

Tutorial-4

Ex. 1] Independent Component Analysis

Mixing statistically independent sources

now, variance of mixture

$$\begin{aligned}
 \text{var}(x) &= \langle (x - \langle x \rangle)^2 \rangle \\
 &= \langle x^2 \rangle - \langle x \rangle^2 \\
 &= \langle (\sum_i w_i s_i)^2 \rangle - \langle \sum_i w_i s_i \rangle^2 \\
 &= \langle (\sum_i w_i s_i) (\sum_j w_j s_j) \rangle - (\sum_i w_i \langle s_i \rangle) (\sum_j w_j \langle s_j \rangle) \\
 &= \langle \sum_{i,j} w_i w_j s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle \\
 &= \sum_i w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) + \sum_{i,j} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) \\
 &= \sum_i w_i^2 (\langle s_i s_i \rangle - \langle s_i \rangle^2) + \sum_{i,j} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle)
 \end{aligned}$$

s_i & s_j are statistically independent for $i \neq j$

$$\Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i s_j \rangle = 0$$

$$\text{and } \text{var}(s_i) = 1$$

$$\therefore \text{var}(x) = \sum_i w_i^2$$

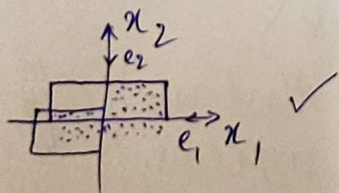
to guarantee that mixture has unit variance,

$$\text{var}(x) = 1$$

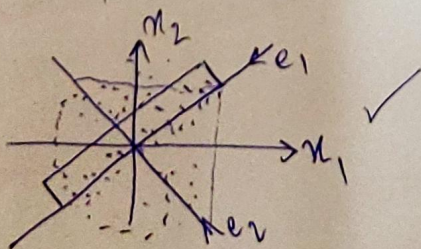
$$\therefore \left[\sum_i w_i^2 = 1 \right]$$

← the following constraint has to be imposed on the weights w_i for the mixture to hv. unit variance.

Ex. 2] (a)



(b)



(c) x - cannot be separated into ~~linear~~ independent components

