdot\frac{\pi}{2} = \pi a^2 \text{ square unit.}$$

60. Given differential equation is $\frac{dy}{dx} + \frac{2}{x}y = x$

Here $P = \frac{2}{x}, Q = x$

$$I \cdot F = e^{\int p dx} = e^{\int \frac{2}{x} dx} = e^{2\log x} = x^2$$

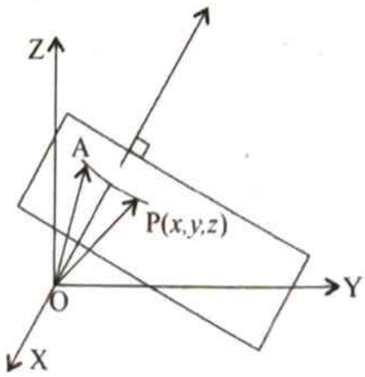
$\therefore$ Required solution is

$$y \cdot x^2 = \int x^2 x\,dx + C = \frac{x^4}{4} + C$$

$$y = \frac{x^2}{4} + Cx^{-2}$$

61. Let a plane passing through a point A with position vector $\vec{a}$ and perpendicular to $\vec{N}$. Let $\vec{r}$ be the position vector of any point $P(x, y, z)$ in the plane. Then the point P lies in the plane if and only if $\overrightarrow{AP}$ is perpendicular to $\vec{N}$.

i.e. $\overrightarrow{AP} \cdot \vec{N} = 0$; but $\overrightarrow{AP} = \vec{r} - \vec{a}$



$\therefore (\vec{r} - \vec{a}) \cdot \vec{N} = 0$ which is the vector equation of the plane Cartesian form.

Let the given point $A \equiv (x_1, y_2, z_1)$, $P \equiv (x, y, z)$ and the direction ratios of $\vec{N}$ are A, B, C.

Then $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and

$\vec{N} = A\hat{i} + y\hat{j} + C\hat{k}$

Now, $(\vec{r}-\vec{a}) \cdot \vec{N} = 0$

$$[(x-x_1)\hat{i}+(y-y_1)\hat{j}+(z-z_1)\hat{k}] \cdot \left(A\hat{i}+B\hat{j}+C\hat{k}\right) = 0$$

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

62. Let X denotes the number of times that the person wins the prize.

Here $n = 50,\ p = \dfrac{1}{100},\ q = \dfrac{99}{100}$

Therefore $P(X=r) = {}^nC_r\, q^{n-r} p^r$

$$= {}^{50}C_r \left(\frac{99}{100}\right)^{50-r} \left(\frac{1}{100}\right)^r$$

Solution: (a) P(he wins a prize atleast once)

$$= 1-P(0) = 1-{}^{50}C_0\, q^{50}\, p^0 = 1-\left(\frac{99}{100}\right)^{50}$$

Solution: (b) P(he wins a prize exactly once)

$$= {}^{50}C_0\, q^{50-1}\, p^1 = 50\left(\frac{99}{100}\right)^{49}\left(\frac{1}{100}\right)^1 = \frac{1}{2} \times \left(\frac{99}{100}\right)^{49}$$

63. Let E :'A solves the problem'

F :'B solves the problem'.

Since E and F are independent evens then

$P(E \cap F) = P(E)P(F)$ ...(1)

Given, $P(E) = \dfrac{1}{2}; P(F) = \dfrac{1}{3}$ and

$$P(E') = \frac{1}{2}, P(F') = \frac{2}{3}.$$

$\therefore$ (1) $\Rightarrow$ $P(E \cap F) = \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$

(i) Problem is solved Þ problem is solved by A or B.

$\therefore$ Required probability $= P(E \text{ or } F) = P(E \cup F)$

$\therefore \quad P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

or $P(E \cup F) = 1 - P((E \cup F)')$

$$= 1 - P(E' \cap F')$$

$$= 1 - P(E') \cdot P(F')$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}$$

(ii) Exactly one of them solves the problem $\Rightarrow E \cap F'$ or $E' \cap F$

$\therefore$ Required probability $= P(E \cap F' \text{ or } E' \cap F)$

$= P$ (A solves and B does not) $+ P$ (A does not B solves)

