PORTFOLIO OPTIMIZATION AND RISK ANALYSIS IN FINANCIAL MARKETS

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Definition and objectives of the portfolio

- **Definition and Composition**: An investment portfolio, also known as an asset portfolio, includes a variety of financial assets like stocks, bonds, cash, foreign exchange, options, precious metals, financial derivatives, real estate, land, antiques, listed business status, art, and wine.
- **Objective:** The goal is to build a portfolio with optimal characteristics such as enhanced liquidity, stability, high returns, and minimal investment risk.
- **Management:** Portfolios can be managed by individual investors, financial professionals, hedge funds, banks, and other financial institutions.
- **Exclusions:** Consumer goods like sports cars, televisions, cosmetics, and apparel are excluded from the portfolio due to their lack of appreciation potential or tendency to depreciate.

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Investment Portfolio

Concept of Investment Portfolio:

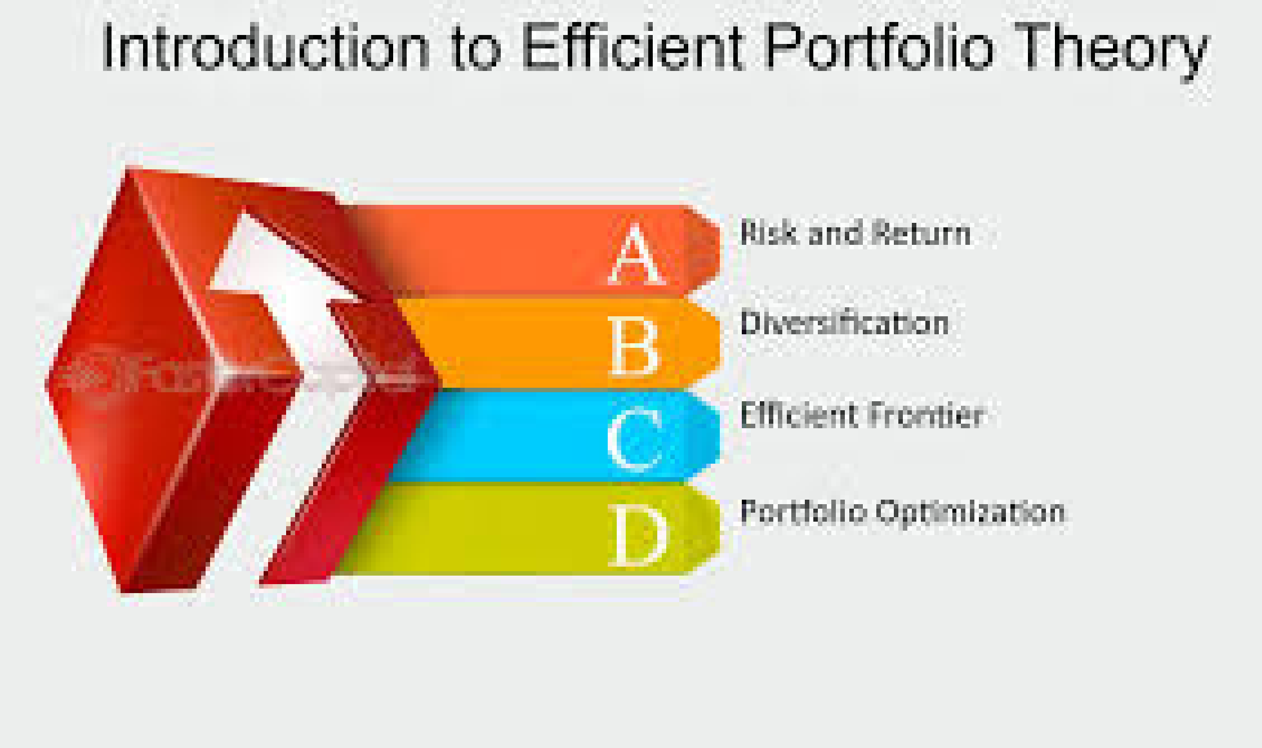
- Investment portfolios are structured to diversify risk across a variety of asset classes such as stocks, bonds, real estate, and commodities.
- The aim is to achieve a balance between maximizing returns and minimizing risks through diversification.
- Key metrics to evaluate portfolio performance include return on investment (ROI), risk-adjusted return, and the Sharpe ratio.

