# Data Foundations: Numpy

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#### Outline

#### Numpy

Everything is a Vector!

Data Objects and Attribute Types

#### Numpy

- Numpy array creation
- Array access and operations
- Basic linear algebra

#### Numpy

- · Stands for Numerical Python
- Is the fundamental package required for high performance computing and data analysis
- It provides
  - · ndarray for creating multiple dimensional arrays
  - Standard math functions for fast operations on entire arrays of data without having to write loops
  - Tools for reading/writing array data
  - · Linear algebra tools
  - · etc.

#### ndarray vs list of lists

- Say you have grades of three exams (2 midterms and 1 final) in a class of 10 students.
  - grades = [[79, 95, 60],
    - [95, 60, 61], [99, 67, 84],
    - [76, 76, 97], [91, 84, 98],
    - [70, 69, 96],
    - [88, 65, 76],
    - [67, 73, 80],
    - [82, 89, 61],
    - [94, 67, 88]]

How to get final exam grade of student 0?

- grades[0][2]
- How to get grades of student 2?
  - grades[2]
- How to get grades of all students in midterm 1?
- · How to get midterm grades of the first three students (or all female students, or those who failed final)?
- How to get mean grade of each exam?
- How to get (weighted) average exam grade for each student?

#### ndarray vs list of lists

• gArray = array(examGrades)

```
In [3]: gArray
Out[3]:
array([[79, 95, 60],
[95, 60, 61],
[99, 67, 84],
...,
[67, 73, 80],
[82, 89, 61],
[94, 67, 88]])
```

```
In [5]: gArray[0,2]
Out[5]: 60

In [7]: gArray[2,:]
Out[7]: array([99, 67, 84])

In [8]: gArray[:, 0]
Out[8]: array([79, 95, 99, 76, 91, 70, 88, 67, 82, 94])

In [9]: gArray[:3, :2]
Out[9]:
```

array([[79, 95],

[95, 60], [99, 67]])

#### ndarray

- ndarray is used for storage of homogeneous data
  - i.e., all elements must be the same type
- Every array must have a shape
- And a dtype
- Supports convenient slicing, indexing and efficient vectorized computation
  - Avoid for loops, and much more efficient

In [15]: type(gArray)
Out[15]: numpy.ndarray

In [16]: gArray.ndim
Out[16]: 2

In [17]: gArray.shape
Out[17]: (10, 3)

In [18]: gArray.dtype
Out[18]: dtype('int32')

#### **Arrays**

```
>>> import numpy as np
>>> a = np.array([1, 2, 3]) # Create a rank 1 array
>>> print(type(a)) # Prints "<class 'numpy.ndarray'>"
>>> print(a.shape) # Prints "(3,)"
>>> print(a[0], a[1], a[2]) # Prints "1 2 3"
>>> a[0] = 5 \# Change an element of the array
>>> print(a) # Prints "[5, 2, 3]"
>>> b = np.array([[1,2,3],[4,5,6]]) # Create a rank 2 array
>>> print(b.shape) # Prints "(2, 3)"
>>> print(b[0, 0], b[0, 1], b[1, 0]) # Prints "1 2 4"
```

#### Exercise 2

Given the provided numpy array, write code that prints the following:

• Print the third row of the array

• Print the second column of the array

 Print the fourth row's third column (this should print a single number).

#### Creating ndarrays

```
In [65]: np.array([[0,1,2],[2,3,4]])
                                                             In [69]: np.eye(3)

    np.array

                     Out[65]:
                                                             Out[69]:
                                                             array([[ 1., 0., 0.],
                     array([[0, 1, 2],
                                                             [0., 1., 0.],

    np.zeros

                     [2, 3, 4]])
                                                             [0., 0., 1.]])
                     In [66]: np.zeros((2,3))
np.ones
                                                             In [70]: np.arange(0, 10, 2)
                     Out[66]:
                                                             Out[70]: array([0, 2, 4, 6, 8])
                     array([[ 0., 0., 0.],

    np.eye

                     [0., 0., 0.]
                                                      In [295]: np.random.randint(0, 10, (3,3))

    np.arrange

                                                      Out[295]:
                     In [67]: np.ones((2,3))
                                                      array([[8, 7, 6],
                     Out[67]:

    np.random

                                                      [0, 8, 9],
                     array([[ 1., 1., 1.],
                                                      [9, 0, 4]])
                     [1., 1., 1.]])
```

```
Numpy data types
```

```
    int8, int16, int32, int64
```