$= P(E \cap F') + P(E' \cap F)$

$= P(E) \cdot P(F') + P(E) \cdot P(F)$

$$= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

PART – E

64. Drawing graph of the system of linear inequalities:

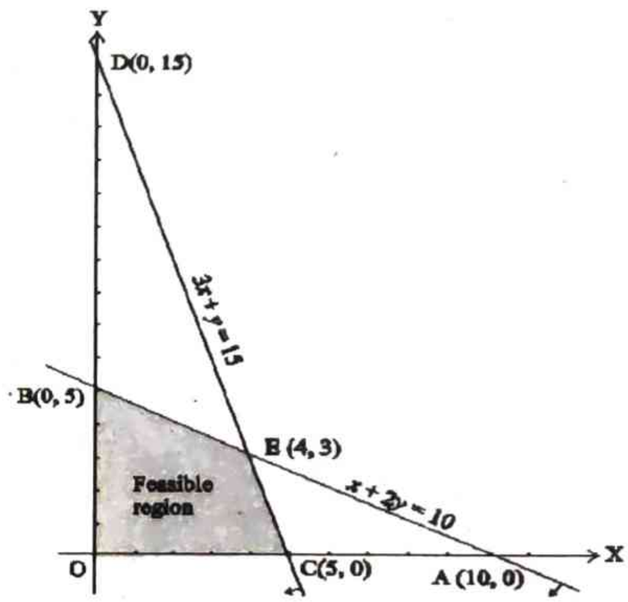
Equation	**x**	**y**	**Conclusion**
$x + 2y = 10 \Rightarrow \frac{x}{10} + \frac{y}{5} = 1$	10	0	The line passes through
	0	5	A (10,0) and B (0,5)
$3x + y = 15 \Rightarrow \frac{x}{5} + \frac{y}{15} = 1$	5	0	Passes through
	0	15	C(5,0) and D (0, 15)

Inequation	Testing	Verification	Inference
$x+y \le 10$	(0, 0)	$0+0 \le 10$ (put x = 0, y= 0 in the inequation)	The solution region contains origin
$3x+y \le 15$	(0, 0)	$0+0 \le 15$ true (put x = 0, y = 0 in the inequation)	The solution region contains origin

Solve (1) and (2)

x = 4, y = 3. $\therefore E\,(4, 3)$

Solution lies in the region OBEC.



Sl. No.	Corner points	Corresponding value of z = 3x + 2y.
3.	0 (0, 0)	O
4.	B (0, 5)	10
5.	E (4, 3)	18 ← Maximum
6.	C (5, 0)	15

Hence, maximum value of z is 18 at the point (4, 3).

b) Now $A^2 - 4A + I = 0$

$\Rightarrow I = 4A - A^2 = A(4 - A)$

Premultiplying by A^{-1} ($\because |A| \neq 0$)

$\Rightarrow A^{-1}I = A^{-1}A(4 - A) = I(4 - A)$

$$\Rightarrow A^{-1} = 4I - A$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

65. Put, $a - x = t \Rightarrow dx = -dt$

When $x = 0, t = a$

When $x = a, t = 0$

$$\therefore \int_0^a f(a-x)\, dx = \int_a^0 f(t)(-dt)$$

Let $$I = \int_b^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \qquad \ldots(1)$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{a-(a-x)}} dx \qquad \ldots(2)$$

$(1) + (2)$ gives,

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx = \int_0^a dx = a$$

$$\therefore I = \frac{a}{2}$$

(b) From data

LHL, $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} 5x + 1 = k(5) + 1 = 5x + 1$ ... (1)

RHL $\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} 3x - 5 = 3(5) - 5 = 15 - 5 = 10$ ... (2)

Given that function is continuous at x = 5.

$\therefore 5k + 1 = 10$ [From (1) and (2)]

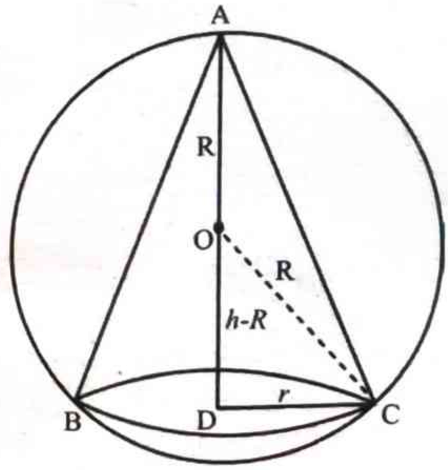
$$5k = 10 - 1 = 9$$

$$k = \frac{9}{5}$$

66.(a) We have, volume of a sphere $=\frac{4}{3}\pi R^3$

Let V, h and r be the volume, height and radius of a cone inscribed in a sphere of radius 'R'.