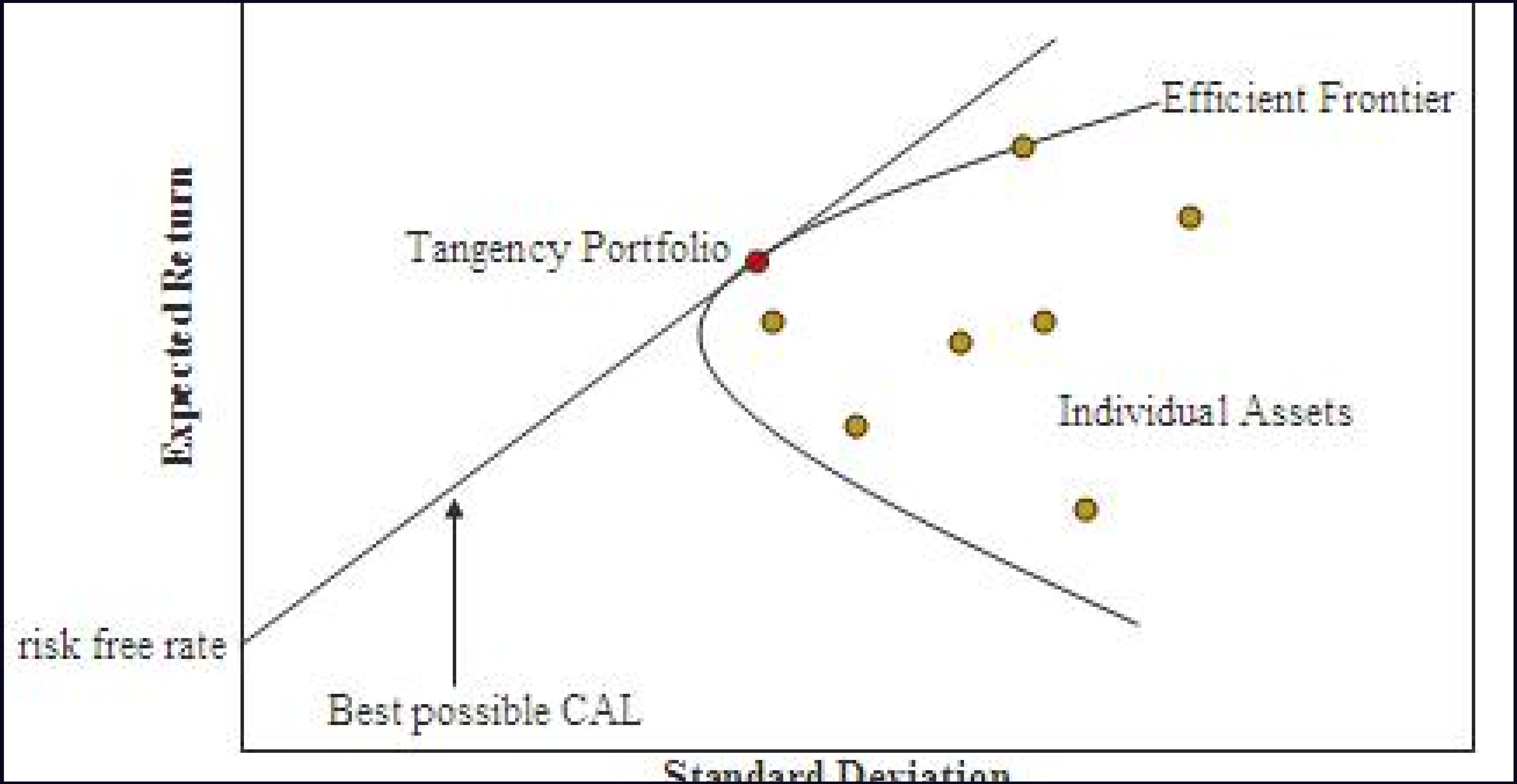
Composition and Types of Investment Portfolio:

- **Market Portfolio**: Typically includes a wide range of asset classes and is used as a benchmark to evaluate the performance of other portfolios.