- float16, float32, float64, float128
- bool
- 64 hits object
- String
- Unicode
- gArray.astvpe

```
In [34]: gArray.astype(float64)
Out[34]:
array([[ 79., 95., 60.],
[ 95., 60., 61.],
[82., 89., 61.],
[ 94., 67., 88.]])
```

In [79]: num string = array(['1.0', '2.05', '3'])

```
In [81]: num string
Out[81]:
array(['1.0', '2.05', '3'],
```

dtype='<U4')

In [82]: num string.astype(float) Out[82]: array([ 1., 2.05, 3.])

#### Array operations

- Between arrays and scalars
- Between equal-sized arrays: elementwise operation

```
In [94]: arr * arr
Out[94]:
array([[ 0, 1, 4],
        [ 9, 16, 25]])

In [95]: arr / (arr+1)
Out[95]:
array([[ 0. , 0.5 , 0.66666667],
        [ 0.75 , 0.8 , 0.83333333]])
```

```
In [87]: arr = array([[0,1,2],[3,4,5]])
In [88]: arr * 2
Out[88]:
array([[ 0, 2, 4],
[6, 8, 10]])
In [90]: arr ** 2
Out[90]:
array([[ 0, 1, 4],
[9, 16, 25]])
In [91]: 2 ** arr
Out[91]:
array([[ 1, 2, 4],
[8, 16, 32]], dtype=int32)
```

#### Sum, Mean, Max

```
>>> print(b.max()) # Prints 6
>>> print(b.mean()) # Prints 3.5
>>> print(b.mean(axis=0)) # Calculates column means, Prints [2.5,
3.5, 4.5]
>>> print(b.mean(axis=1)) # Calculates row means, Prints [2., 5.]
```

>>> print(b.max(axis=1) # Calculates row max, Prints [3, 6]

>>> b = np.array([[1,2,3],[4,5,6]]) # Create a rank 2 array

#### Exercise 3

Given the provided array in the Lab file, write code the prints the following:

- The average grade per test (column) (this should print an array with 3 items)
- The average grade per row, i.e., the average test grade per student
- Print the max grade per test
- Print the average across all students (rows) and tests (columns)

## Speed difference between for loop and vectorized computation

```
In [118]: a = np.random.rand(1000000,1)
...: %timeit a**2
...: %timeit [a[i]**2 for i in range(1000000)]
```

100 loops, best of 3: 4.02 ms per loop

1 loop, best of 3: 1.25 s per loop

Vectorization is more than 300 times faster!

```
In [151]: timeit map(lambda x: x**2, a) 1000000 loops, best of 3: 270 ns per loop
```

map appears to be very fast, but it is just because it is lazy – actual calculation has not been done yet.

```
def mySum(inputList):
    s = 0
    for i in range(len(inputList)):
        s += inputList[i]
    return s
```

```
In [148]: timeit mySum(a)
1 loop, best of 3: 605 ms per loop
```

```
In [149]: timeit np.sum(a)
1000 loops, best of 3: 1.15 ms per loop
```

```
In [150]: timeit reduce(lambda x, y: x+y, a)
```

```
1 loop, best of 3: 791 ms per loop
```

Vectorization is 500 times faster than for loop. Reduce is even slower than for loop here.