Then $V=\frac{1}{3}\pi r^2 h$ ...(1)



From the figure,

$OC^2 = OD^2 + DC^2$

$\Rightarrow R^2 = (h-R)^2 + r^2 \Rightarrow r^2 = R^2 - (h-R)^2$

$\Rightarrow r^2 = 2hR - h^2 \;\therefore (1) \Rightarrow V = \frac{1}{3}(2hR - h^2)h$

$\Rightarrow V = \frac{\pi}{3}(2h^2R - h^3)$ ...(2)

We have $\frac{dV}{dh} = \frac{\pi}{3}(4hR - 3h^2)$

and $\frac{d^2V}{dh^2} = \frac{\pi}{3}(4R - 6h)$

For maxima, $\frac{dV}{dh} = 0$

$\Rightarrow \frac{\pi}{3}(4hR - 3h^2) = 0 \Rightarrow 4hR - 3h^2 = 0$

$\Rightarrow h(4R - 3h) = 0 \Rightarrow h = \frac{4R}{3}$ Q $h > 0$

When $h = \frac{4R}{3}, \frac{d^2V}{dh^2} = \frac{\pi}{3}(4R - 8R) = \frac{-8R}{3} < 0$

$\therefore$ V is maximum at $h = \frac{4R}{3}$.

$\therefore V)_{\max} = \frac{\pi}{3}\left(2 \times \frac{16R^2}{9} \times R - \frac{64R^3}{27}\right)$, (using (2))

$$=\frac{\pi}{3}\left(\frac{32R^3}{9}-\frac{64R^3}{27}\right)=\frac{\pi}{3}\left(\frac{96-64}{27}\right)R^3$$

$$=\frac{\pi}{3}\left(\frac{32}{27}\right)R^3=\frac{\pi R^3}{3}\times\frac{8\times 4}{27}$$

$$=\frac{8}{27}\times\frac{4}{3}\pi R^3=\frac{8}{27}\times \text{ volume of a sphere.}$$

b) $\text{LHS}=\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}=\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Q $R_1 \to R_1+R_2+R_3$

Take out $a+b+c$ from R_1

$$=(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$=(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

Q $c_2 \to c_2-c_1 \, c_3 \to c_3-c_1$

$$=(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

Take out $(a+b+c)$ from c_1 and c_3

$$=(a+b+c)(a+b+c)(a+b+c)$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}=(a+b+c)^3[1(1-0)-0+0]$$

$$=(a+b+c)^3=\text{RHS}$$

* * *

MOST FREQUENTLY ASKED QUESTIONS

(One mark questions)

RELATIONS AND FUNCTIONS

1. Define a reflexive relation.
2. Define a symmetric relation.
3. Define a transitive relation.
4. Define an equivalence relation.
5. A relation R on set $A = \{1,2,3\}$ is defined as $R = \{(1,1),(1,2),(3,3)\}$ is not symmetric. Why?
6. A relation R on set $A = \{1,2,3\}$ is defined as $R = \{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ is not transitive. Why?
7. Give an example of a relation which is symmetric but neither reflexive nor transitive.
8. Give an example of a relation which is transitive but neither reflexive and nor symmetric.
9. Give an example of a relation which is reflexive and symmetric but not transitive.
10. Give an example of a relation which is symmetric and transitive but not reflexive.
11. Define a one-one function.
12. Define a onto function.
13. Define a bijective function.
14. Let $A = \{1,2,3\}$, $B = \{4,5,6,7\}$ and $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. Show that f is one-one.
15. Write the number of all one-one function from the set A={a,b,c} to itself.
16. Define a binary operation.
17. On N, show that subtraction is not a binary operation.
18. On Z^+ (The set of positive integers) defined $*$ *by* $a*b = a-b$. Determine whether * is a binary operation or not.
19. On Z^+ (The set of positive integers) defined $*$ by $a*b = ab$. Determine whether * is a binary operation or not.

20. On Z^{+} (The set of positive integers) defined $*$ by $a*b=a^{b}$. Determine whether * is a binary operation or not.
21. On Z^{+} (The set of positive integers) defined $*$ by $a*b=|a-b|$. Determine whether * is a binary operation or not.
22. Let * be a binary operation on N given by $a*b=L.C.M.\ of\ a\ and\ b$. Find $5*7$.
23. Let * be a binary operation on N given by $a*b=L.C.M.\ of\ a\ and\ b$. Find $20*16$.