Efficient Market Portfolios:

- **Minimum Variance Portfolio**: Focuses on the lowest possible risk for a given return by considering the variances and covariances of asset returns.
- **Tangential Portfolio:** Lies on the efficient frontier and maximizes the Sharpe ratio, representing the best possible risk-return trade-off.
- **Efficient Frontier Portfolio:** Consists of portfolios that offer the highest expected return for each level of risk, forming the efficient frontier.




Efficient Frontier
Tangency Portfolio
Expected Return
Individual Assets
risk free rate
Best possible CAL

Theoretical basis of portfolio optimization

1. **Introduction of Portfolio Theory:** In 1952, economist Markowitz introduced portfolio theory through his publication "Portfolio Selection," which emphasizes balancing portfolio risks and rewards using a mean-variance model.
2. **Mean-Variance Model:** This model helps in choosing the best portfolio based on asset correlation, aiding financial institutions and investors in managing risks and boosting returns.
3. **Yang Ming’s Hypothesis:** Grounded in Markowitz's model, it includes six assumptions about investor behavior and market conditions, including risk aversion, decision-making based on expected return and variance, a single investment period, efficient markets, infinite divisibility of securities, and supply elasticity.

4. **Improved Decision-Making:** The approach makes financial investment decision-making more scientific and reasonable, enhancing the efficiency of the financial industry.
5. **Diversification Benefits:** A multi-asset portfolio reduces the risks associated with single-asset investing by diversifying assets among various investment vehicles.
6. **Correlation Coefficients:** The correlation coefficient measures how asset values move in relation to each other, with perfect positive correlation at 1 and perfect negative correlation at -1.
7. **Impact of Correlation on Risk:** With less than perfect positive correlation, high-risk portfolios can have lower total risk than their individual components, and assets with perfect negative correlation can reduce portfolio variance to zero, maximizing returns.

<u>Models for Portfolio Optimization</u>

- **<u>Model Objective:</u>** The model aims to identify a portfolio with high investment efficiency by minimizing variance while meeting constraints on expected return and total investment.
- **<u>Covariance and Expected Return:</u>** It is crucial to estimate the covariance between the rates of return of assets (σij) and their expected returns (ri). Positive σij indicates correlated returns, negative σij indicates inverse correlation, and σij = 0 indicates no correlation.
- **<u>Risk Mitigation and Asset Selection:</u>** Combining uncorrelated assets can reduce risk without affecting returns. While negative correlation is ideal for optimizing portfolio performance, real returns often vary, complicating portfolio construction. Investors can include diverse assets like stocks and bonds to enhance portfolio efficiency.

$$\min\left(\sigma^2 = \sum_{i=1}^{n} \sigma_i^2 x_i^2 + \sum_{\substack{i,j=1 \\ i \neq 1}} \sigma_{ij} x_i x_j\right)$$

$$\text{s.t} \sum_{i=1}^{n} x_i r_i = r, \qquad \sum_{i=1}^{n} x_i = 1$$

Definition and Classification of Risk and Risk Measurement Index

- **Risk Definition and Components:** Risk is defined as the likelihood and severity of undesirable consequences, encompassing the likelihood, potential magnitude, and overall probability of adverse outcomes, driven by the interaction of risk events, consequences, and variables.
- **Risk Quantification Tools:** Volatility measures the extent of asset price fluctuations, with lower volatility indicating reduced risk. Beta indicates the volatility of a portfolio or stock relative to the broader market, with a negative beta indicating lower volatility and risk, and a beta greater than 1 indicating higher risk.

