DISTRIBUTIONS----PS 5

1. The mean and variance of a binomial variate X with parameters n and p are 16 and 8. Find (i) P (X = 0), (ii) P(X = 1), (iii) $P(X \geq 2)$.

2. In 256 sets of twelve tosses of a fair coin, in how many cases may one expect eight heads and four tails?

3. In 100 sets of ten tosses of an unbiased coin, in how many cases should we expect (i) Seven heads and three tails, (ii) at least seven heads?

4. During war 1 ship out of 9 was sunk out on an average in making a certain voyage. What was the probability that exactly 3 out of a convoy of 6 ships would arrive safely?

5. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers chosen at random, four or more will suffer from the disease?

6. With the usual notations find p for a BD if n = 6 and if 9 P(X = 4) = P(X = 2).Hence mean and variance of X

7. X is a random variable following BD with mean 2.4 and variance 1.44.Find P (X≥ 5) and P (1<X≤ 4).

8. If X is a Poisson variable such that $p(X = 2) = 9p(X = 4) + 90p(X = 6)$. Find the mean of X and P(X>2) and P(X = 1)

9. Assuming that the chance of a traffic accident in a day in a street of a city is 0.001, on how many days out of a trial of 1,000 days can we expect (i) no accident (ii) more than three accidents if there are 1000 such streets in the whole city?

10. Patients arrive randomly and independently at a doctor's surgery from 8.00 AM at an average rate of one in 5 minutes. The waiting room holds 2 persons. What is the probability that the room will be full when the doctor arrives at 9.30 AM?

11. An office switchboard receives telephone calls at the rate of 3 calls per minute on an average. What is the probability of receiving (i) no calls in a one-minute interval, (ii) at the most 3 calls in a five-minute interval?

12. If X is a Poisson variable such that $p(X = 2) = \frac{2}{3}p(X = 4)$. Find the mean and variance of X and P (X>2) and P (X = 0).

13. If a boy is throwing at a target, what is the probability that his 10^{th} throw is his 6^{th} hit if the probability of hitting the target at any trail is 0.4?

14. Subway trains on a certain line run every half hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20 minutes?

15. If X has exponential distribution with mean 2, find p (X < 1 / X < 2).

16. Suppose that during rainy season on a tropical island the length of the shower has an exponential distribution, with parameter $\lambda = 2,$ time being measured in minutes. What is the probability that a shower will last more than three minutes? If a shower has already lasted for 2 Minutes, what is the probability that it will last for at least one more minutes?

17. If X has exponential distribution with mean 2, find P (X < 1/ X < 2).
18. If X ~Expo (λ) with $P(X \le 1) = P(X > 1)$ find its mean and variance.
19. If 95% of certain high-performance radial tires last at least 30.000 miles, find the mean and the standard deviation of the distribution of the number of these tires, among 20 selected at random, that last at least 30,000 miles find the mean and variance.
20. If 0.8% of the fuses delivered to an arsenal are defective, use the Poisson approximation to determine the probability that 4 fuses will be defective in a random sample 400.
21. The number of gamma rays emitted per second by a certain radioactive substance is a random variable having the Poisson distribution with $\lambda = 5.8$. If a recording instrument becomes inoperative when there are more than 12 rays per second, what is the probability that this instrument becomes inoperative during any given second?
22. Given that the switchboard of a consultant's office receives on the average 0.6 calls per minute, find the probabilities that (a) in a given minute there will be at least 1 call. (b) in a 4-minute interval there will be at least 3 calls.
23. At a checkout counter customers arrive at an average of 1.5 per minute. Find the probabilities that (a) at most 4 will arrive in any given minute; (b) at least 3 will arrive during an interval of 2 minutes; (c) at most 15 will arrive during an interval of 6 minutes.
24. A pool company's records show that the probability is 0.20 that one of its new pools will require repairs within a year. What is the probability that the sixth pool it builds in a given year will be the first one to require repairs within a year?
25. Suppose that the lifetime of a certain kind of an emergency backup battery (in hours) is a random variable X having the Weibull distribution with $\alpha = 0.1$ and $\beta = 0.5$. Find (a) the mean lifetime of these batteries; (b) the probability that such a battery will last more than 300 hours.
26. In a certain experiments, the error made in determining the solubility of a substance is a random variable having the uniform density with $\alpha = -0.025$ and $\beta = 0.025$. What are the probabilities that such an error will be (a) between 0.010 and 0.015; (b) between –0.012 and 0.0127?
27. In a certain city, the daily consumption of electric power (in millions of kilowatt hours) can be treated as a random variable having a gamma distribution with $\alpha = 3$ and $\beta = 2$. If power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

28. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with $\beta = 50$ days. Find the

probabilities that such a camera will (a) have to be reset in less than 20 days; (b) not have to reset in at least 60 days.