INVERSE TRIGONOMETRIC FUNCTIONS

1. Write the domain of $f(x)=cos^{-1}x$.
2. Write the domain of $f(x)=sin^{-1}x$.
3. Write the range of $f(x)=cos^{-1}x$.
4. Write the range of $f(x)=sin^{-1}x$.
5. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.
6. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$
7. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.
8. Find the principal value of $\operatorname{cosec}^{-1}(2)$.
9. Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$
10. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.
11. Find the principal value of $\tan^{-1}(-1)$
12. Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$
13. Write the values of x for which $2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, holds.
14. Find the value of $\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right)$, $|x|\geq 1$.

15. Find the value of $\sin\left(\sin^{-1} x + \cos^{-1} x\right)$.

16. Find the value of $\cot\left(\tan^{-1} x + \cot^{-1} x\right)$.

17. If $\sin\left[\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x)\right] = 1$, then find the value of x.

18. Find the value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$

MATRICES

1. Define a diagonal matrix.
2. Define a scalar matrix.
3. Define aidentity matrix.
4. If a matrix has 5 elements, what are the possible orders it can have?
5. If a matrix has 8 elements, what are the possible orders it can have?
6. Construct a 2×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}\,|\,i-3j\,|$.
7. Construct a 2×2 matrix, $A=\left[a_{ij}\right]$, whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$.
8. Construct a 2×2 matrix, $A=\left[a_{ij}\right]$, whose elements are given by $a_{ij} = \frac{i}{j}$.
9. Construct a 2×2 matrix, $A=\left[a_{ij}\right]$, whose elements are given by $a_{ij} = |\,i-j\,|$.
10. Find the number of all possible matrices of order 3×3 with each entry 0 or 1.
11. If $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$, find the values of x, y and z.

12. If $A=\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}$, find $(A')'$.

DETERMINANTS

1. Evaluate $\begin{vmatrix}3 & -1 & -2\\0 & 0 & -1\\3 & -5 & 0\end{vmatrix}$.
2. If $A=\begin{bmatrix}1 & 1 & -2\\2 & 1 & -3\\5 & 4 & -9\end{bmatrix}$, find $|A|$.
3. Find the values of x for which $\begin{vmatrix}3 & x\\x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\4 & 1\end{vmatrix}$.
4. If $A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, find $|2A|$.
5. Find the value of x, if $\begin{vmatrix}2 & 3\\4 & 5\end{vmatrix}=\begin{vmatrix}x & 3\\2x & 5\end{vmatrix}$.
6. Find $|3A|$, if $\cot^2 \cot^2 x$.
7. If $A=\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}$, then show that $|2A| = 4|A|$.
8. Evaluate $\begin{vmatrix}102 & 18 & 36\\1 & 3 & 4\\17 & 3 & 6\end{vmatrix}$.
9. If A is an invertible matrix of order 2 then find $|A^{-1}|$.
10. If A is a square matrices of order 3 and |A|=4, find the value of |2A|.
11. If A is a square matrix with $|A|=8$, then find the value of $|AA'|$.
12. If A is a square matrices of order 3 and |A|=4, find the value |adj A|.

DIFFERENTIABILITY

1. Find the derivative of $y=\tan(2x+3)$.
2. If $y=\sin(x^2+5)$, find .

3. If $y=\cos(\sin x)$, find $\frac{dy}{dx}.\frac{dy}{dx}$
4. If $y=\sin(ax+b)$, find $\frac{dy}{dx}$.
5. If $y=\cos(\sqrt{x})$, find $\frac{dy}{dx}$.
6. Find $\frac{dy}{dx}$, if $y=\cos(1-x)$.
7. If $y=\log(\sin x)$, find $\frac{dy}{dx}$.
8. Find $x-y=\pi$.
9. Find $\frac{dy}{dx}$, if $y=\sin(\log x),\quad x>0$.
10. If $y=e^{\cos x}$,find $\frac{dy}{dx}$.
11. Find $\frac{dy}{dx}$, if $y=e^{\sin^{-1}x}$.
12. Find $\frac{dy}{dx}$, if $y=\log(\log x),\ x>0$.
13. Find $\frac{dy}{dx}$, if $y=x^2+3x+2$.
14. If $y=e^{\log x}$, prove that $\frac{dy}{dx}=1$.
15. Find $\frac{dy}{dx}$, if $y=5^x$.

