RANDOM VARIABLES 2-D-------P. S -6

1. Find the value of k, if f(x,y) =k(1-x)(1-y) for 0<x,y<1, is to be a joint density function.
2. If the joint pdf of (X,Y) is $f(x,y) = 6e^{-2x-3y}$, $x \geq 0, y \geq 0$, find the marginal density of X and conditional density of Y given X.
3. The input to a binary communication system, denoted by a RV X, takes on one of two values 0 or 1 with the probabilities ¾ & ¼ respectively. Because of errors caused by noise in the system, the output Y differs from the input occasionally. The behavior of the communication system is modeled by the conditional probabilities given below: P(Y=1/X=1) = 3/4 and P(Y=0/X=0) = 7/8. Find i) P(Y=1), ii)P(Y=0) and iii)P(X=1/Y=1).

4. The following table represents the joint probability distribution of the discrete RV (X, Y). Find all the marginal and conditional distributions.

Y	X		
	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	¼	2/15

5. The joint distribution of X_1, X_2 is given by $f(x_1,x_2) = (x_1+x_2)/21$, $x_1 = 1, 2$ and 3; $x_2 = 1,2$.

   Find the marginal distributions of X_1 and X_2.

6. If the joint pdf of a two-dimensional RV (X, Y) is given by $f(x,y) = x^2+(xy)/3$;

   $0<x<1, 0<y<2$ 0 elsewhere, find (i) P (x>1/2) (ii) P(Y<X) and (iii) P(Y<1/2/X<1/2)

7. If the joint pdf of a two – dimensional RV (X, Y) is given by f(x,y) = k(6-x-y);

   0<x<2,2<y<4; 0 elsewhere, Find i) P(X<1,Y<3), ii) P(X+Y < 3) and iii) P(X<1/Y<3).

8. The joint probability function two RV's X and Y is given by f(x,y) = c(2x+y), where x and y can assume all integers such that $0 \leq x \leq 2$ and $0 \leq y \leq 3$, and f(x,y) = 0 otherwise.

   Find $P(X \geq 1, Y \leq 2)$.

9. The joint density function of the RV's X and Y is given by f(x,y) = 8xy; 0<x<1,0<y<x

   = 0 elsewhere, find P(y<1/8/X<1/2).

10. Given that the joint pdf of (X,Y) is $f(x,y) = ke^{-y}$; x>0,y>x; =0 elsewhere.Find (i) P(X>1/Y<5) and ii) marginal distributions of X and Y.

11. If the joint pdf of a two – dimensional RV (X,Y) is given by f(x,y) = k; $0<x<1$, $0<y<x$ = 0, elsewhere. Find the marginal density function of X and Y.

12. The joint density function of a RV (X,Y) is f(X,Y) = 8xy, $0<x<1$, $0<y<x$. find the conditional density function f(y/x).

13. The joint density function of a RV (X,Y) is given by f(x,y) = axy, $1 \leq x \leq 3, 2 \leq y \leq 4$, and =0, elsewhere find i) the marginal densities of X and Y ii)the conditional densities of X and Y, given Y and X respectively.

14. The joint pdf of the RV's X and Y is given by $p(x,y) = xe^{-x(y+1)}$ where $0 \leq x, y < \infty$.
Find p(x) and p(y) and (ii) Are the RV's independent?

15. The two – dimensional RV(X,Y) has the joint density f(x,y) = 8xy, $0<x<y<1$
=0,otherwise. i)Find $P(X<1/2 \cap Y < 1/4)$, ii)Find the marginal and conditional distributions, and iii) Are X and Y independent?

16. If two RV's have the joint density, $f(x,y) = (6/5)(x+y^2)$ for $0<x<1, 0<y<1$ = 0 elsewhere, find the probability that $0.2<X<0.5$ and $0.4<Y<0.6$.

17. If two RV's have the joint density, $f(x_1,x_2)=x_1x_2$ for $0<x_1<1, 0<x_2<2$ = 0 elsewhere. Find the probabilities that a) both RV's will take on values less than 1;b) The sum of the values taken on by the two RV's will be less than 1.

18. For the bivariate probability distribution of (X, Y) given below, find $p(x \leq 1)$, $p(Y \leq 3)$, $P(X \leq 1, Y \leq 3)$, $P(X \leq 1/Y \leq 3)$, $P(Y \leq 3/X \leq 1)$ and $P(X + Y \leq 4)$. Find E(x+y) E(xy) E(2x-3y+8)

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

19. The joint probability function of (X, Y) is given by p(x,y) = k(2x + 3y), x = 0,1,2; y = 1,2,3. Find all the marginal and conditional probability distributions. Also find E(X + Y).
20. The joint probability distribution of (X, Y) is given below. The relevant probabilities have been computed by using the given law. Find cov(X,Y)

X	Y		
	0	1	2
0	0.1	0.04	0.06
1	0.2	0.08	0.12
2	0.2	0.08	0.12

21. A machine is used for a particular job in the forenoon and for a different job in the afternoon. The joint probability distribution of (X, Y), where X and Y represent the number of times the machine breaks down in the forenoon and in the afternoon respectively, is given in the following table. Examine if X and Y are independent RVs.
Find the correlation coefficient of X,Y