#### Array indexing and slicing

• Somewhat similar to python list, but much more flexible

```
In [157]: gArray[:, 2]
Out[157]: array([60, 61, 84, 97, 98, 96,
76, 80, 61, 88])
```

#### Array indexing and slicing (cont'd)

```
In [177]: gArray[[0, 2], [0, 1, 2]]
Traceback (most recent call last):
...
IndexError: shape mismatch: ...
In [178]: gArray[[0, 2], [0, 2]]
Out[178]: array([79, 84])
```

#### Array slices are views

Use .copy() to make a copy of an array explicitly.

#### Boolean indexing

```
# select record for those who had
\# \le 70 in final
In [265]: gArray[gArray[:,
21<70,:1
Out[265]:
array([[95, 60, 61],
       [82, 89, 61]])
# anything < 70 is changed to 70
In [2671: gArrav[gArrav < 701 = 70]
In [268]: gArray
Out[268]:
array([[100, 100, 100],
       [ 95, 70, 70],
       [ 99, 70, 84],
       . . . ,
       [70, 73, 80],
       [ 82, 89, 70],
       [ 94, 70, 88]])
```

#### Exercise 4

Write code that prints the grades of all students that have an average grade less than 90.

#### Reshaping and transposing

```
axis 0 1 2 0,0 0,1 0,2 axis 0 1 1,0 1,1 1,2 2 2 2,0 2,1 2,2
```

Fast element-wise functions

#### Table 4-3. Unary ufuncs

ast element-wise functions	Function	Description
	abs, fabs	Compute the absolute value element-wise for integer, floating point, or complex values. Use fabs as a faster alternative for non-complex-valued data
	sqrt	Compute the square root of each element. Equivalent to arr ** 0.5
	square	Compute the square of each element. Equivalent to arr ** 2
	exp	Compute the exponent e <sup>x</sup> of each element
	log, log10, log2, log1p	Natural logarithm (base $e$ ), log base 10, log base 2, and log(1 + x), respectively
	sign	Compute the sign of each element: 1 (positive), 0 (zero), or -1 (negative)
	ceil	Compute the ceiling of each element, i.e. the smallest integer greater than or equal to each element
	floor	Compute the floor of each element, i.e. the largest integer less than or equal to each element
	rint	Round elements to the nearest integer, preserving the dtype
	modf	Return fractional and integral parts of array as separate array
	isnan	Return boolean array indicating whether each value is NaN (Not a Number)
	isfinite, isinf	Return boolean array indicating whether each element is finite (non-1nf, non-NaN) or infinite, respectively
	cos, cosh, sin, sinh, tan, tanh	Regular and hyperbolic trigonometric functions
	arccos, arccosh, arcsin, arcsinh, arctan, arctanh	Inverse trigonometric functions
_	logical_not	Compute truth value of not x element-wise. Equivalent to -arr.

Table 4-4. Binary universal functions

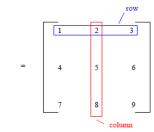
Table 4-4. Binary universal functions		
Function	Description	
add	Add corresponding elements in arrays	
subtract	Subtract elements in second array from first array	
multiply	Multiply array elements	
divide, floor_divide	Divide or floor divide (truncating the remainder)	
power	Raise elements in first array to powers indicated in second array	
maximum, fmax	Element-wise maximum. fmax ignores NaN	
minimum, fmin	Element-wise minimum. fmin ignores NaN	
mod	Element-wise modulus (remainder of division)	
copysign	Copy sign of values in second argument to values in first argument	
<pre>greater, greater_equal, less, less_equal, equal, not_equal</pre>	Perform element-wise comparison, yielding boolean array. Equivalent to infix operators >, >=, <, <=, ==, !=	
logical_and, logical_or, logical_xor	Compute element-wise truth value of logical operation. Equivalent to infix operators & $\ \ ,\ ^\wedge$	

#### https://docs.scipy.org/doc/numpy/reference/

#### Matrix

 A matrix is a rectangular array of numbers organized in rows and columns

 If a matrix A has m rows and n columns, then we say that A is an m x n matrix



#### Matrix

▶ In general, a matrix A of the order  $m \times n$  means:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

- $ightharpoonup a_{ii}$  is the element of A in row i and column j.
- **row** *i* is the elements  $a_{i1}a_{i2}\cdots a_{in}$ .
- **column** j is the elements  $a_{1j}a_{2j}\cdots a_{mj}$ .

