QUEUING THEORY

1. A T.V. repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average of 10 per 8-hour day, what is repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?
2. Let on an average 96 patients per 24-hour day require the service of an emergency clinic. Also on an average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 patients, and that each minute of decrease in this average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from one and one third patients to half a patient.
3. In a super market the average arrival rate of customers is 5 every 30 minutes. The average time it takes to list and calculate the customer's purchase at the cash desk is 4.5 minutes, and this time is exponentially distributed.(a) How long will the customer expect to wait for service at the cash desk? (b) What is the chance that the queue length will exceed 5? (c) What is the probability that the cashier is working?
4. At a one-man barber shop, customer arrive according to Poisson distribution with a mean arrival rate of 5 per hour and his hair cutting time was exponentially distributed with an average hair cut taking 10 minutes. It is assumed that because of his excellent reputation, customers were always willing to wait. Calculate the following: (i) average number of customers in the shop and the average number of customers waiting for a haircut? (ii) The percent of time an arrival can walk right in without having to wait? (iii) The percentage of customers who have to wait prior to getting into the barber's chair.
5. Cars arrive at a petrol pump with exponential inter-arrival times having mean 1/2 minute. The attendant takes an average of 1/5 minute per car to supply petrol, the service times being exponentially distributed. Determine (i) the average number of cars waiting to be served, (ii) the average number of cars in the queue and (iii) the proportion of time for which the pump attendant is idle.
6. Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponential, with mean 3 minutes. (a) What is the probability that a person arriving at the booth will have to wait? (b) The telephone department will install a second booth when convinced that an arrival would expect waiting for at least 3 minutes for phone. By how

much should the flow of arrivals increase in order to justify a second booth? (c) Find the average number of units in the system. (d) Estimate the fraction of a day that the phone will be in use. (e) What is the probability that it will take him more than 10 minutes altogether to wait for phone and complete his call?

7. People arrive at a theatre ticket booth in a Poisson distributed arrival rate of 25 per hour; Service time is constant at 2 minutes. Calculate (i) the mean number in the waiting line. (ii) The mean waiting time.
8. A drive-in bank window has a mean service time of 2 minutes, while the customers arrive at a rate of 20 per hour. Assuming that these represent rates with a Poisson distribution: (i) what percentages of time will the teller is idle? (ii) After driving up, how long will it take the average customer to wait in line and be served? (iii) What fraction of customers will have to wait in line?
9. At a public telephone booth in a post office arrivals are considered to be Poisson with an average inter-arrival time of 12 minutes. The length of a phone call may be assumed to be distributed exponentially with an average of 4 minutes. Calculate the following: (a) what is the probability that a fresh arrival will not have to wait for the phone? (b) What is the probability that an arrival will have to wait more than 10 minutes before the phone is free? (c) What is the average length of queues that form from time to time?
10. At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various numbers of trains in the system. Also find the average waiting time of a new train coming into the yard.
11. Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines, one of which is reserved for shunting purposes), calculate the probability that the yard is empty and find the average queue length.
12. At a one-man barbershop, the customers arrive according to Poisson fashion at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. Suppose that the customers do not wait if they find no seat available (assuming that only 5 seats are available, then find the average number of customers in the system, average queue length and average waiting time a customer spends in system.
13. Patients arrive at a clinic according to a Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14

patients. Examination time per patient is exponential with mean rate 20 per hour. (a) Find the effective arrival rate at the clinic. (b) What is the probability that an arriving patient will not wait? (c) What is the expected waiting time until a patient is discharged from the clinic?

14. Customers arrive at a one-window drive-in bank according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The car space in front of the window, including that for the service can accommodate a maximum of 3 cars. Other cars can wait outside this space. (a) What is the probability that an arriving customer can drive directly to the space in front of the window? (b) What is the probability that an arriving customer will have to wait outside the indicated space? (c) How long is an arriving customer expected to wait before starting service?
15. A super market has two girls running up sales at the counters. If the service time for each customer is exponential with mean 4 minutes, and if people arrive in a Poisson fashion at the rate of 10 an hour, (a) what is the probability of having to wait for service? (b) What is the expected percentage of idle time for each girl?
16. A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles depositors only. It has been found that the service time distribution of both deposits and withdrawals are exponential with a mean service time of 3 minutes per customer. Depositors and withdrawers are found to arrive in a Poisson fashion throughout the day with mean arrival rate of 16 and 14 per hour. What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both withdrawals and deposits? What would be the effect if this could only be accomplished by increasing the service time to 3.5 minutes?
17. A petrol pump station has two pumps. The service time follows the exponential distribution with a mean of 4 minutes and cars arrive for service in a Poisson process at the rate of ten cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pumps remain idle?
18. A two-channel waiting line with Poisson arrival has a mean arrival rate of 50 per hour and exponential service with a mean service rate of 75 per hour for each channel. Find: (i) The probability of an empty system. (ii) The probability that an arrival in the system will have to wait.
19. A bank has two counters for withdrawals. Counter I handle withdrawals of value less than Rs.300 and counter II Rs.300 and above. Analysis if service time of 6 minutes per customer for both the Counters. Arrival of customers follows Poisson distribution with mean 8 per hour for Counter I and 5 per hour for Counter II. (a) What are the average waiting times per customer of

each counter? (b) If each counter could handle all withdrawals irrespective of their value, how would the average waiting time change?