Formula:

$$\text{Beta Coefficient } (\beta) = \frac{\text{Covariance}(Re, Rm)}{\text{Variance}(Rm)}$$

Where, Re = return on individual stock

Rm = return on the overall market

Covariance = changes in stock's returns in relation to changes in market returns

Variance = how far the markets data point spread out from their average value

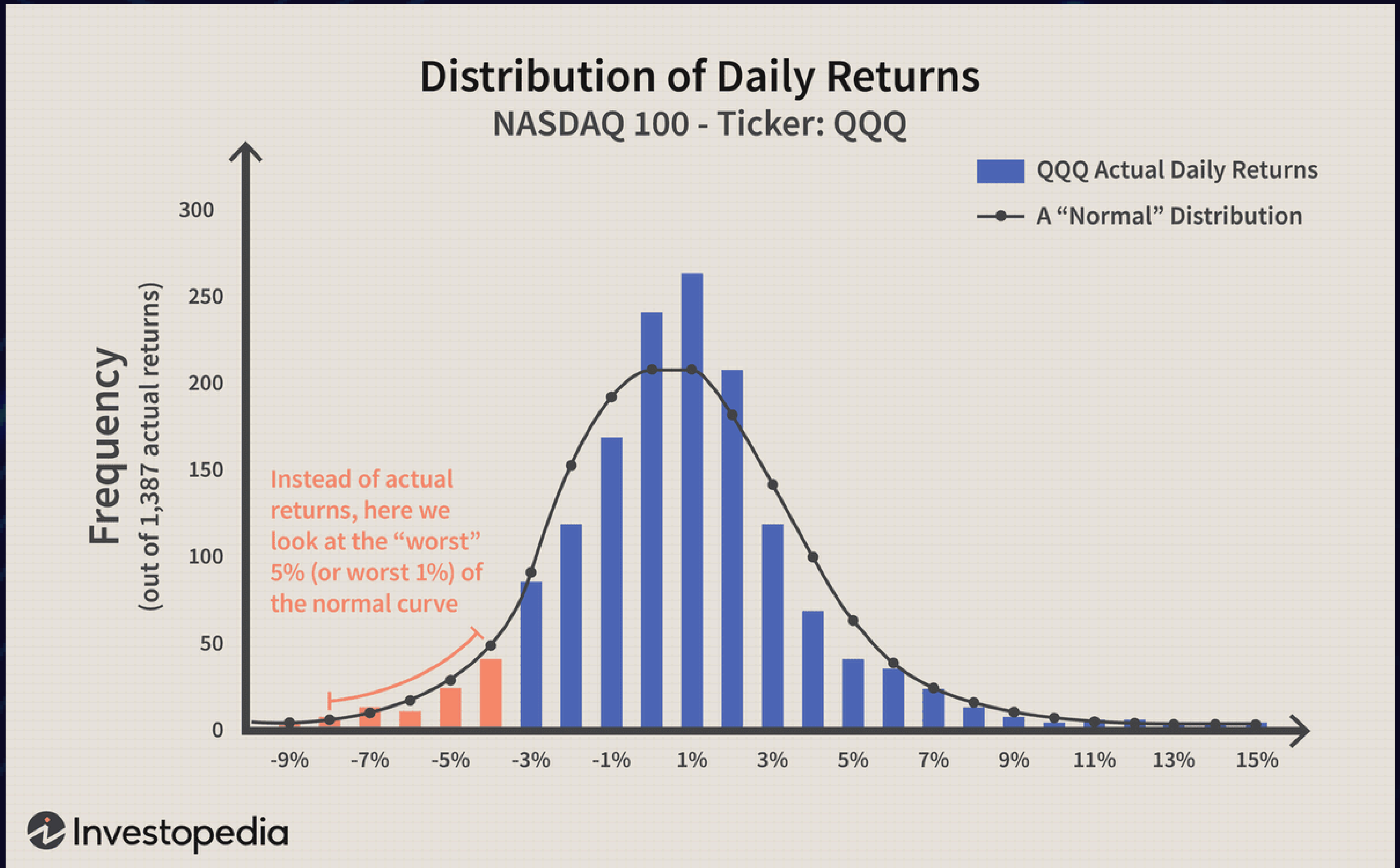
Risk Analysis in Portfolio Management:

