MARKOV AND POISSON PROCESSES

1. The tpm of a Markov chain with three states 0,1,2 is P = $\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and the initial state distribution of the chain is $P\{X_0 = i\} = \frac{1}{3}$, i = 0,1,2. Find (i) $P\{X_2 = 2\}$ and (ii) $P\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$
2. A man is at an integral point on the x-axis between the origin and the point 3. He takes a unit step to the right with probability $\frac{1}{3}$ or to the left with probability $\frac{2}{3}$, unless he is at the origin, where he takes a step to the right to reach the point 1 or is at the point 3, where he takes a step to the left to reach the point 2. What is the probability that (i) he is at the point 1 after 3 walks? And (ii) he is at the point 1 in the long run?
3. Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $1/3$ and that probability of a rainy day following a dry day is $1/2$. Given that May 1 is a dry day. Find the probability that (i) May 3 is also a dry day and (ii) May 5 also a dry day.
4. A communication source can generate 1 of 3 possible messages 1,2 and 3. Assume that the generation can be described by a homogeneous Markov chain with the following transition probability matrix

Current message		Next message	
	1	2	3
1	0.5	0.3	0.2
2	0.4	0.2	0.4
3	0.3	0.3	0.4

and the initial state probability distribution $P^{(0)} = (0.3, 0.3, 0.4)$. Find $P^{(3)}$.

5. Assume that the weather in a certain locality can be modeled as the homogeneous Markov chain whose transition probability matrix is given below.

		Tomorrow's weather	
Todays weather	Fair	Cloudy	Rainy
Fair	0.8	0.15	0.05
Cloudy	0.5	0.3	0.2
Rainy	0.6	0.3	0.1

If the initial state distribution is given by $P^{(0)}$ = (0.7, 0.2, 0.1), find $P^{(2)}$ and $\lim_{n\to\infty} P^{(n)}$.

6. There are 2 white marbles in urn A and 4 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. The state of the

related Markov chain is the number of red balls in A after the interchange. What is the probability that there are 2 red balls in urn A (i) after 3 steps and (ii) in the long run?

7. A student's study habits are as follows: If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night as well. In the long run, how often does he study?
8. A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state?
9. A housewife buys 3 kinds of cereals, A, B and C. she never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys a cereal B. However, if she buys B or C, the next week she is 3 times likely to buy A as other cereal. In the long run how often she buys each of the three cereals?
10. Two boys B1 and B2 and two girls G1 and G2 are throwing a ball form one to another. Each boy throws ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand, each girl throws the ball to each boy with probability 1/2 and never to the other girl. In the long run, how often does each receive the ball?
11. A gambler's luck follows a pattern. If he wins a game, the probability of his winning the next game is 0.6. However if he loses a game, the probability of his losing the next game is 0.7. There is an even chance that the gambler wins the first game. What is the probability that he wins (i) the second game, (ii) the third game and (iii) in the long run?
12. The three-state Markov chain is given by the tpm P = $\begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ Prove that the chain is irreducible and all the states are a periodic and non-null persistent. Find also the steady – state distribution of the chain.
13. Three boys A,B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.
14. Find the nature of the states of the Markov chain with tpm $P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix} \end{array}$
15. The transition probability matrix of a Markov chain $\{X_n\}$, n = 1,2,3… having 3 states 1, 2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$. Find (i) $P\{X_2 = 3\}$ and (ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

16. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run.

17. If particles are emitted from a radioactive source at the rate of 20 per hour, find the probability that exactly 5 particles are emitted during a 15-min period.
18. On the average, a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of ships sighted in a given length of time is a Poisson variate, find the probability of sighting (i) 6 ships in the next half-an-hour, (ii) 4 ships in the next 2 hour (iii) at least 1 ship in the next 15 min and (iv) at least 2 ships in the next 20 min.
19. Patients arrive randomly and independently at a doctor's consulting room from 8 A.M. at an average rate of one in 5 min. The waiting room can hold 12 persons. What is the probability that the room will be full when the doctor arrives at 9 A.M.?
20. A radioactive source emits particles at a rate of 6 per minute in accordance with Poisson process. Each particle emitted has a probability of 1/3 of being recorded. Find the probability that at least 5 particles are recorded in a 5-min period.
21. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute; find the probability that during a time interval of 2 min (i) exactly 4 customers arrive and (ii) more than 4 customers arrive.
22. A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks.
23. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (i) more than 1 min, (ii) between 1 min and 2 min (iii) 4 min. or less.