Random Processes

1. Consider a random process X(t) defined by X(t) = U cost + V sint , where U and V are independent random variables each of which assumes the values -2 and 1 with probabilities 1/3 and 2/3 respectively. Show that X (t) is wide sense stationary
2. Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R(z) = 25 + \dfrac{4}{1+6z^2}$. Find the mean and variance of the process {X (t)}.
3. Show that the process X(t) = A cosλ t + B sin λ t (where A and B are random variables) is wide sense stationary, if (1) E (A) = E (B) = 0 (2) $E(A^2) = E(B^2)$ and $\mathrm{E}(\mathrm{AB}) = 0$
4. Consider the random process $X(t) = \cos(\omega_0 t + \theta)$ where θ uniformly distributed in the interval –π to π. Check whether X (t) is stationary or not.
5. Consider a random process X (t) = B cos (50t + ϕ) where B and ϕ are independent random variables. B is a random variable with mean 0 and variance 1. ϕ is uniformly distributed in the interval [-π , π] . Find mean and autocorrelation of the process.
6. Find the mean of the stationary process {X(t)} whose auto correlation function is $R(z) = \dfrac{25z^2 + 36}{6.55z^2 + 4}$.
7. Show that the random process X (t) = Acos (ω t + θ) is wide sense stationary if A and ω are constant and $\square$ is uniformly distributed random variable in (0 , 2π).
8. The process {X(t)}whose probability distribution under certain condition is given by $P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}} & n = 1,2,\ldots\ldots \\ = \dfrac{at}{1+at} & n = 0 \end{cases}$ show that it is not stationary.
9. Two random process X (t) and Y (t) are defined by X(t) = A cosω t + B sinω t and Y(t) = B cosω t - A sinω t. Show that X (t) and Y (t) are jointly wide-sense stationary if A and B are uncorrelated random variables with zero means and the same variables and ω is constant.
10. Given a random variable Y with characteristic function ϕ(ω) = $E\left|e^{iwy}\right|$ and a random process defined by X(t) = cos (λ t + y) , show that {X(t)} is stationary in the wide sense if ϕ(1) =ϕ(2) = 0.
11. For the process {X(t):t≥ 0},X(t) is given by X(t) = a cos$\boxed{w}$t + b sin$\boxed{w}$t. Here a and b are two independent normal variables with E(a) = E(b) = 0 and var(a) = var(b) = σ^2 and $\boxed{w}$ is a constant. Obtain the mean, variance, correlation.
12. Let X (t) = cos ($\square$t + y) for t≥ 0 where $\square$ is a constant. Show that {X (t)} is stationary in the wide sense if and only if ϕ(1) = 0 = ϕ(2) where ϕ is the characteristic function of the random variable Y.

13. Prove that the random process X (t) and Y (t) defined by $X(t) = A\cos\omega_0 t + B\sin\omega_0 t$, $Y(t) = B\cos\omega_0 t - A\sin\omega_0 t$ are jointly wide-sense stationary if A and B are uncorrelated zero mean random variables with the same variance.
14. If {X(t)} is a WSS process with autocorrelation function Rxx (τ) and Y (t) = (t + a) – X (t-a), show that Ryy (τ) = 2Rxx (τ) – Rxx (τ + 2a) – Rxx (τ -2a).
15. Consider two random processes X (t) = 3 cos (ω t + $\square$) and Y (t) = 2 cos (ω t + $\square$-π/2) where $\square$ is a random variable uniformly distributed in (0, 2π). Prove that $\sqrt{Rxx(0)Ryy(0)} \geq |Rxy(\tau)|$.
16. If $X(t) = \sin(\omega t + y)$, where Y is uniformly distributed in $(0, 2\pi)$, prove that {X(t)}is a wide-sense stationary process.
17. Calculate the autocorrelation function of the process X(t) = A $\sin(\omega_0 t + \phi)$, where A and ω_0 are constants and ϕ ~U(0, 2π).
18. Consider the random process $V(t) = \cos(\omega t + \theta)$, where θ is a RV with probability density $P(\theta) = \begin{cases} \frac{1}{2\pi} & -\pi \leq \theta \leq \pi \\ 0 & \text{elsewhere} \end{cases}$

Show that the first and second moments of V (t) are independent of time. If θ = constant, will the ensemble mean of V (t) be time-independent?

19. A stochastic process is described by x(t) = A sin t + B cos t , where A and B are independent RVs with zero means and equal standard deviations. Show that the process is stationary of the second order.
20. Consider a random process $Z(t) = X_1 \cos\omega_0 t - X_2 \sin\omega_0 t$, Where X_1 and X_2 are independent Gaussian RVs with zero mean and variance σ^2. Find $E\{z\}$ and $E\{z^2\}$. .
21. If U (t) = X cos t + Y sin t and V (t) = Y cos t + X sin t , where X and Y are independent RVs such that E(X) = 0 = E(Y), $E(X^2) = E(Y^2) = 1$, show that {U(t)} and {V(t)} are individually stationary in the wide sense , but they are not jointly wide-sense stationary.
22. If $X(t) = 5\cos(10t + \theta)$ and $Y(t) = 20\sin(10t + \theta)$, where θ is a RV uniformly distributed in (0, 2π), prove that the processes {X (t)} and {Y (t)} are jointly wide-sense stationary.
23. If $X(t) = A\sin(\omega t + \theta)$, where A and ω are constants and θ is RV uniformly distributed (-$\square$,$\square$).find the autocorrelation of {Y (t)}, where Y(t) = X^2(t)
24. A stationary process has an autocorrelation function given by $R(\tau) = \frac{144\tau^2 + 36}{625\tau^2 + 4}$.
Find the mean value, mean-square value and variance of the process. f the autocorrelation of a process {X (t)} is R_{xx}. and if Y (t) = X (t + a)–X(a) where a is a constant, express R_{YY} in terms of R_{xx}.

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