29. Suppose that the time to failure (in minutes) of certain electronic components subjected to continuous vibrations may be looked upon as a random variable having the Weibull distribution with $\alpha = \frac{1}{3}$ and $\beta = \frac{1}{3}$ (a) How long can such a component be expected to last? (b) What is the probability that such a component will fail in less than 5 hours?
30. Suppose that the service life (in hours) of a semiconductor is a random variable having the Weibull distribution with $\alpha = 0.025$ and $\beta = 0.500$. What is the probability that such a semiconductor will still be in operating condition after 4,000 hours?
31. Five fair coins are flipped. If the outcomes are assumed independent, find the probability mass function of the number of heads obtained.
32. Suppose that the number of typographical errors on a single page of this book has a Poisson distribution with parameter $\lambda = \frac{1}{2}$. Calculate the probability that there is at least one error on this page.
33. Suppose that the probability that an item produced by a certain machine will be defective is 0.1. Find the probability that a sample of 10 items will contain at most 1 defective item.
34. Consider an experiment that consists of counting the number of α - particles given off in a 1- second interval by 1 gram of radioactive material. If we know from past experience that, on the average, 3.2 such α - particles are given off, what is a good approximation to the probability that no more than 2 α - particles will appear.
35. Find the expected value and the variance of the number of times one must throw a die until the outcome 1 has occurred 4 times.
36. A sample of 33 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.
37. The expected number of typographical errors on a page of a certain magazine is 0.2. What is the probability that the next page you read contains (a) 0 and (b) 2 or more typographical errors?
38. The monthly worldwide average number of airplane crashes of commercial airlines 3.5. What is the probability that there will be (i) at least 2 such accidents in the next month; (ii) at most 1 accident in the next month?
39. If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $\frac{1}{100}$, what is the (approximate) probability that you will win a prize.

    (a) at least one; (b) exactly once; (c) at least twice?

40. When three friends go for coffee, they decide who will pay the check by each flipping a coin and then letting the "odd person" pay. If all three flips are the same (so there is no odd person), then they make a second round of flips, and continue to do so until there is an odd person. What is the probability that exactly 3 rounds of flips are made; more than 4 rounds are needed?

41. If X is U~(0, 10), calculate the probability that (a) X < 3, (b) X > 6, and (c) 3 < X < 8.

42. Buses arrive at a specified stop at 15-minutes intervals starting at 7 a.m. That is, they arrive at 7, 7:15, 7:30, 7:45; and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that waits less than 5 minutes for a bus; more than 10 minutes for a bus.

43. If X is a normal random variable with parameters $\mu = 3\ \sigma^2 = 9$, find the (a) p {2 < X < 5}; (b) p{X > 0}; (c) p {|X – 3 | > 6}.

44. Let X is the number of times that a fair coin flipped 40 times, lands heads. Find the probability that X = 20. Use the normal approximation and then compare it to the exact solution.

45. Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at a public telephone booth, Interpret and find p{X > 10} p{10 < X < 20}

46. Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 miles. If a person desires to take a 5000-mile trip, what is the probability that he or she will be able to complete the trip without having to replace the car battery? What can be said when the distribution is not exponential?

47. If X is a normal random variable with parameter $\mu = 10\ \sigma^2 = 36$, compute (a) p{ X > 5}; (b) p{4 < X < 16}; (c) p{X < 8};(d) p{ X< 20}; (e) p{X > 16}.

48. Suppose that X is a normal random variable with mean 5. If p {X > 9} =0 .2, approximately what is Var (X)?

49. If X is uniformly distributed over (-1 , 1), find (a) $P\left\{|X| > \frac{1}{2}\right\}$;

50. A random chosen IQ test taker obtains a score that is approximately a normal random variable with mean 100 and standard deviation 15. What is the probability that the test score of such person is (a) above 125; (b) between 90 and 110?

51. The mean weight of 500 male students at a certain college is 151 pounds (lb), and the standard deviation is 15lb. Assuming that the weights are normally distributed, find how many students weigh (a) between 120 and 155lb and (b) more than 185lb.

52. Determine how many of the 500 students in problem 7.20 weigh (a) less than 128 lb, (b) 128 lb, and (c) less than or equal to 128 lb.
53. A fair coin is tossed 500 times. Find the probability that the number of heads will not differ from 250 by (a) more than 10 and (b) more than 30.
54. Find (a) the mean and (b) the standard deviation on an examination in which grades of 70 and 88 correspond to standard scores of – 0.6 and 1.4 respectively.
55. Find the area under the normal curve between (a) $z = -1.20$ and $z = 2.40$, (b) $z= 1.23$ and $z = 1.87$, and (c) $z = - 2.35$ and $z= -0.50$.
56. Find the area under the normal curve (a) to the left $z = - 1.78$. (b) to the left of $z = 0.56$, (c) to the right of $z = -1.45$, (d) corresponding to $z \geq 2.16$, (e) corresponding to $-0.80 \leq z \leq 1.53$, and (f) to the left of $z = -2.52$ and to the right of $z = 1.83$.
57. If z is normally distributed with mean 0 and variance 1, find (a) $\Pr\{z \geq -1.64\}$, (b) $\Pr\{-1.96 \leq z \leq 1.96\}$, and (c) $\Pr\{|z| \geq 1\}$.
58. Find the value of z such that (a) the area to the right of z is 0.2266, (b) the area to the left of z is 0.0314, (c) the area between –0.23 and z is 0.5722, (d) the area between 1.15 and z is 0.0730, and (e) the area between –z and z is 0.9000.
59. If the heights of 300 students are normally distributed with mean 68.0 in and standard deviation 3.0 in, how many students have heights (a) greater than 72in, (b) less than or equal to 64in, (c) between 65 and 71in inclusive, and (d) equal to 68 in? Assume the measurements to be recorded to the nearest inch.
60. If the diameters of ball bearings are normally distributed with mean 0.6140in and standard deviation 0.0025in, determine the percentage of ball bearings with diameters (a) between 0.6010 and 0.618 in inclusive, (b) greater than 0.617in, (c) less than 0.608in, and (d) equal to 0.615.
61. In a BD consisting of 5 independent trials probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the probability of getting at most 3 successes.
62. In a distribution exactly normal 7% of the items are under 35 and 89% are under 63.What are the parameters of the distribution?
63. Of a large group of men 5% are under 180 cms in height and 40% are between 180and 195 cms. Assuming a normal distribution find the mean and the SD.