

RANDOM VARIABLES 1-D PS—4

- 1) If the probability distribution of X is given as:
Find $P[1/2 < X < 7/2 / X > 1]$

x:	1	2	3	4
P(x):	0.4	0.3	0.2	0.1.

- 2) Verify whether $f(x) = |x|$ in $-1 \leq x \leq 1$
0 else can be the pdf a RV.

- 3) If $f(x) = kx^2$, $0 < x < 3$ is to be the density function, find the value of k .

- 4) Find the value of k , if $f(x) = kxe^{-x}$ $x > 0$
0 else is the pdf of a RV X .

- 5) If the pdf of a RV X is $f(x) = k(x/2)$ in $0 < X < 2$, find $p(X > 1.5 / X > 1)$.

- 6) A discrete RV X has the following probability distribution.

X	0	1	2	3	4	5	6	7	8
p(x)	a	3a	5a	7a	9a	1a	13a	15a	17a

Find $p(x < 3)$, $p(X < 7 / X > 3)$

- 7) The probability function of a RV X is defined as,

X	0	1	2	3	4	5	6	7
p(x)	0	a	2a	2a	3a	a^2	$2a^2$	$7a^2 + a$

Find $p(X \leq 2)$ $p(X > 2 / X \leq 5)$ $p(X < \frac{7}{2} / X > 0)$

- 8) If the probability mass function of a RV X is given by $p(X=r) = Kr^3$; $r = 1, 2, 3, 4$, find $P(1/2 < X < 5/2 / X > 1)$. Also find mean and variance of X

- 9) The diameter of an electric cable X is a RV with pdf $f(x) = kx(1-x)$, $0 \leq x \leq 1$. Find (i) the value of a such that $P(X < a) = 2P(X > a)$ and

- (ii) $P(X \leq \frac{1}{2} / \frac{1}{3} < X < \frac{2}{3})$.

- 10) A RV that can assume values between $x=2$ and $x=5$ has a density function given by $f(x) = 2(1+x)/27$. Find $P(3 < X < 4)$.

11) An experiment consists of four tosses of a coin. Denoting the outcomes HHTH, THTT... and assuming that all 16 outcomes are equally likely, find the probability distribution for the total number of heads.

12) Check whether the following can define probability distribution, and explain your answers.

a) $f(x) = x/15$ for $x=0,1,2,3,4,5$; b) $f(x)=(5-x^2)/6$ for $x=0,1,2,3$; c) $f(x)= 1/4$ for $x=3,4,5,6$

d) $f(x)=(x+1)/25$ for $x = 1,2,3,4,5$

13) Given that $f(x) = K/2^x$ is a probability distribution for a RV that can take on the values $x=0, 1, 2, 3, 4$ find K.

14) A manufacturer of digital phones has the following probability distribution for the number of defects per phone

x	:	0	1	2	3
f(x)	:	0.89	0.07	0.03	0.01

a) Determine the probability of 2 or more defects. b) Is a randomly selected phone more likely to have 0 defects or 1 or more defects?

15) If the probability density of a random variable is given by $f(x) = \begin{cases} Kx & 0 < x < 1 \\ 0 & \text{else} \end{cases}$. Find the probability that the RV takes on a value a) b/w $1/4$ & $3/4$. b) Greater than $2/3$.

16) If the probability density of a RV is given by $f(x) = x$ $0 < x < 1$, $2-x$ $1 \leq x < 2$, 0 elsewhere. Find the probability that a RV having this probability density will take on a value a) b/W 0.2 & 0.8 ; b) b/w 0.6 & 1.2 .

17) Given that probability density $f(x) = K/(1+x^2)$ for $-\infty < x < \infty$, find K.

18) Let the phrase error in a tracking device have probability density $f(x) = \cos x$ $0 < x < \pi/2$, = 0 elsewhere. Find the probability that the phrase error is a) b/w 0 and $\pi/4$. b) Greater than $\pi/3$.