20. An oil company is constructing a service station on a highway. Traffic analysis indicates that customer's arrivals over most of the day would approximate a Poisson distribution with a mean of 30 automobiles per hour. Previous studies show that one pump could service a mean of 10 automobiles per hour, with the service time distribution approximating the negative exponential if 4 pumps are installed: (a) What is the probability that an arrival would have to wait in line? (b) Find out the average waiting time, average time spent in the system and the average number of automobiles in the system. (c) For what percentage of time would a pump be idle on an average?
21. A post office has 3 windows providing the same services. It receives on an average 30 customer/hr. Arrivals are Poisson distributed and service time exponentially distributed. The post office serves on average 12 customer/hr. (a) What is the probability that a customer will be served immediately? (b) What is the probability that a customer will have to wait? (c) What is the average number of customers in the system? (d) What is the average total time that a customer must spend in the post office?
22. Let there be an automobile inspection situation with there inspection stalls. Assume that cars wait in such a way that when a stall becomes vacant, the car at the head of the line pulls up to it. The station can accommodate at most four cars waiting (seven in the station) at one time. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with mean 6 minutes. Find the average number of customers in the system during peak hours, the average waiting time and the average number per hour that cannot enter the station because of full capacity.
23. A barbershop has two barbers and three chairs for waiting customers. Assume that customers arrive in a Poisson fashion at a rate of 5 per hour and that each barber services customers according to an exponential distribution with mean 15 minutes. Further, if a customer arrives and there are no empty chairs in the shops he will leave. Find the steady state probabilities. What is the probability that the shop is empty? What is the expected number of customers in the shop?
24. In a heavy machine shop, the overhead crane is 75 per cent utilized. Time study observations gave the average slinging time as 10.5 minutes with a standard deviation of 8.8 minutes. What is the average calling rate for the services of the crane, and what is the average delay in getting service? If the average service time is cut to 8.0 minutes, with standard deviation of 6.0 minutes, how much reduction will occur, on average, in the delay of getting served?

25. In a railway marshalling yard, goods trains arrive at a rate of 3.0 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average minutes. Calculate the following: (i) the mean queue size (line length). (ii) the probability that the queue size exceeds 10.
26. The mean arrival rate to a service center is 3 per hour. The mean service time is found to be 10 minutes per service. Assuming Poisson arrival and exponential service time, find (a) Utilization factor for this service facility, (b) Probability of two units in the system, (c) Expected number of units in the queue, (d) Expected time in minutes that a customer has to spend in the system.
27. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be exponential with mean 4 minutes. (a) Find the average number of persons waiting in the system; (b) what is the probability that the person arriving at the booth will have to wait in the queue? (c) What is the probability that it will take him more than 10 minutes altogether to wait for the phone will be in use? (d) Estimate the fraction of the day when the phone will be in use (e) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for at least 3 minutes for phone. By how much the flow of arrivals should increase in order to justify a second booth? (f) What is the average length of the queue that forms from time to time?
28. An office assistant who earns Rs. 5 per hour operates a duplicating m/c maintained for office use. The time to complete each job varies according to an exponential distribution with mean 6 min. Assume Poisson input with an average arrival rate of 5 jobs per hour. If an 8-h day is used as a base, find (a) the average time a job in the system, (b) the average earning per day of the assistant, (c) the percentage idle time of the m/c.
29. There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. (a) What fraction of the time all the typists will be busy? (b) What is the average number of letters waiting to be typed? (c) What is the average time a letter has to spend for waiting and for being typed?
30. Given an average arrival rate of 20 per hour, is if better for a customer to get service at a single channel with mean service rate of 22 customers per hour or at one of two channels in parallel with mean service rate of 11 customers per hour for each of two channels.
31. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. (a) Find the effective arrival rate at the clinic. (b) What is the

probability that an arriving patient will not wait? (c) What is the expected waiting time until a patient is discharged from the clinic?

