

# Assignment-1

## Chapter-2

## Complex Number

AI24BTECH11029- Rudrax Garwa

### SECTION-B JEE MAIN / AIEEE

- 14) If  $z^2 + z + 1 = 0$ , where  $z$  is an imaginary number, then the value of  $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \dots + \left(z^6 + \frac{1}{z^6}\right)^2$  is (1) [2006]
- a) 18      b) 54      c) 6      d) 12
- 15) If  $|z+4| \leq 3$ , then the maximum value of  $|z+1|$  is [2007]
- a) 6      b) 0      c) 4      d) 10
- 16) The conjugate of a complex is  $\frac{1}{i-1}$  then that complex number is [2008]
- a)  $\frac{-1}{i-1}$       b)  $\frac{1}{i+1}$       c)  $\frac{-1}{i+1}$       d)  $\frac{1}{i-1}$
- 17) Let  $R$  be the real line. Consider the following subsets of the real plane:
- $$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\} \quad (2)$$
- $$T = \{(x, y) : x - y \text{ is an integer}\}, \quad (3)$$
- Which one of the following is true ? [2008]
- Neither  $S$  nor  $T$  is an equivalence relation on  $R$   
 Both  $S$  and  $T$  are equivalence relation on  $R$   
 $S$  is an equivalence relation on  $R$  but  $T$  is not  
 $T$  is an equivalence relation on  $R$  but  $S$  is not
- 18) The number of complex numbers  $z$  such that  $|z-1| = |z+1| = |z-i|$  equals [2010]
- a) 1      b) 2      c)  $\infty$       d) 0
- 19) Let  $\alpha, \beta$  be real and  $z$  be a complex number. If  $z^2 + z\alpha + \beta = 0$  has two distinct roots on the line  $\operatorname{Re} z = 1$ , then it is necessary that: [2011]
- a)  $\beta \in (-1, 0)$       b)  $|\beta| = 1$   
 c)  $\beta \in (1, \infty)$       d)  $\beta \in (0, 1)$
- 20) If  $\omega (\neq 1)$  is the cube root of unity, and  $(1 + \omega)^7 = A + B\omega$ . Then  $(A, B)$  equals : [2011]
- a) (1, 1)      b) (1, 0)  
 c) (-1, 1)      d) (0, 1)
- 21) If  $z \neq 1$  and  $\frac{z^2}{z-1}$  is real, then the point represented by the complex number  $z$  lies: [2012]
- either on the real axis or on a circle not passing through the origin  
 either on the real axis or on a circle not passing through the origin on imaginary axis
- 22) If  $z$  is a complex number of unit modulus and argument  $\theta$ , then the  $\arg\left(\frac{1+z}{1+\bar{z}}\right)$  equals: [JEE M 2013]
- a)  $-\theta$       b)  $\frac{\pi}{2} - \theta$       c)  $\theta$       d)  $\pi - \theta$
- 23) If  $z$  is a complex number such that  $|z| \geq 2$ , then the minimum value of  $|z + \frac{1}{z}|$  : [JEE M 2014]
- a) is strictly greater than  $\frac{5}{2}$   
 b) is strictly greater than  $\frac{3}{2}$  but less than  $\frac{5}{2}$   
 c) is equal to  $\frac{5}{2}$   
 d) lies in the interval  $(1, 2)$
- 24) A complex number  $z$  is said to be unimodular if  $|z| = 1$ . Suppose  $z_1$  and  $z_2$  are complex numbers such that  $\frac{z_1 - 2z_2}{2 - z_1 \bar{z}_2}$  is unimodular and  $z_2$  is not unimodular. Then the point  $z_1$  lies on a: [JEE M 2016]
- a) circle of radius 2  
 b) circle of radius  $\sqrt{2}$   
 c) straight line parallel to x-axis  
 d) straight line parallel to y-axis
- 25) A value of  $\theta$  for which  $\frac{2+3i \sin(\theta)}{1-2i \sin(\theta)}$  is purely imaginary is : [JEE M 2016]

a)  $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$       b)  $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

c)  $\frac{\pi}{3}$       d)  $\frac{\pi}{6}$

- 26) Let  $\{ A = \theta \in \left(\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin(\theta)}{1-2i\sin(\theta)} \text{ is purely imaginary. Then the sum of element in A is: [JEE M 2019-9 Jan (M)]$

a)  $\frac{5\pi}{6}$       b)  $\pi$       c)  $\frac{3\pi}{4}$       d)  $\frac{2\pi}{3}$

- 27) let  $\alpha$  and  $\beta$  be two roots of the equation  $x^2 + 2x + 2 = 0$ , then  $\alpha^{15} + \beta^{15}$  is equal to : [JEE M 2019-Jan (M)]

a) -256      b) 512      c) -512      d) 256

- 28) All the points in the set  $\left( S = \left\{ \frac{\alpha-i}{\alpha+i} : \alpha \in R \right\} \right)$  [JEE M 2019-9 April (M)]

- a) straight line whose slope is 1
- b) circle whose radius is 1
- c) circle whose radius is  $\sqrt{2}$
- d) straight line whose slope is -1