- **Diversification:** Spreading investments across various asset classes to reduce risk.
- **Risk Tolerance Assessment:** Evaluating an investor's ability to endure market volatility and potential losses.
- **Risk Metrics:** Using tools like Value at Risk (VaR), Beta, and standard deviation to measure risk levels
- **Scenario Analysis and Stress Testing:** This involves examining how a portfolio might perform under various hypothetical scenarios, including extreme market conditions. It helps in understanding the potential impact of adverse events on the portfolio's performance.

- **Value at Risk (VaR):**
- VaR measures the potential loss in portfolio value over a defined period for a given confidence level.
- Calculation methods include:
- **Historical Simulation:** Uses past return data to simulate potential future losses.
- **Monte Carlo Simulation:** Employs random sampling to generate a wide range of possible future scenarios and estimate potential losses.

Conditional Value at Risk (CVaR):

- CVaR, also known as Expected Shortfall, focuses on the tail end of the loss distribution, providing a measure of the average loss beyond the VaR threshold.
- It offers a more comprehensive risk assessment by considering extreme loss scenarios.



Risk Constraints of Portfolio Optimization

- **Portfolio Optimization with Risk Constraints:** The study discusses portfolio optimization techniques incorporating risk constraints, highlighting the prohibition of short selling to reduce risk in nascent markets like China.
- **Model Description:** An extended mean-variance optimization approach using VaR within a no-short-selling risk constraint is presented, focusing on minimizing portfolio variance without accounting for transaction costs and taxes, and assuming endlessly divisible risky assets.
- **Empirical Analysis:** Based on Zhang Peng's research, an analysis of six weighted stocks on the Shanghai Stock Exchange demonstrates the effectiveness of integrating VaR constraints into the mean-variance model to limit potential investment losses.
- **Investor Implications:** The model offers guidance for investors aiming to mitigate risk, though the restriction on short selling reduces the range of investment options available.

- x is the vector of asset weights.
- G is the covariance matrix of asset returns.
- r is the vector of expected returns.
- r0 is the minimum acceptable return.
- Φ represents a risk-free rate or other benchmark.
- c is a constant factor.
- VaR0 is the Value at Risk threshold.
- e is a vector of ones ensuring the sum of weights equals 1.

- x≥0 enforces the no-short-selling constraint.

$$\min x^T G x / 2$$

$$\text{s.t.} \begin{cases} r^T x \gg r_0 \\ r^T x \gg \emptyset^{-1}(c)\sqrt{x^T G x} - V_a R_0 \\ e^T x = 1;\ x \gg 0 \end{cases}$$

Comprehensive methods and models for portfolio optimization and risk analysis

- **Minimizing Variance:** The models aim to minimize portfolio variance using theories like Markowitz's, with a focus on determining optimal asset weights.
- **Model Description:** The model minimizes the total risk subject to constraints such as the sum of asset weights equaling 1 and achieving the expected rate of return μ.
- **Expected Return Calculation:** The expected return on investment for a security is calculated using local integral means to obtain the valuation model of the return.
- **Risk Categorization:** Risks are categorized into non-systematic risks, (which can be mitigated through diversification), and systematic risks, (which are intrinsic to the market and cannot be mitigated).

