Tutorial-6 (Machine Learning)

1) a) $P(H) = \lambda$
$P(T) = 1 - \lambda$ $\quad P(\text{first head at } K+1) = (1-\lambda)^K \cdot \lambda$

b) Let M be no. of tosses reqd. to get first head,
Let $S = E[M]$
as tosses are independent & eqn. is additive,

$$S = \lambda \times 1 + (1-\lambda)(S+1)$$
$$= \lambda + S + 1 - \lambda S - \lambda$$
$$\therefore \lambda S = 1 \quad \therefore S = \frac{1}{\lambda} \text{ //}$$

2) $X \rightarrow$ random var.

a) $var(X) = E[X - E(X)^2]$

to prove: $var(X) = E[X^2] - E[X]^2$

now, we have,

$$var(X) = E[(X - E[X])^2]$$
$$= E[X^2 - 2XE[X] + E[X]^2]$$
$$= E[X^2] - 2E[XE[X]] + E[X]^2$$
$$= E[X^2] - 2E[X]^2 + E[X]^2$$
$$= E[X^2] - (E[X])^2 \quad \text{thus proved //}$$

b) $E[X] = 0$, $E[X^2] = 1$

(i) $var(X) = E[X^2] - E[X]^2$
$= 1 - 0^2 = 1$

(ii) $Y = a + bX$

$$E[Y^2] = E[(a+bX)^2] = E[a^2 + 2abX + b^2X^2]$$
$$= E[X] \cdot 2ab + a^2 + b^2 E[X^2]$$
$$= a^2 + 2ab(0) + b^2$$
$$= a^2 + b^2 \text{ //}$$

$$E[Y] = E[a+bX] = a + bE[X]$$
$$= a + b(0) = a \quad //$$

$$\therefore var(Y) = -E[X]^2 + E[X^2]$$
$$= a^2 + b^2 - a^2$$
$$= b^2 \quad //$$

3) A ⇒ Aku predicts given horse is winning horse
$\sim A$ ⇒ " " " " is not " "

B ⇒ event that given horse wins
$\sim B$ ⇒ " " " " doesnot win

a) given a horse, prob. it wins.

$$P(B) = P(B,A) + P(B \cap A)$$
$$= P(B/A).P(A) + P(B/\sim A).P(\sim A)$$
$$= 0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5})$$

$$\therefore P(B) \approx 1.99 \times 10^{-5} \quad //$$

b) prob. that Aku predicts black beauty is winning

$$P(A/B) = \frac{P(A,B)}{P(B)} = \frac{P(A/B)P(A)}{P(B)} = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$$

$$\therefore P(A/B) = 0.497 \quad //$$