Lektion 5

Träge Operatoren

$S := Sum\left(\frac{1}{j^2}, j = 1 ..infinity\right)$

$$\sum_{j=1}^{\infty} \frac{1}{j^2} \tag{1.1}$$

$S = value(S)$

$$\sum_{j=1}^{\infty} \frac{1}{j^2} = \frac{1}{6}\pi^2 \tag{1.2}$$

$is(\mathbf{(1.2)})$

$$\mathit{false} \tag{1.3}$$

$S := Sum\left(\frac{1}{j^2 \cdot \log(j+1)}, j = 1 ..infinity\right)$

$$\sum_{j=1}^{\infty} \frac{1}{j^2 \ln(j+1)} \tag{1.4}$$

$S = value(S)$

$$\sum_{j=1}^{\infty} \frac{1}{j^2 \ln(j+1)} = \sum_{j=1}^{\infty} \frac{1}{j^2 \ln(j+1)} \tag{1.5}$$

$I1 := Int(\exp(-x) \cdot x^t, x = 0 ..infinity)$

$$\int_0^{\infty} e^{-x} x^t \, dx \tag{1.6}$$

$I1 = value(I1)$

$$\int_0^{\infty} e^{-x} x^t \, dx = \Gamma(t+1) \tag{1.7}$$

$f := \sin(x^2)$

$$\sin(x^2) \tag{1.8}$$

$Df := Diff(f, x)$

$$\frac{d}{dx} \sin(x^2) \tag{1.9}$$

$Df = value(Df)$

$$\frac{d}{dx} \sin(x^2) = 2\cos(x^2)\, x \tag{1.10}$$

for n **from** 0 **to** 5 **do**
  $Df := Diff(f, [x\$n]);$

print(*Df* = *value*(*Df*))
end do :

$$Diff(\sin(x^2), [\ ]) = \sin(x^2)$$

$$\frac{d}{dx} \sin(x^2) = 2\cos(x^2)\, x$$

$$\frac{d^2}{dx^2} \sin(x^2) = -4\sin(x^2)\, x^2 + 2\cos(x^2)$$

$$\frac{d^3}{dx^3} \sin(x^2) = -8\cos(x^2)\, x^3 - 12\sin(x^2)\, x$$

$$\frac{d^4}{dx^4} \sin(x^2) = 16\sin(x^2)\, x^4 - 48\cos(x^2)\, x^2 - 12\sin(x^2)$$

$$\frac{d^5}{dx^5} \sin(x^2) = 32\cos(x^2)\, x^5 + 160\sin(x^2)\, x^3 - 120\cos(x^2)\, x \qquad \textbf{(1.11)}$$

Zusammenfassen

$a := (x^4 - 3\cdot x^2 + 5)\cdot \exp(x^2)\cdot \sin(x)$

$$(x^4 - 3x^2 + 5)\, e^{x^2} \sin(x) \qquad \textbf{(2.1)}$$

$da := diff(a, x, x, x)$

$$24\, x\, e^{x^2} \sin(x) + 6\,(12\, x^2 - 6)\, x\, e^{x^2} \sin(x) + 3\,(12\, x^2 - 6)\, e^{x^2} \cos(x) + 3\,(4\, x^3 - 6\, x)\, e^{x^2} \sin(x) + 12\,(4\, x^3 - 6\, x)\, x^2\, e^{x^2} \sin(x) + 12\,(4\, x^3 - 6\, x)\, x\, e^{x^2} \cos(x) + 6\,(x^4 - 3\, x^2 + 5)\, x\, e^{x^2} \sin(x) + 5\,(x^4 - 3\, x^2 + 5)\, e^{x^2} \cos(x) + 8\,(x^4 - 3\, x^2 + 5)\, x^3\, e^{x^2} \sin(x) + 12\,(x^4 - 3\, x^2 + 5)\, x^2\, e^{x^2} \cos(x) \qquad \textbf{(2.2)}$$

expand(*da*)

$$34\, x^3\, e^{x^2} \sin(x) + 9\, e^{x^2} \cos(x)\, x^2 + 7\, e^{x^2} \cos(x) + 30\, x^5\, e^{x^2} \sin(x) + 17\, x^4\, e^{x^2} \cos(x) + 8\, x^7\, e^{x^2} \sin(x) + 12\, e^{x^2} \cos(x)\, x^6 \qquad \textbf{(2.3)}$$

collect(*expand*(*da*), $\exp(x^2)$)