Minimizing Total Risk:

$$\min \sigma_p^2 = Z^T M Z$$

- Z: Vector of asset weights
- M: Covariance matrix of asset returns

$$Q^T Z = x_1 + x_2 + \cdots + x_n = 1$$

$$E(R_P) = E(k)^T \cdot Z = \mu$$

- **Covariance Calculation:** The estimated covariance between securities is calculated (σ_{ij}).
- **Optimization Objective:** The objective is to determine the optimal weight allocation for an investment portfolio to minimize risk while considering specific projected returns and constraints, aiming to maximize the rate of return.
- σ_p^2: Portfolio variance.
- Z: Vector of portfolio weights.
- M: Covariance matrix of asset returns
- μ: Vector of expected returns for each asset.
- σ_{ij}: Covariance between securities i and j.
- $r_{i,k}$: Return of security i at time k.
- Q: Constraint matrix
- σ^2: Variance of security returns.
- r_i: Individual returns of the security.
- E(k): Expected return of the security.
- m: Number of return observations.

Expected Return Calculation:

$$E(k) = \frac{\sum r_i}{m}$$

Risk Measurement (Variance):

$$\sigma^2 = \frac{1}{m-1}\sum_{i=1}^{m}(r_i - E(k))^2$$

Covariance Calculation:

$$\sigma_{ij} = \frac{1}{m-1}\sum_{k=1}^{m}(r_i - E(R_i))(r_j - E(R_j))$$

Optimization for Maximum Return:

$$\max R_P = R^T Z$$

$$\sigma_P^2 = Z^T M Z = \sigma_0^2$$

$$Q^T Z = x_1 + x_2 + \cdots + x_n = 1$$

Portfolio optimization and risk analysis in Chinese stock market

- **Objective:** The section aims to empirically analyze and measure risk in portfolio optimization using real securities, examining the validity and applicability of the models discussed earlier.
- **CVaR:** Conditional Value-at-Risk (CVaR) is presented as an enhanced method over Value-at-Risk (VaR), quantifying the expected loss beyond the VaR threshold at a certain confidence level. CVaR provides a more comprehensive measure of extreme value risk by focusing on tail losses.
- **Empirical Application:** The study applies Ni Yaoqi's findings and models to analyze 251 non-financial stocks from the Shanghai and Shenzhen stock exchanges. These stocks have been listed for at least two years and are selected based on their 24-month BETA value and volatility data.
- CVaR Formula: The formula $\mathbf{CVaR\alpha=E(X \mid X>VaR\alpha)}$ represents the expected loss given that the loss exceeds the VaR threshold. This emphasizes the importance of CVaR in capturing the tail risk, providing a more accurate and reliable measurement of extreme losses compared to VaR alone. This makes CVaR particularly useful for assessing the risk of large losses in financial portfolios.

- Results: Using a multi-factor methodology, the stocks are sorted by factor value from 12 variables. The top 50 combinations are analyzed for annual average return and correlation with the overall factor, with differential returns calculated by comparing the last 50 combinations. The findings are summarized in Table 1.

Table 1: Statistical Results of Candidate Factors in Multifactor Model [6]

Factor Name	Average return	Excess returns	Difference income	correlation coeffcient
EPS growth rate	61.03%	12.30%	17.19%	0.417
Growth rate of main revenue	33.54%	-15.19%	-17.00%	0.242
Main profit growth rate	46.43%	-2.3%	5.21%	0.136
Net asset growth rate	35.77%	-12.96%	-28.01%	0.253
Total asset growth rate	37.23%	-11.5%	-19.77%	0.240

Table 1: (continued)

Growth rate of shareholders	39.01%	-9.72%	-14.82%	-0.190
Return on equity	34.69%	-14.04%	-20.72%	-0.069
Number of outstanding A-shares	91.03%	42.30%	69.12%	0.071
12 month dividend yield	56.70%	7.97%	9.48%	0.052
24 month annualized volatility	86.75%	38.02%	57.6%	0.154
24 month BETA value	105.95%	57.22%	93.33%	0.638
Annual average turnover rate	55.44%	6.71%	1.96%	0.271

Table 2: Correlation between various standardized factors and annual growth rate [6]