32. A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with m = 8 cars per day per bay. Find the average number of cars in the service station; the average number of cars waiting for service and the average time a car spends in the system.
33. A service station expects a customer every 4 min on an average. Service takes 3 min. Assume Poisson input and exponential service. (a) What is the average number of customers waiting for service? (b) How long can customer expect to wait for service? (c) What is the probability that a customer will spend less than 15 minutes waiting and getting service? (d) What is the probability that a customer will spend longer than 10 minutes waiting for and getting service?
34. Patients arrive at a hospital for emergency service at the rate of one every hour. Currently only one emergency can be handled at a time. Patients spend an average of 20 min for receiving emergency service. How much the average time to wait and receive the service less than 25 min?
35. An insurance company has 3 claim registers in its branch office. People with claims against the company are found to arrive in a Poisson fashion at an average rate of 20/8-h day. The amount of time that an adjuster spends with a claimant is found to have exponential distribution with mean service time 40 min. Claimants are processed in the order of their appearance. (a) How many hours a week can an adjuster expect to spend with claimants? (b) How much time, on the average, does a claimant spend in the branch office?
36. A petrol pump station has 4 pumps. The service times follow the exponential distribution with mean of 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour. (a) What is the probability that an arrival would have to wait in line? (b) Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system. (c) For what percentage of time would a pump be idle on an average?

<u>Model –I</u> (M/M/1:∞/FIF0)

1) $P_0 = 1 - \rho = 1 - \dfrac{\lambda}{\mu}$
2) $P_n = p(\ 'n'\ \text{ customers in the system}) = \rho^n (1-\rho) = \rho^n P_0$

   P_0 = p(No customers in the system)
3) $E(n) = \dfrac{\rho}{1-\rho} = \dfrac{\lambda}{\mu - \lambda}$
4) $E(m) = \dfrac{\rho^2}{1-\rho} = \dfrac{\lambda^2}{\mu(\mu - \lambda)}$
5) $E(W) = \dfrac{1}{\lambda} E(m)$
6) $E(v) = \dfrac{E(n)}{\lambda}$
7) If W is a RV which denotes the total time that a unit has to spend in the system in the system then pdf of W is $f(w) = \begin{cases} (\mu - \lambda)\, e^{-(\mu - \lambda)} & w > 0 \\ 0 & \text{elsewhere} \end{cases}$

<u>Model –II</u> (M/M/1:N/FIF0)

1) $P_0 = \begin{cases} \dfrac{1-\rho}{1-\rho^{N+1}} & \rho \neq 1 \\ \dfrac{1}{N+1} & \rho = 1 \end{cases}$

Steady State Probabilities are $P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}} = P_0 \rho^{\text{n}} & \rho \neq 1 \\ \dfrac{1}{N+1} & \rho = 1 \end{cases}$

Here $P_n = \rho_0 \rho^{\text{n}}$ $\quad$ $\text{P}_1 = P_0 \rho$, $\text{P}_2 = P_1 \rho$, $P_3 = P_2 \rho$ and so on…

2) $E(n) = \sum_{n=0}^{N} n P_n$
3) $E(m) = \sum_{n=1}^{N} (n-1) P_n$

Find $\lambda' = \lambda\,(1 - P_N)$ or $\mu\,(1 - \text{P}_0)$

4) $E(v) = \dfrac{E(n)}{\lambda'}$ And $E(w) = \dfrac{E(m)}{\lambda'}$

Model –III (M/M/C:∞/FIF0)

1) $P_n = \begin{cases} \frac{1}{n!}\rho^n \rho_0 & 1 \le n < C \\ \frac{1}{C^{n-c}C!}\rho^n \rho_0 & n \ge C \end{cases}$

$$Where \quad P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu - \lambda} \right]^{-1}$$

2) $P(n \ge c) = \text{P(that an arrival has to wait)} = \frac{P_0}{C!}\left(\frac{\lambda}{\mu}\right)^C \left(\frac{C\mu}{C\mu - \lambda}\right)$

3) $E(m) = \frac{1}{(C-1)!}\frac{\lambda\mu}{(C\mu - \lambda)^2}\left(\frac{\lambda}{\mu}\right)^C P_0$

4) $E(n) = E(m) + \frac{\lambda}{\mu}$

5) $E(w) = \frac{1}{\lambda} E(m)$

6) $E(v) = \frac{E(n)}{\lambda}$

Model –IV (M/M/C:N/FIF0)

1) $P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & 0 \le n < c \\ \frac{1}{C^{n-C}C!}\left(\frac{\lambda}{\mu}\right)^n P_0 & C \le n \le N \end{cases}$

2) $$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=c}^{N} \frac{1}{C^{n-c}C!}\left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

3) $E(m) = \sum_{n=c}^{N} (n-c)P_n$

4) $E(n) = \sum_{n=0}^{N} nP_n$ *where* $P_0, P_1, P_2, \ldots .P_N$ are steady state Probabilities.

Find $\lambda' = \lambda(1 - P_N)$ or $\mu(1 - P_0)$ and then find $E(v) = \frac{E(n)}{\lambda'}$ and $E(w) = \frac{E(m)}{\lambda'}$