#### **Vectors**

- ▶ If a matrix has only one row, then it is a row vector.
  - ► Example: (1 10 11 12 7)
- ▶ If a matrix has only one column, then it is a **column vector**.
  - Example:  $\begin{pmatrix} 7 \\ -2 \\ 5 \\ 11 \end{pmatrix}$

#### Identity matrix

- ► The **identity matrix** is a square matrix that has 1s on the main diagonal and 0s everywhere else.
- ▶ An identity matrix of order  $n \times n$  is denoted  $I_n$ .

$$Example: I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

#### Diagonal matrix

- ▶ A diagonal matrix is a square matrix such that every element that is not on the main diagonal is 0 (elements on the main diagonal can be 0 or non-zero).
- An  $n \times n$  diagonal matrix is denoted by  $D_n$ .

Example: 
$$D_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

► Example: 
$$D_3 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

#### Dot product

- A dot product is a multiplication of a row vector of order 1 × n with a column vector of order n × 1. The result is a scalar
- It is obtained by multiplying the ith element of the row vector with the ith element of the row vector and then summing these products.

$$A = (a_1 a_2 \cdots a_n), B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

 $A \cdot B = a_1b_1 + a_2b_2 + \cdots + a_nb_n$ 

Matrix multiplication the number of columns of A is the same as the number of rows of B). Then the matrix multiplication  $A \cdot B$  is defined. ▶ The result is an  $m \times n$  matrix (resulting matrix has the same

number of rows as A and the same number of columns as B). The (i, j)th entry of the resulting matrix is the dot product of row i of A and column j of B.

▶ Suppose A is an  $m \times p$  matrix and B is a  $p \times n$  matrix (note

 $a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} + a_{24}b_{41}$ 

is the same as the number of rows of B. Result will be a 2 x 2 matrix. Entry in position (2,1) in the resulting matrix will be the dot product of the 2nd row in A with the 1st column of B:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix}, B = \begin{pmatrix} b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{pmatrix}$$

$$Multiplication is defined because the number of columns of A$$

```
In [152]: qArray
                              79
                                   95
                                       60
                                                         76.2
Out[152]:
                              95
                                   60
                                       61
                                                         70.9
array([[79, 95, 60],
                                                 0.3
                              99
                                   60
                                       61
                                                         83.4
        [95, 60, 61],
                                            Χ
                                                 0.3
        [99, 67, 84],
                                                 0.4
                              67 73
                                       80
                                                         74.0
       [67, 73, 80],
                              82
                                   89
                                       61
                                                         75.7
       [82, 89, 61],
                              94
                                   67
                                                         83.5
                                       88
       [94, 67, 88]])
In [321]: gArray.dot([0.3, 0.3, 0.4])
```

What is being calculated here?

75.7, 83.5**1**)

Out[321]: array([ 76.2, 70.9, 83.4, 84.4, 91.7, 80.1, 76.3, 74.,

```
In [152]: qArray
                      In [329]: scaling = [1.1, 1.05, 1.03]
Out[152]:
array([[79, 95, 60],
                    In [330]: diag(scaling)
      [95, 60, 61], Out[330]:
      [99, 67, 84], array([[ 1.1 , 0. , 0. ],
                             [ 0. , 1.05, 0. ],
      . . . ,
      [67, 73, 80],
                             [0., 0., 1.03]
      [82, 89, 61],
      [94, 67, 88]])
                      In [331]: gArray.dot(diag(scaling))
                      Out[331]:
                      array([[ 86.9 , 99.75, 61.8],
                        [ 104.5 , 63. , 62.83],
 What are we doing here?
                             [ 108.9 , 70.35, 86.52],
                             [ 73.7 , 76.65, 82.4 ],
                             [ 90.2 , 93.45, 62.83],
                             [ 103.4 , 70.35, 90.64]])
```

```
In [152]: gArray
                        In [337]: qArray.max(axis=0)
Out[1521:
                        Out[337]: array([99, 95, 98])
array([[79, 95, 60],
     [95, 60, 61],
     [99, 67, 84],
                        In [338]: maxInExam = qArray.max(axis=0)
     [67, 73, 80],
                        In [339]:
     [82, 89, 61],
     [94, 67, 88]])
                        gArray.dot(diag(100/maxInExam)).round()
                        Out[3391:
                        array([[ 80., 100., 61.],
                                [ 96., 63., 62.],
```