	ZX1	ZX2	ZX3	ZX4	ZX5	ZX6	Annual increase
ZX1	1.000						
ZX2	-0.094	1.000					
ZX3	-0.022	0.223	1.000				
ZX4	0.096	-0.009	-0.258	1.000			
ZX5	-0.121	0.449	0.124	0.202	1.000		
Zx6	0.116	-0.307	-0.244	0.395	-0.151	1.000	
Annual increase	0.143	0.361	0.079	0.287	0.624	0.065	1.000

- **Stock Scoring and Selection:** Each stock is scored using the formula
- $Y1=\beta1ZX1+\beta2ZX2+\beta3ZX3+\beta4ZX4+\beta5ZX5+\beta6ZX6$, where the correlation coefficient represents the relationship between the standardized factor x and the annual increase. This scoring system helps identify stocks with potential for future growth.
- **Multi-Factor Model:** The study utilizes a multi-factor quantitative stock selection methodology to assess the relative performance of 251 stocks. Based on this model, the top 10 stocks are selected for further evaluation.
- **Performance and Excess Returns:** The selected portfolio of the top 10 stocks generated a return of 119.62% over the specified period, with an effective excess return rate of 15.29%. This indicates the efficacy of the multi-factor model in identifying high-performing stocks.

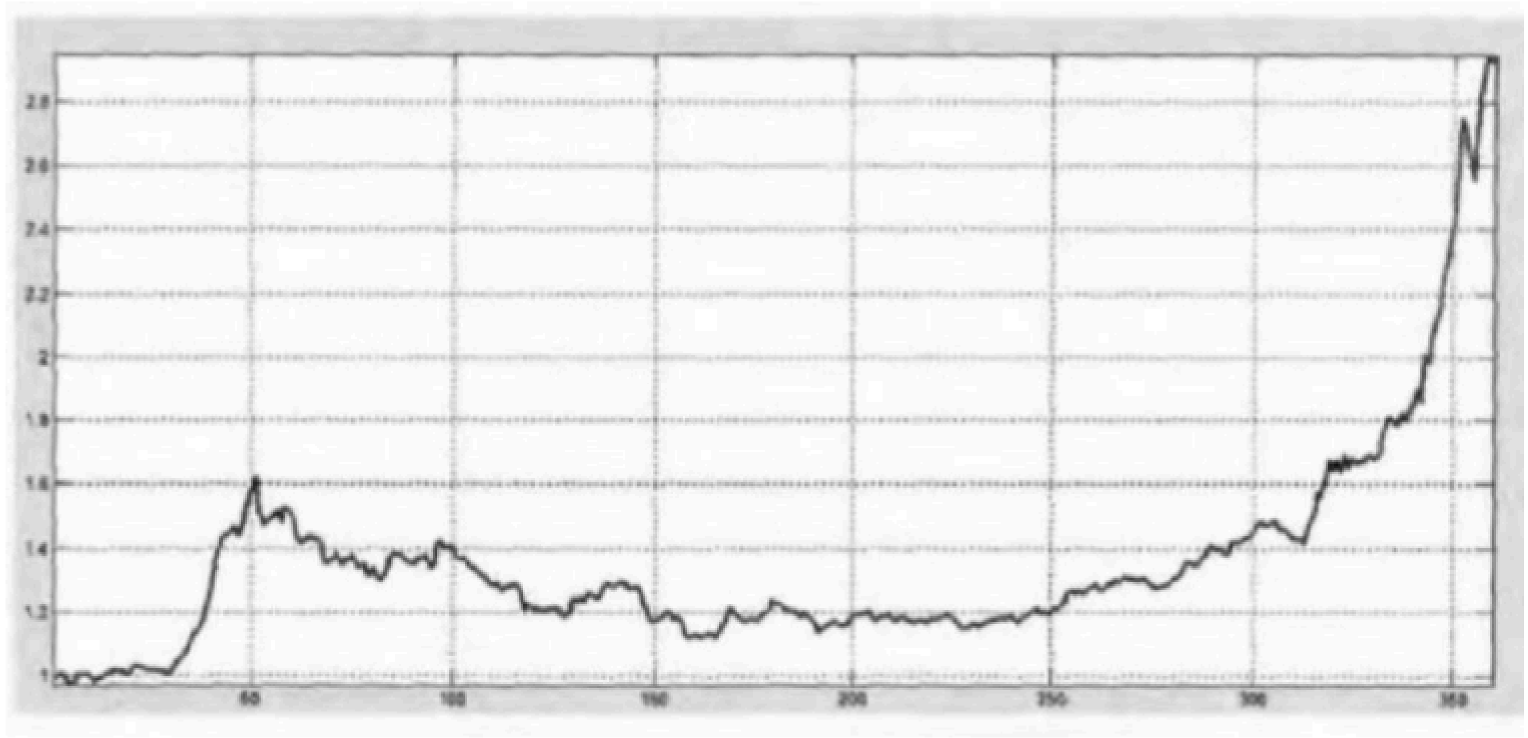


Figure 1: 360 trading day return curve of investment portfolio [6]

- **Risk Analysis and Maximum Retracement:** The portfolio undergoes risk analysis, focusing on the maximum retracement, which measures the potential maximum loss from the highest to the lowest point within a specific period. In Ni Yaoqi's study, a maximum retracement of 30.71% was identified, indicating a significant potential loss.
- Mean-CVaR Model for Portfolio Optimization: The mean-CVaR model is utilized to optimize the portfolio. The optimization involves minimizing CVaR while ensuring the portfolio meets specific return and weight constraints. This model is particularly effective in measuring and reflecting risks in the Chinese stock market.

$$\text{Maximum Retracement} = \frac{\text{Highest Point} - \text{Lowest Point}}{\text{Highest Point}}$$

- **Calculation of CVaR:** The Conditional Value at Risk (CVaR) for
- This method provides a more comprehensive risk measure by capturing tail risks.
- **Empirical Analysis and Results:** The analysis of the return series of ten stocks over 359 trading days reveals that the optimized portfolio, using the mean-CVaR model, significantly enhances performance. The distribution of portfolio weights approaches zero, indicating an improved risk-return profile.