INTEGRATION:

1. Evaluate $\int \tan^2(2x).\,dx$
2. Find an anti-derivative of $\cot^2 x$ with respect to x.
3. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(sin^3x)\,dx.\int \tan^2(2x).\,dx$
4. Find an anti-derivative of $\sqrt{1+\sin 2x}$ with respect to x.
5. Evaluate$\int\left(\frac{d}{dx}e^{5x}\right)dx.$
6. Evaluate $\int(1-x)\sqrt{x}\,dx$.
7. Find the following integrals:

i) $\int (\sin x + \cos x)\, dx$

ii) $\int cosec\, x\, (\, cose\, x + \cot x)\, dx$

iii) $\int \frac{1-\sin x}{\cos^2 x}\, dx$

8. Find i) $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx$ ii) $\int \frac{(x^2+1)e^x}{(x+1)^2}\, dx$

9. Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\, dx$

10. Find $\int \frac{dx}{\sin^2 x \cos^2 x}$

VECTOR ALGEBRA

Definition : Position vector, Null vector, unit vector, cointial, collinear, equal vectors, negative of vector, coplanar.

1. Write the direction ratio's of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ its direction cosines.
2. Find the vector joining the points $P(2, 3, 0)$ and $Q(-1, -2, -4)$ directed from P to Q.
3. Find unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$
4. Let $\vec{a} = \hat{\imath} + 2\hat{\jmath}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath}$. is $|\vec{a}| = |\vec{b}|$? Are the vectors $\vec{a}$ and $\vec{b}$. Equal?
5. Find the unit vector in the direction of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$
6. Show that the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear.
7. Find the direction cosines of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

THREE DIMENSIONAL GEOMETRY

1. If a line makes angle $90^0, 60^0$ and 30^0 with the positive direction of x, y and z –axis respectively, find its direction cosines.
1. If a line has direction ratios $2, -1, -2$ determine its direction cosines.
3. Find the direction cosines of x, y and z –axis.
4. Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z-axis respectively.
5. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

LINEAR PROGRAMMING

1. Definitions:LPP, Objective functions, optical value, constraint, optimization problem, feasible region, feasible solution, infeasible region, infeasible solution, optimal solution, corner point.

PROBABILITY

1. Given that E and F are events such that $P(E) = 0.6, P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E|F)$ and $P(F|E)$
2. Compute $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$
3. If $P(A) = 0.8, P(B) = 0.5$ and $P(B|A) = 0.4$, find (i) $P(A \cap B)$ (ii) P(A | B) (iii) $P(A \cup B)$
4. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$
5. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find (i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(B|A)$
6. If $P(A) = \frac{1}{2}, P(B) = 0$, then $P(A|B)$ is?
7. If A and B are event such that $P(A|B) + P(B|A)$, then?
8. If $P(A) = \frac{3}{5}$ and $(B) = \frac{1}{5}$, find $P\ (A \cap B)$ if A and B are independent events.
9. Let E and F be events with $P(E) = \frac{3}{5}, P(F) = \frac{3}{10}$and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?
10. If A and B are two events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ And $P(A \cap B) = \frac{1}{8}$, find P (not A and not B).
11. Given two independent events A and B such that $P(A) = 0.3, P(B) = 0.6$.
    Find(i) P(A and B) (ii) P(A and not B)
    (iii) P(A or B) (iv) P(neither A nor B

5 Marks Questions

RELATIONS AND FUNCTIONS

1. Prove that the function $f : R \rightarrow R$ defined by $f(x) = 4x + 3$, where $Y = \{y : y = 4x + 3, x \in N\}$ is invertible. Also write the inverse of $f(x)$.

2. Let $f: N \to Y$ be a function defined as $f(x) = 4x+3$, where, $Y = \{y \in N : y = 4x+3 \textit{ for some } x \in N\}$. Show that f is invertible. Find the inverse.

3. Prove that the function, $f: N \to Y$ defined by $f(x) = x^2$,where $Y = \{y : y = x^2, x \in N\}$ is invertible. Also find the inverse of f .

4. Let $f: N \to R$ be defined by $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$ where, S is the range of function f , is invertible. Find the inverse of f .

5. Let R_+ be the set of all non-negative real numbers. Show that the function $f: R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$ is invertible and write the inverse of f .

6. Consider $f: R_+ \to [-5, \infty)$, given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$.

MATRICES :

1. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$, find $A(BC), (AB)C$ and show that $(AB)C = A(BC)$.

2. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ calculate AC, BC and $(A+B)C$. Also, verify that $(A+B)C = AC + BC$.

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40\,I = O$

4. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$, verify that $(AB)' = B'A'$

5. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A + (B-C) = (A+B) - C$.

6. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

7. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$
S.T. $A(B + C) = AB + AC)$

8. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 3 & 0 \end{bmatrix}$
verify $A(BC) = (AB)C$

9. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$
S.T $A + (B - C) = (A + B) - C$

DETERMINANTS

1. Solve the system of equations by matrix method:
$3x - 2y + 3z = 8,\ \ 2x + y - z = 1,\ \ 4x - 3y + 2z = 4.$

2. Solve the system of equations by matrix method:
$x - y + z = 4\ ,\ \ 2x + y - 3z = 0\ ,\ \ x + y + z = 2.$

3. Solve the system of equations by matrix method:
$2x + 3y + 3z = 5,\ \ x - 2y + z = -4,\ \ 3x - y - 2z = 3.$

4. Solve the system of equations by matrix method:
$4x + 3y + 2z = 60,\ 2x + 4y + 6z = 90,\ 6x + 2y + 3z = 70$

5. Solve the system of equations by matrix method:
$x + y + z = 6\ \ y + 3z = 11,\ \ x + z = 2y.$

6. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations
$2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3.$

7. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve system of equations
$x - y + 2z = 1;\ \ 2y - 3z = 1$ and $3x - 2y + 4z = 2.$

CONTINUITY AND DIFFERENTIABILITY

1. If $y=3e^{2x}+2e^{3x}$, then prove that $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0.$

2. If $y=\sin^{-1}x$, then prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0.$

3. If $y=\cos^{-1}x$, *find* $\frac{d^2y}{dx^2}$ in terms if y alone.

4. If $y=3\cos(\log x)+4\sin(\log x)$, show that $x^2y_2+xy_1+y=0.$

5. If $y=Ae^{mx}+Be^{nx}$, prove that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+(mn)y=0.$

6. If $y=500e^{7x}+600e^{-7x}$, show that $y_2=49y.$

7. If $e^y(x+1)=1$, Prove that $\frac{dy}{dx}=-e^y$ hence prove that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

8. If $y=\left(\tan^{-1}x\right)^2$, show that $(1+x^2)^2y_2+2x(1+x^2)y_1=2.$

9. If $y=e^{a\cos^{-1}x}, -1\le x\le 1$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0.$

10. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\frac{d^2y}{dx^2}$.

APPLICATION OF DERIVATIVES

1. The volume of a cube is increasing at a rate of 9 cubic centimeters persecond. How fast is the surface area increasing when the length of an edge is 10centimeters?

2. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is $10\ cm$, how fast is the enclosed area increasing?

3. The length x of a rectangle is decreasing at the rate of $3\ cm/minute$ and the width y is increasing at the rate of $2\ cm/minute$. When $x=10\ cm$ and $y=6\ cm$, find the rate of change of (i) the perimeter and (ii) the area of the rectangle.

4. The length x of a rectangle is decreasing at the rate of $5\ cm/minute$ and the width y is increasing at the rate of $4\ cm/minute$. When

$x = 8\ cm$ and $y = 6\ cm$, find the rates of change of (i) the perimeter, and (ii) the area of the rectangle.

5. A ladder $5m$ long is leaning against a wall. The bottom of the ladder is Pulled along the ground, away from the wall at the rate of $2m/sec$. How fast is its height on the wall decreasing when the foot of the ladder is $4m$ away from the wall?
6. A ladder 24 feet long leans against a vertical wall. The lower end is moving away at the rate of 3 feet/sec. find the rate at which the top of the ladder is moving downwards, if its foot is 8 feet from the wall.
7. A man of height 2 meters walks at a uniform speed of 5 km/hour, away from a lamp post which is 6 meters high. Find the rate at which the length of the his shadow increases.
8. A sand is pouring from a pipe at the rate of $12 cm^3/s$. The falling sand form a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is $4cm$?
9. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the $y-$coordinate is changing 8 times as fast as the $x-$coordinate.
10. The volume of a cube is increasing at the rate of $8\ cm^3/s$. How fast is the surface area increasing when the length of an edge is $12\ cm$?
11. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.
12. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at the rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 metre.