What are we doing here?

```
[ 68., 77., 82.],
[ 83., 94., 62.],
[ 95., 71., 90.]])
```

[ 100., 71., 86.],

### Speed difference between for loop and matrix

```
multiplication
```

```
In [361]: maxInCol = a.max(axis=0)
In [362]: maxInCol.shape
Out[362]: (100,)
```

```
In [355]: a = rand(10000, 100)
```

```
In [363]: timeit [[a[j,i]*100/maxInCol[i]
for i in range(100)]
for j in range(10000)]
1 loop, best of 3: 673 ms per loop
```

```
In [356]: timeit a.dot(100/a.max(0))
100 loops, best of 3: 1.77 ms per loop
```

```
In [357]: timeit [a[:,i]*100/max(a[:,i]) for i in range(100)] 10 loops, best of 3: 72.9 ms per loop
```

```
In [358]: timeit [[a[j,i]*100/max(a[:,i]) for i in range(100)] for j in range(10000)]
```

Ctrl-C

Table 4-5. Basic array statistical methods

	Method	Description		
	sum	Sum of all the elements in the	array or along an axis. Zero-length arrays have sum 0.	
	mean	Arithmetic mean. Zero-length arrays have NaN mean.		
	std, var	Standard deviation and variance ment (default denominator n).	ce, respectively, with optional degrees of freedom adjust-	
	min, max	Minimum and maximum.		
	argmin, argmax	Indices of minimum and maxin	num elements, respectively.	
	cumsum	Cumulative sum of elements st	tarting from 0	
	cumprod	Cumulative product of element	ts starting from 1	
In [394	!]: a=randint(0, 5	, size=(3,3))	In [397]: a.sum(axis=0)	
In [395	i]: a		Out[397]: array([7, 4, 7])	
Out[395	5]:			
array(	[[4, 1, 2],		In [398]: a.sum(1)	
	[2, 2, 1],		Out[398]: array([7, 5, 6])	
	[1, 1, 4]])			
In [396 Out[396	5]: a.sum() 5]: 18		<pre>In [405]: (a &gt; 2).any(1) Out[405]: array([ True, False, Tru dtype=bool)</pre>	

#### numpy.sort()

#### Exercise 5

Write code to adds 10 points every student's test grade **if** their average test grade is greater than 90.

#### Exercise 6

Write a function that takes two 1-dimensional arrays as input and returns the euclidean distance between the two arrays.

Euclidean distance is defined as

EDist = 
$$\sqrt{(x_0 - v_0)^2 + (x_1 - v_1)^2 + \dots + (x_{D-1} - v_{D-1})^2}$$

The square root of a number in numpy can be calculated as np.sqrt(x), where x is a number or array.

Try to complete this exercise with for loops and with vector notation.

#### Numpy

Everything is a Vector!

Data Objects and Attribute Types

# **Data Objects and Attribute Types**

A data object represents an entity. Examples include...