$$\text{NDays' CVaR} = \text{One day's CVaR} \times \sqrt{N}$$

Mean-CVaR Model:

$$\min \mathrm{CVaR}_\alpha$$

$$\text{s.t.} \sum_{i=1}^{n} x_i r_i = C$$

$$\sum_{i=1}^{n} x_i = 1, \quad i \in N^+$$

Limitations and future development of portfolio optimization and risk analysis

- **Limitations of VaR:** Value at Risk (VaR) has two main drawbacks: it fails sub-additivity, making it inconsistent, and its measurement of tail loss is inadequate as it only considers points within a certain confidence level, ignoring the risk of major losses.
- **CVaR as an Alternative:** Conditional Value at Risk (CVaR) is introduced as an improved technique over VaR, addressing some limitations but still lacking robustness due to sub-additivity. Experts suggest using both VaR and CVaR together for their complementary capabilities.
- **Critique of Markowitz Model:** The Markowitz securities investment model's strict assumptions are deemed unrealistic, limiting its practical application. Flexibility with conditional assumptions could improve its relevance and usability in real-world market dynamics.
- **Omission of Transaction Costs:** The current model often ignores significant trading costs, especially for frequent, short-term traders, leading to potential portfolio management failures. Including transaction costs is crucial for more accurate and successful portfolio optimization.

CONCLUSION

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Study Focus:

The study centers on Markowitz’s portfolio optimization model and introduces the concept of unrelated assets for investors' portfolios.

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Risk Assessment Methods:

It covers two primary risk assessment approaches, Value at Risk (VaR) and Conditional Value at Risk (CVaR), presenting their respective models.

03

Empirical Analysis:

Using the mean CVaR model, an empirical analysis of the Chinese stock market was conducted, resulting in an optimal investment portfolio.

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Scope and Recommendations:

The research is limited to representative stocks in China, and findings may not be universally applicable. Future research should include empirical examinations of stock markets in various countries and explore the methodologies' suitability and constraints.

THANK YOU