Tutorial - 5

(Evaluation & Measurement of Hypothesis Testing)

1) $H_0 : p = 0.7$

$H_1 : p \neq 0.7$

Level of significance $= \alpha = 0.1$

test stat. : binomial var. X with $p = 0.7$, $n = 15$

$X = 8$ & $np_0 = 15 \times 0.7 = 10.5$

$\therefore P = 2P(X \leq 8 \text{ when } p = 0.7)$

$= 2 \sum_{x=0}^{8} b(x; 15, 0.7)$

$= 2 \times 0.1311$ (from binomial prob. table)

$= 0.2622$

$\because P > 0.1$ i.e. $P > \alpha$

$\therefore$ do not reject H_0. Conclude that there is insufficient reason to doubt the builders claim.

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2) $H_0 : P = 0.6$

$H_1 : P > 0.6$

$\alpha = 0.05$

given, $x = 70$, $n = 100$, $p = 0.6$

$$z = \frac{x - np_0}{\sqrt{np_0q_0}}$$

$$= \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}} = 2.04$$

$P = P(z > 2.04)$

$= 0.0207$ (from table)

as $P < \alpha$, reject H_0 & conclude that new drug is superior.

3) Let P_1 be proportion of Mumbai voters
P_2 be proportion of surrounding area residents.

$\hat{P}_1 = \frac{120}{200} = 0.6$, $\hat{P}_2 = \frac{240}{500} = 0.48$, $\hat{P}_p = \frac{120+240}{200+500} = 0.514$

$\alpha = 5\% = 0.05$

Hypothesis: $H_0 : P_1 \leq P_2$
$H_1 : P_1 > P_2$

$$z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_p(1-\hat{P}_p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.6 - 0.48}{\sqrt{0.514(1-0.514)\left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$\therefore z = 2.869 \Rightarrow P(z > 2.869) = 0.0044$

now, as $P < \alpha$, reject H_0 and conclude that the proportion of Mumbai voters favouring the proposal is higher than proportion of surrounding area voters.

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4) a) $H_0 : p = 0.2$
$H_1 : p > 0.2$ the critical region is in right tail.

b) $H_0 : \mu = 3$
$H_1 : \mu \neq 3$ the critical region is in both tails.

c) $H_0 : p \, \cancel{\geq} = 0.15$ the critical region is in left tail.
$H_1 : p < 0.15$

d) $H_0 : \mu = 500$ the critical region is in right tail.
$H_1 : \mu > 500$

e) $H_0 : \mu = 15$ the critical region is in both tails.
$H_1 : \mu \neq 15$

5) Let μ_1 = population mean "robustness" - laptops companyA
μ_2 = " " " - company B

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$ $\quad \alpha = 0.05$

$$\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i} = \frac{9.3+8.8+6.8+8.7+8.5+6.7+8+6.5+9.2+7}{10}$$

$\therefore \bar{x}_1 = 7.95$

$$\bar{x}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} x_{2i} = \frac{11+9.8+9.9+10.2+10.1+9.7+11+11.1+10.2+9.6}{10}$$

$\therefore \bar{x}_2 = 10.26$

$$s_1^2 = \frac{1}{n_1-1}\left(\sum_{i=1}^{n_1} x_{1i}^2 - n_1\bar{x}_1^2\right) = \frac{10.865}{9} = 1.207$$

$$s_2^2 = \frac{1}{n_2-1}\left(\sum_{1}^{n_2} x_{2i}^2 - n_2\bar{x}_2^2\right) = \frac{2.924}{9} = 0.325$$

as sample variances are very different, we cannot assume population variances equal, so use the "unpooled t-test"

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{9}\left(\frac{1.207}{10}\right)^2 + \frac{1}{9}\left(\frac{0.325}{10}\right)^2}$$

$\therefore v = 10.30 \approx 10$

the test statistic used to test hypo. is $T = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

which under the null hypothesis, follows approximately t-distribution with $v = 10$ degrees of freedom. Under null hypo, $(\mu_1 - \mu_2) = 0$

$$\therefore \text{value of } T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.90$$

Since the test is two-sided,
the value of test is the doubled area under density curve of t-distribution with $(v=10)$, right of the absolute value of test stat

$|t| = |-5.9| = 5.9$ i.e. $p\text{-value} = 2P(T \geq |t|)$
$= 2P(T \geq 5.9)$

$t_{0.0005}(10) = 4.587$ & since $|t| = 5.9$ is even greater than $P(T \geq 5.9) < 0.0005$ so,

$p\text{-value} < 0.001$

as $p < \alpha$, reject null hypo., conclude that the mean "robustness" of laptop is not same for both companies.