- Customers
- Students/Professors/Courses
- Tweets
- Images
- Genes, Drugs, Procedures

A attribute is a data field (synonyms dimension/feature/variable)

## **Types of Attributes**

https://www.youtube.com/watch?v=N9fDIAf1CMY

- Nominal hair\_color (black, blonde, red, green?), martial\_status (single, married, divorced, widowed,...), occupation (teacher, dentist, programmer,...)
- Binary (Boolean) smoker (yes or no), medical tests (positive or negative)
- Ordinal drink\_size (short, tall, grande, venti), grade (A, B, C, D, F), professional\_rank (assistant, associate, full)
  - Numeric temperature  $(70^{\circ}F)$ , speed (400 mph)

## Nominals: How should we represent a nominal?

#### As a number?

- As integers? e.g., hair\_color (black = 0, blonde = 1, red = 2)
  - Not the best choice because there is no inherent order to the categories?
- As a vector?

e.g., hair\_color (black 
$$= [1,0,0]$$
, blonde  $= [0,1,0]$ , red  $= [0,0,1]$ 

► This is referred to as one-hot encoding.

```
>>> import numpy as np

>>> vec = np.zeros((3,)) # Assume 4 colors: Blue, Red, Green

>>> classes = ["blue", "red", "green"]

>>> vec[classes.index("green")] = 1

>>> vec
```

For multiple **objects** (examples), the vectors above can be generated for each, appended to a list, then cast to a numpy array.

```
>>> from sklearn.feature_extraction import DictVectorizer
>>> datasetDicts = [{'age':1, 'hair_color':'blue'},
                 {'age':2, 'hair_color':'green'}]
>>> vec = DictVectorizer(sparse=False) # In general it is better to
use sparse=True
>>> dataset = vec.fit_transform(datasetDicts) Takes a list of dicts
>>> dataset # dataset is a numpy array with shape (2, 3)
array([[1., 1., 0.],
     [2., 0., 1.]]
>>> vec.feature_names_
['age', 'hair_color=blue', 'hair_color=green']
```

How do we vectorize new data? Use the .transform() method without  $\operatorname{fit}_{\scriptscriptstyle{-}}$ 

DictVectorizer handles nominal, binary, and numeric features

```
>>> data = [{'age':3, 'hair_color':'purple', 'is_smoker':0}]
```

Store **nominals** as a string, **binary** variables as a 0 (absent) or 1 (present), and **numeric** features as an integer/float.

# What about multiple attributes?

What if someone has multiple hair colors?



# What about multiple attributes? Method 1

What if someone has multiple hair colors?

The person has red, green AND blue hair.

## What about multiple attributes? Method 2

What if someone has multiple hair colors?

>>> from sklearn.preprocessing import MultiLabelBinarizer

```
>>> mlb = MultiLabelBinarizer()
```

MultiLabelBinarizer takes a list of sets as input to the fit transform method.

#### How can we combine multiple sets of attributes?

Assume two sets of attributes in the form of 2 matrices:

```
>>> atts1
array([[0., 1., 1.],
      [1., 0., 0.]
>>> atts2
array([[1., 1., 0.],
      [2., 0., 1.]])
>>> combine_atts = np.hstack([atts1, atts2]) # Horizontal stack
>>> combine_atts
array([[0., 1., 1., 1., 1., 0.],
      [1., 0., 0., 2., 0., 1.]])
```

# Loading Data from a CSV: All Numeric data

```
myData.csv
label, feature 1, feature 2
positive, 0.1,1
positive, 1,7
positive, 3,4
example.py
import csv
import numpy as np
X = [] # Will be a list of lists
y = [] # will be a list
with open('myData.csv') as inFile:
       iCSV = csv.reader(inFile, delimiter=',')
       next(iCSV)
      for row in iCSV:
             X.append([float(x) for x in row[1:]]) # get features
             y.append(row[0]) # get class
X = np.array(X) \# convert to numpy array for scikit-learn
y = np.array(y) # convert to numpy array for scikit-learn
```

#### Exercise 1

Write code to load the data in the "iris.csv" into Numpy arrays.

The first 4 columns are the features/attributes. The last column is the class. Simply load the class as a list of strings. Don't forget to convert the dataset into a numpy array. You can use either DictVectorizer or the CSV method on the previous slide to load the features.

# The End

The End