Rudresh Khandani
35, AI & DS

Q.1 What do you mean by Minimum Spanning Tree? What are the applications of MST?

Ans.1 Minimum Spanning Tree is a of edges of a connected edge-weighted undirected graph that ~~construct~~ connects all the vertices together without any cycles and with minimum possible edge weighted.

APPLICATIONS:

1. Consider and stations are to be linked using a communication network and lying of communication link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.
2. Designing LAN
3. Suppose you want to construct highways or railroads spanning several cities, then we can use the concept of MST.
4. Laying pipeline connecting offshore drilling sites, refineries and consumer market.

Q.2. Analyze time and space complexity of Prim, Krushal, Dijksta and Billman Ford Algorithm.

Time Complexity of Prim's Algorithm: $O(|E| \log |V|)$

Space Complexity of Prim's Algorithm: $O|V|$

Time Complexity of Krushal Alogorithm: $O(|E| \log |E|$

Space complexity of Krushal Alogorithm: $O|V|$

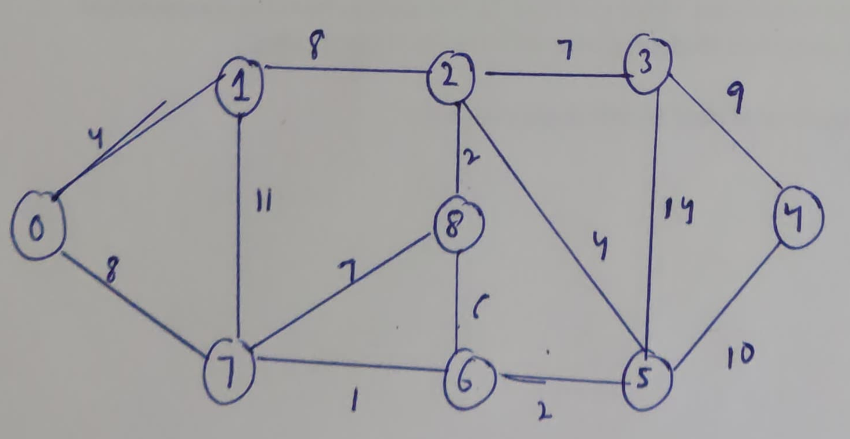
Time Complexity of Dijksta Algorithm: $O(V^2)$

Space Complexity of Dijksta Alogorithm: $O(V^2)$

Time Complexity of Billman Alogorithm: $O(VE)$

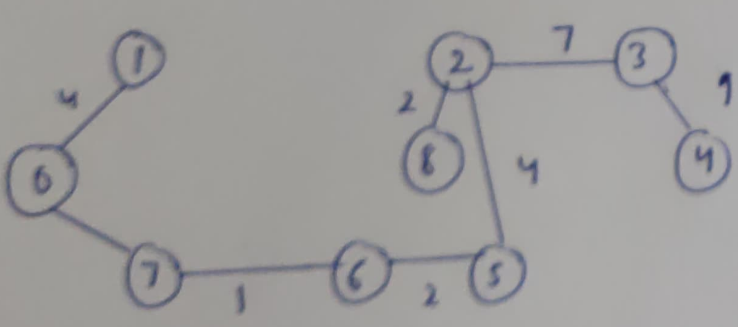
Space Complexity of Billman Alogorithm: $O(E)$

Q.3. Apply Krushal and Prim's Algorithm on a given graph to compute MST and its weight.



Kruskal's Algorithim

U	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	X
3	5	14	X



weight = 1+2+2+4+4+7+8+9

Prim's Algorithim

weight = 4+8+12+4+2

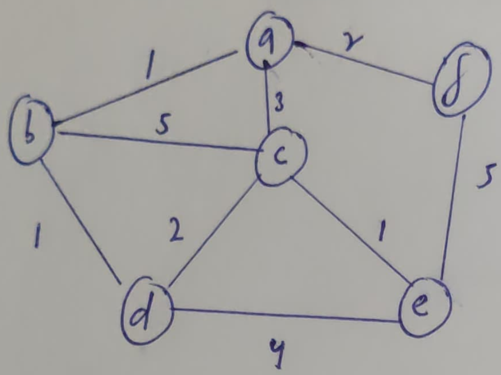
+7+9+3

= 37

Given a directed weighted graph, you are also given the shortest path from a source vertex 's' to a destination vertex 't'. and the shortest distance path remain same in following cases: (4)

