1E3101

Roll No. Total No. of Pages: 4

1E3101

B. Tech. I - Sem. (Main / Back) Exam., - 2025

1FY2-01 Engineering Mathematics - I

Time: 3 Hours **Maximum Marks: 70**

Instructions to Candidates:

Attempt all ten questions from Part A, five questions out of seven questions from Part B and three questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL ______ 2. NIL ______

PART – A [10×2=20]

(Answer should be given up to 25 words only)

All questions are compulsory

Q.1 Evaluate - $\int_0^1 x^2(1-x)^3dx$

Q.2 Test the convergence of $\int_1^\infty \frac{dx}{x^{3/2}}$.

Q.3 What is Convergence and Divergence of a sequence?

Q.4 Find the interval of convergence of Exponential and Logarithmic series.

Q.5 Write Euler's formula of Fourier Series.

Q.6 Find half range sine series for the function $f(x) = x$ in the interval $0 < x < z$.

Q.7 If $u = e^{xyz}$, then find $\frac{\partial^3 u}{\partial x\, \partial y\, \partial z}$.

Q.8 Write the equation of the tangent plane to the surface $z = f(x, y)$.

Q.9 Change the order of integration and then evaluate - $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$.

Q.10 Write the statement of Green theorem.

PART – B

[5×4=20]

(Analytical/Problem solving questions)

Attempt any five questions

Q.1 Show that $\int_0^\infty \frac{x^2\, dx}{(1+x^4)^3} = \frac{5\pi\sqrt{2}}{128}$.

Q.2 Test for convergence of the series $\sum \frac{1}{\sqrt{n} + \sqrt{n+1}}$.

Q.3 Find the Fourier series to represent $f(x) = |x|$ for $-\pi < x < \pi$.

Q.4 Find the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ at $(1, 1, -1)$ in the direction of the vector $2\hat{i} + \hat{j} - \hat{k}$. Also find the direction of maximum directional derivative at $(1, 1, -1)$ and its max value.

Q.5 Find the limit and test for continuity of the function

$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x + y} & \text{if} \quad x + y \neq 0 \\ 0 & \text{if} \quad x + y = 0 \end{cases} \text{ at the point } (0, 0).$$

Q.6 Evaluate $\iint_R (x^2 + y^2)\, dx\, dy$ where R is the region bounded by $y = x$ and $y^2 = 4x$.

Q.7 Evaluate $\iiint_V f dV$ where $f = 2x + y$, V is the closed region bounded by the cylinder $z = 4 - x^2$ and the plane $x = y = z = 0$ and $y = z$.

PART – C

[3×10=30]

(Descriptive/Analytical/Problem Solving/Design Questions)

Attempt any three questions

Q.1 Find the Fourier Series to represent $f(x) = x - x^2$ in the interval $-1 < x < 1$.

Q.2 Test the convergence of the series $\frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \ldots\ldots\ldots\ldots$

Q.3 If $u = f(r)$, $r^2 = x^2 + y^2$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

Q.4 Find the volume of greatest rectangular parallelepiped inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Q.5 Verify Stokes theorem for $F = (x^2 + y - 4)\,\hat{i} + 3xy\,\hat{j} + (2xz + z^2)\hat{k}$ over the surface of the hemisphere $x^2 + y^2 + z^2 = 16$ above the xy plane.