19) The mileage (in thousands of miles) that car owners get with a certain kind of tire is a random variable having the probability density, $f(x) = \{(1/20) e^{-x/20}$ for $x > 0$, 0 $x \leq 0$. find the probability that one of these tires will that a) at most 10,000 miles; b) Anywhere from 16,000 to 24,000 miles; c) at least 30,000 miles.

20) In a certain city, the daily consumption of electric power (in million kilowatt) is random variables having the probability density, $f(x) = (1/9) x e^{-x/3}$ for $x > 0$, and 0 for $x \leq 0$. If the city's power plant has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

21) If the probability density of a RV I given by $f(x) = \{K (1-x^2) \text{ for } 0 < x < 1, 0 \text{ elsewhere.}$ Find the values of K and the probabilities that a RV having the probability density will take on a value a) b/w 0.1 & 0.2; b) greater than 0.5.

22) In certain experiments, the error made in determining the density of a silicon compound is a RV having the probability density $f(x) = \{25 \text{ for } -0.02 < x < 0.02, 0 \text{ elsewhere.}$ Find the probability that such an error will be a) b/w -0.03 & 0.04; b) b/w -0.005 & 0.005.

23).A random variable X has the following probability distribution.

X:	-2	-1	0	1	2	3
P (x):	0.1	k	0.2	2k	0.3	3k.

Evaluate $P(X < 2)$ and $(-2 < X < 2)$, also evaluate the mean and variance of X.

24).The Probability function of an infinite discrete distribution is given by $P(X = j) = (1/2)^j$ ($j = 1, 2, \dots, \infty$). Find the mean and variance of the distribution. Find also $P(X \geq 5)$ and $P(X \text{ is divisible by } 3)$.

25).A Random variable X has the following probability distribution.

X :	0	1	2	3	4	5	6	7
P(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find mean and variance of X. Also find $E(2x - 9)$ $E(x^2 + x - 10)$ $V(2x + 5)$

26) If $p(x) = \begin{cases} x e^{-x^2/2} & x \geq 0 \\ 0 & x < 0 \end{cases}$ Show that p (x) is a pdf of a RV X) and find mean and variance of X.

27). A RV X has a pdf $f(x) = Kx^2 e^{-x}$; $x \geq 0$. Find mean and Variance.

28).The RVX has the following probability distribution:

X:	-2	-1	0	1
P (x):	0.4	k	0.2	0.3

Find the mean value of X.

29). In a distribution, the probability density is given by $f(x) = kx(2-x)$, $0 < x < 2$. Find mean, variance

30). X is a RV with pdf given by $f(x) = kx$, in $0 \leq x \leq 2$; $= 2k$, in $2 \leq x \leq 4$, and $= 6k - kx$, in $4 \leq x \leq 6$. Find the value mean and variance of X

31). Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1 that there will be 0, 1, 2, or 3 power failures in a certain city during the month of July. Find μ and σ^2 .

32). The following table gives the probabilities that a certain computer will malfunction 0, 1, 2, 3, 4, 5 or 6 times on any one-day: Find μ and σ^2

Number of malfunctions: x	0	1	2	3	4	5	6
Probability: f(x)	0.17	0.29	0.27	0.16	0.07	0.03	0.01

33). Find the mean and the variance of the uniform probability distribution given by

$$f(x) = \frac{1}{n} \quad \text{for } x = 1, 2, 3, \dots, n$$

34). Find μ and σ^2 for the probability density of $f(x) = \begin{cases} kx^3 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$

35). Find μ and σ^2 for the probability density of $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x < 2 \\ 0 & \text{elsewhere} \end{cases}$

36). Given the following table:

X	-3	-2	-1	0	1	2	3
P(x)	0.05	0.10	0.30	0	0.30	0.15	0.10

Compute (i) $E(X)$, (ii) $E(2X \pm 3)$, (iii) $E(4X + 5)$, (iv) $E(X^2)$ (v) $V(X)$, and (vi) $V(2X \pm 3)$.

37). Let X be a random variable with p.d.f. as given below:

Find expected value of $Y = (X-1)^2$

X	0	1	2	3
P(x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{24}$	$\frac{1}{8}$