INTEGRALS

1. Find the integral of $\sqrt{x^2 + a^2}$ with respect to x and evaluate $\int \sqrt{4x^2 + 9}\, dx$.

2. Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2+2x+2}}dx$

3. Find the integral of $\frac{1}{x^2-a^2}$ with respect to x and hence evaluate $\int \frac{x}{x^4-16}dx$.

4. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect to x and hence evaluate $\int \frac{dx}{\sqrt{7-6x-x^2}}$.

5. Find the integral of $\sqrt{a^2-x^2}$ with respect to x and hence evaluate $\int \sqrt{1+4x-x^2}dx$.

6. Find the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2-25}}dx$.

7. Find the integral of $\int \sqrt{x^2+a^2}$ with respect to x and hence evaluate $\int \sqrt{1+x^2}$ dx

8. Find the integral of $\int \sqrt{x^2-a^2}$ with respect to x and hence evaluate $\int \sqrt{x^2+4x-5}$ dx

APPLICATION OF INTEGRALS

1. Using integration find the area of the region bounded by the triangle whose vertices are $(-1,0)$, $(1,3)$ and $(3,2)$.

2. Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$.

3. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a>b)$ by the method of integration and hence find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

4. Find the area of the smaller region enclosed by the circle $x^2+y^2=4$ and the line x + y = 2.

5. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

6. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle, $x^2 + y^2 = 32$.

7. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.

DIFFERENTIAL EQUATIONS

1. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2(x\neq 0)$.

2. Find the general solution of the differential equation $ydx-(x+2y^2)dy=0$.

3. Find the general solution of the differential equation $\frac{dy}{dx}+2y=\sin x$

4. Find the general solution of the differential equation $\frac{dy}{dx}+3y=e^{-2x}$.

5. Find the general solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$.

6. Find the general solution of the differential equation $\frac{dy}{dx}+(\sec x)y=\tan x(0\leq x\leq \pi/2)$.

7. Find the general solution of the differential equation $\cos^2 x\frac{dy}{dx}+y=\tan x(0\leq x\leq \pi/2)$.

8. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$.

9. Find the general solution of the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$.

10. Find the general solution of the differential equation $(1+x^2)dy+2xydx=\cot xdx(x\neq 0)$.

11. Find the general solution of the differential equation $x\frac{dy}{dx}+y-x+xy\cot x=0(x\neq 0)$.

12. Find the general solution of the differential equation $(x+y)\frac{dy}{dx}=1$.

13. Find the general solution of the differential equation $ydx+(x-y^2)dy=0$.

14. Find the general solution of the differential equation $(x+3y^2)\frac{dy}{dx}=y(y>0)$.

15. For, $\frac{dy}{dx}+2y\tan x=\sin x$, find the particular solution satisfying the given condition y=0 when x=$\frac{\pi}{3}$.

16. For, $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$, find the particular solution satisfying the given condition y=0 when x=1.

17. For, $\frac{dy}{dx}-3y\cot x=\sin 2x$ find the particular solution satisfying the given condition y=2 when x=$\frac{\pi}{2}$

THREE DIMENSIONAL GEOMETRY

1. Derive the equation of the line in space passing through the point and parallel to the vector both in the vector form and cartesian form.
2. Derive the equation of the line in space passing through two given points both in vector and cartesian form.
3. Derive the equation of the plane in normal form. (both in vector and cartesian form).
4. Derive the equation of the plane perpendicular to the given vector and passing through a given point both in cartesian and vector form.
5. Derive equation of a plane passing through three non-collinear with position vectors $\vec{a},\vec{b},\vec{c}$(both in vector and cartesian form).

PROBABILITY

1. If a coin is tossed 8 times. Find the probability of
   i) At least five heads ii)At most five heads
2. IF a coin is tossed 10 times. Find the probability of
   i) Exactly six heads ii) At most six heads iii) At least six heads
3. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of i) 5 successes ii) At most 5 successes
   iii) At least 5 successes
4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that i) All the 5 cards are spades? ii) Only 3 cards are spades? iii) None is a spade?
5. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize i) At least once ii) Exactly once iii) At least twice
6. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs
   i) None ii) More than one iii) Not more than one iv) At least one
   Will fuse after 150 days of use.
7. On a multiple choice examination with 3 possible answer for each of the 5 questions. What is the probability that a candidate would getting 4 or more correct answers just by guessing.

6 Mark Questions

INTEGRALS

1. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence integrate the following.