(i) If weight of every edge is increased by 10 unit

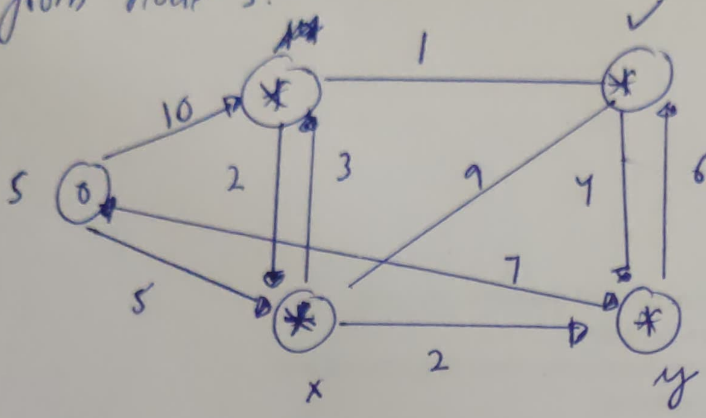
(ii) If weight of every edge is multiplied by 10 unit



The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'.
For example - has the shortest path of weight 15 and has edges 5. let them there be another path with 2 edges and the total weight 25. The weight of the shortest path is increased by 5*10 and becomes 15+50. weight of others path is increased by 2*10. It becomes 25+20. So, the shortest path to other path weight as 45.

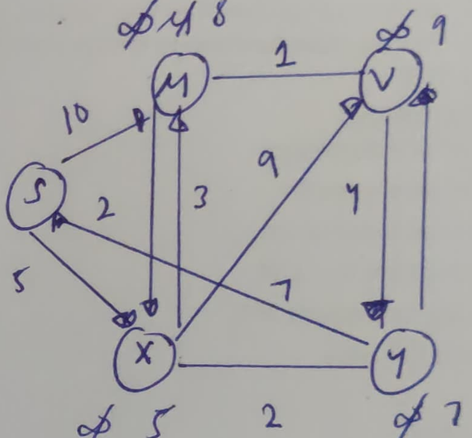
(ii) If we multiply all edges weight by 10, the shortest path does not change. The reason is that weight of all paths from 's' to 't' get multiplied by the same unit. The number of edges and path does not matter.

Apply Dijkstra & Bellman Ford algorithm on graph given the right side to compute shortest path to all node from node s. (5)



Dijkstra's Algorithm

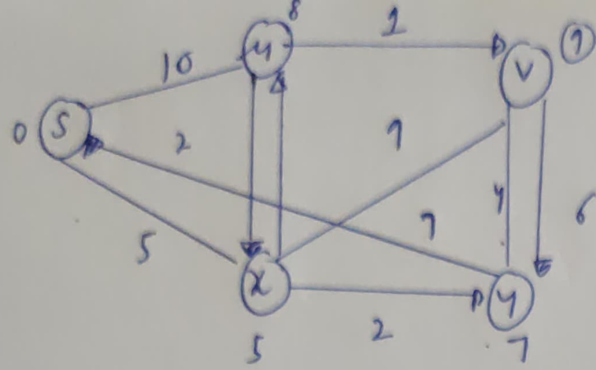
NODE	SHORTEST DISTANCE FROM SOURCE NODE
u	8
x	5
v	9
y	7



Bellman Ford Algorithm

	s	u	v	x	y
1st →	0	~~0~~ ∞	∞	~~∅~~ 5	∞
2nd →	0	10	~~∅~~ 11	5	∞
3rd →	0	~~10~~ 8	~~11~~ 9	5	9
4th →	0	8	9	5	7
5th →					

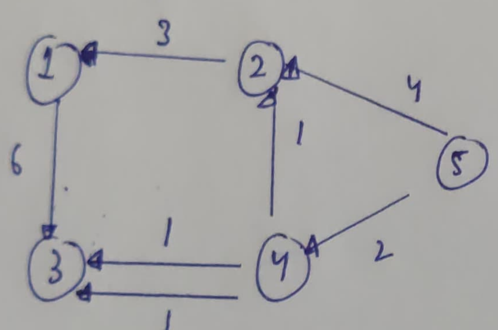
graph does not have negative cycle.



- final graph.

©

Q.6 Apply all pair shortest path algorithm.- Floyd warshall on the maintained graph. Also analyze space & time complexity of it.



$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
3 & 1 & 1 & 0 & \infty \\
6 & 4 & 12 & 2 & 0
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
3 & 1 & 1 & 0 & \infty \\
6 & 4 & 12 & 2 & 0
\end{bmatrix}
\end{array}
$$

AN.

Time capacity – $O(|V|^3)$

Space capacity – $O(|V|^2)$.