X	Y		
	1	2	3
0	3k	6k	9k
1	5k	8k	11k
2	7k	10k	13k

22. If the joint density function of (X , Y) is given by f(x, y) = 2 – x – y, $0 \le x, y \le 1$, find E(X), E(Y), Var(X), Var(Y), ρ_{xy}.
23. If the two- dimensional RV(X, Y) is uniformly distributed in $0 \le x < y \le 1$, find E(X),E(Y), Var(X), Var(Y), ρ_{xy}.
24. If the joint pdf of (X, Y) is given by f(x , y) = x + y, $0 \le x, y \le 1$, find ρ_{xy}.
25. If X, Y, Z are uncorrelated RV s having the same variance, find the correlation co-efficient between (X + Y) and (Y + Z).
26. If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation co-efficient between (X + Y) and (X – Y) .

27. If X and Y are 2 uncorrelated RVs with zero means, prove that U = X cos α + Y sin α and V = X sin α - Y cos α are also uncorrected. Assume equal variance for X and Y.
28. X and Y are 2 RVs with variance σ^2_x and $\sigma^2 y$ respectively. Find the value of k,if U = X + kY and V = X + $\frac{\sigma_x}{\sigma_y}$ Y are un correlated.
29. If the joint pdf of (X, Y) is given by f (x, y) = 2, $0 \le x < y \le 1$, find the conditional Mean and conditional variance of X, given that Y = y.
30. If the joint pdf of (X, Y) is given by $f(x,y) = 21x^2y^3$, $0 \le x < y \le 1$, find the Conditional mean and variance of X , given that Y = y
31. X and Y are independent variables with means 10 and 20, and variances 2 and 3 respectively. Find the variance of 3X + 4 Y.
32. Suppose that X is a random variable for which E(X) = 10 and Var(X) = 25. Find the positive values of a and b such that Y = aX – b, has expectation 0 and variance 1.
33. Let X_1 *and* X_2 be two stochastic random variables having variances k and 2 respectively. If the variance of Y = $3X_2 - X_1$ is 25, find k.
34. Let X and Y be two random variables each taking three values –1,0 and 1, and having the joint probability distribution : (i) show that X and Y have different expectations.

X / Y	-1	0	1
1	0	.1	.1
0	.2	.2	.2
1	0	.1	.1

(ii) Prove that X and Y are uncorrelated , (iii) Find Var X and Var Y. (iv) Given that Y = 0, what is the conditional probability distribution of X ? (v) Find $V(Y \setminus X = -1)$.
35. Two random variables X and Y have the following joint probability density function f(x, y) = 2 – x – y ; $0 \le x \le 1$, $0 \le y \le 1$ = 0, otherwise find (i) Marginal probability density function of X and Y, (ii) Conditional density functions, (iii) Var(X) and Var (Y). (iv) Co-variance between X and Y.
36. Let f (x, y) = 8xy, $0 < x < y < 1$; f(x,y) = 0 elsewhere. Find (a) E(Y\X = x) (b) E(X\Y = y), (c) Var (Y\X = x).
37. X_1 and X_2 have a bivariate distribution given by $P(X_1 = x_1 \cap X_2 = x_2) = k(x_1 + 3x_2)$, Where (x_1, x_2) = (1,1),(1,2),(2,1),(2,2).Find the conditional mean and variance of X_1, given $X_2 = 2$.
38. Two random variables X and Y have the following joint probability density Function: $f(x,y) = k(4 - x - y)$; $0 \le x \le 2$; $0 \le y \le 2 = 0$, otherwise Find (i) the constant k, (ii) marginal density functions of X and Y, (iii) Conditional density functions, and(iv) Var(X),Var(Y) and Cov(X,Y).
39. Let the joint probability density function of the random variables X and Y be $f(x,y) = 2(x + y - 3xy^2)$; $0 < x < 1$, $0 < y < 1$ and zero otherwise. Find the marginal distributions of X and Y. Is E (XY) = E (X) E (Y)? Find E (X + Y) and E (X – Y).
40. Let X and Y have the joint probability density function $f(x,y) = 2, 0 < x < y < 1$ and zero otherwise. Find the conditional mean and variance of X given Y = y

41. If f(x, y) = 2; $0 < x < y, 0 < y < 1$. Find (i) E (Y/X), (ii) E(X/Y).
42. Let $f_{xy}(x, y) = e^{-(x+y)}$; $0 < x < \infty, 0 < y < \infty$. Find (a) P (X>1),(b) P(1<X + Y < 2) (c) P(X < Y/X<2Y) (d) ρ_{xy}.
43. The joint p.d.f. of X and Y is given by: $f(x, y) = 3(x + y); 0 \le y \le 1; \quad 0 \le x + y \le 1$. Find (a) marginal density of X, (b) P (X + Y < ½), (c) E (Y/X = x) and (d) Cov (X, Y).