   1) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ 2) $\int_0^{\frac{\pi}{2}} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x}\, dx$

   3) $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}\, dx$ 4) $\int_0^{\frac{\pi}{2}} \frac{sinx-cosx}{1+sinxcosx}\, dx$

   5) $\int_0^2 x\sqrt{2-x}\, dx$ 6) $\int_0^1 \tan^{-1}\left[\frac{2x-1}{1+x-x^2}\right] dx$

   7) $\int_o^{\pi/2} 2\log\sin x - \log\sin 2x\, dx$

   8) $\int_0^{\pi/2} \log(\sin x)\, dx$ 9)$\int_0^{\frac{\pi}{4}} \log(1+tanx)\, dx$

2. Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ & hence integrate the following.

1) $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}-\sqrt{x}}dx$ 2) $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx$

3) $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}dx$ 4) $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

3. Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if \quad f(2a-x)=f(x) \\ 0 & if \quad f(2a-x)=-f(x) \end{cases}$

and integrate the following

1) $\int_0^{\pi} |\cos x| dx$ 2) $\int_0^{\pi/2} \sqrt{1-\sin 2x}\, dx$

3) $\int_0^{2\pi} \cos^5 x\, dx$ 4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$

4. Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if \quad f(x)\ is\ even \\ 0 & if \quad f(x)\ is\ odd \end{cases}$ and integrate the following.

1) $\int_{-1}^{1} \sin^5 x \cos^4 x\, dx$ 2) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1-x}{1+x}\right) dx$

3) $\int_{-\pi/2}^{\pi/2} \sin^7 x\, dx$4)$\int_{-\pi/2}^{\pi/2} (x^3 + x\, cosx + tan^5 x + 1)dx$

LINEAR PROGRAMMING:

1. Maximize and Minimize the following:

$Z = 4x + y$, Subject to the constraints: $x + y \le 50$, $3x + y \le 90$, $x \ge 0 , y \ge 0$

$Z = 200x + 500y$, Subject to the constraints: $x + 2y \ge 10 , 3x + 4y \le 24 , x \ge 0 , y \ge 0$

$Z = 3x + 9y$, Subject to the constraints: $x + 3y \le 60 , x + y \ge 10$, $x \le y$, $x \ge 0 , y \ge 0$

$Z = 3x + 2y$, Subject to the constraints, $x + 2y \le 10$, $3x + y \le 15$, $x, y \ge 0$

$Z = -3x + 4y$, Subject to the constraints, $x + y \le 8, 3x + 2y \le 12, x \ge 0, y \ge 0$

$Z = 5x + 3y$, Subject to the constraints, $3x + 5y \le 15, 5x + 2y \le 10, x \ge 0, y \ge 0$

$Z = 5x + 10y$, Subject to the constraints, $x + 2y \le 120, x + y \ge 60, x - 2y \ge 0, x, y \ge 0$

$Z = x + 2y$, Subject to the constraints, $x + 2y \ge 100, 2x - y \le 0, 2x + y \le 200, x, y \ge 0$

CONTINUITY AND DIFFERENTIABLITY

1. Find the relationship between 'a' and 'b' so that the function 'f' defined by $f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & if\ x > 3 \end{cases}$ is continuous at $x = 3$.

2. Determine the value of k, if

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & if\ x \ne \dfrac{\pi}{2} \\ 3, & if\ x = \dfrac{\pi}{2} \end{cases}$$ is continuous at $x = \frac{\pi}{2}$.

3. Find the value of k if $f(x) = \begin{cases} kx^2, & if\ x \le 2 \\ 3, & if\ x > 2 \end{cases}$ is continuous at $x = 2$.

4. Find the value of k so that the function

$f(x) = \begin{cases} kx+1, & if\ x \le 5 \\ 3x-5, & if\ x > 5 \end{cases}$ at $x = 5$ is a continuous function.

5. Find the values of $a\ and\ b$ such that

$$f(x) = \begin{cases} 5, & if\ x \le 2 \\ ax+b, & if\ 2 < x < 10 \\ 21, & if\ x \ge 10 \end{cases}$$ is a continuous functions.

DETERMINANTS:

1. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

2. Prove that $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$.

3. Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

4. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$.

5. Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

6. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

7. Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$.

8. Prove that $\begin{vmatrix} 1+a^2 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix} = 1+a^2+b^2+c^2$.

9. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$.

10. Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$.

11. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+ab+bc+ca$

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