$$(8\sin(x)\, x^7 + 12\cos(x)\, x^6 + 30\sin(x)\, x^5 + 17\cos(x)\, x^4 + 34\sin(x)\, x^3 + 9\cos(x)\, x^2 + 7\cos(x))\, e^{x^2} \qquad \textbf{(2.4)}$$

collect(*expand*(*da*), [$\exp(x^2)$, $\sin(x)$, $\cos(x)$])

$$((8\, x^7 + 30\, x^5 + 34\, x^3)\sin(x) + (12\, x^6 + 17\, x^4 + 9\, x^2 + 7)\cos(x))\, e^{x^2} \qquad \textbf{(2.5)}$$

collect(*expand*(*da*), [$\cos(x)$, $\sin(x)$, $\exp(x^2)$])

$$(12\, x^6 + 17\, x^4 + 9\, x^2 + 7)\, e^{x^2} \cos(x) + (8\, x^7 + 30\, x^5 + 34\, x^3)\, e^{x^2} \sin(x) \qquad \textbf{(2.6)}$$

Trigonometrische Funktionen

$a := \cos(x+y)$

$$\cos(x+y) \tag{3.1}$$

$expand(a)$

$$\cos(x)\cos(y) - \sin(x)\sin(y) \tag{3.2}$$

$A := \sin(x)\cdot\cos(y)$

$$\sin(x)\cos(y) \tag{3.3}$$

$combine(A)$

$$\frac{1}{2}\sin(x+y) + \frac{1}{2}\sin(x-y) \tag{3.4}$$

$expand(\mathbf{(3.4)})$

$$\sin(x)\cos(y) \tag{3.5}$$

$C := \sin(x) + \sin(y)$

$$\sin(x) + \sin(y) \tag{3.6}$$

$trigsubs(C)$

$$\left[2\sin\left(\frac{1}{2}x + \frac{1}{2}y\right)\cos\left(\frac{1}{2}x - \frac{1}{2}y\right)\right] \tag{3.7}$$

$C = trigsubs(C)[1]$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{1}{2}x + \frac{1}{2}y\right)\cos\left(\frac{1}{2}x - \frac{1}{2}y\right) \tag{3.8}$$

$is(\mathbf{(3.8)})$

$$\mathit{true} \tag{3.9}$$

$L := trigsubs(a)$

$$\left[\cos(-x-y),\ \cos\left(\frac{1}{2}x + \frac{1}{2}y\right)^2 - \sin\left(\frac{1}{2}x + \frac{1}{2}y\right)^2,\ \frac{1}{\sec(x+y)},\ \frac{1-\tan\left(\frac{1}{2}x + \frac{1}{2}y\right)^2}{1+\tan\left(\frac{1}{2}x + \frac{1}{2}y\right)^2},\ \frac{1}{2}e^{I(x+y)} + \frac{1}{2}e^{-I(x+y)},\ \cos(x)\cos(y) - \sin(x)\sin(y)\right] \tag{3.10}$$

$nops(L)$

$$6 \tag{3.11}$$

$a = L[6]$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \tag{3.12}$$

$D := irgendwas$

Error, attempting to assign to `D` which is protected. Try declaring `local D`; see ?protect for details.

$F := \tan(x)^2 + 1$

$$\tan(x)^2 + 1 \qquad (3.13)$$

$simplify(F)$

$$\frac{1}{\cos(x)^2} \qquad (3.14)$$

$convert(F, sincos)$

$$\frac{\sin(x)^2}{\cos(x)^2} + 1 \qquad (3.15)$$

$G := \tan(3 \cdot x)$

$$\tan(3\,x) \qquad (3.16)$$

$expand(G)$

$$\frac{3\tan(x) - \tan(x)^3}{1 - 3\tan(x)^2} \qquad (3.17)$$

$H := \tan(x) + \cot(y)$

$$\tan(x) + \cot(y) \qquad (3.18)$$

$trigsubs(H)$

Error, (in trigsubs) sum not found in table

$H1 := convert(H, sincos)$

$$\frac{\sin(x)}{\cos(x)} + \frac{\cos(y)}{\sin(y)} \qquad (3.19)$$

$H2 := normal(H1)$

$$\frac{\sin(x)\sin(y) + \cos(x)\cos(y)}{\cos(x)\sin(y)} \qquad (3.20)$$

$H3 := combine(H2)$

$$\frac{2\cos(x-y)}{\sin(x+y) - \sin(x-y)} \qquad (3.21)$$

$H3 := map(combine, H2)$

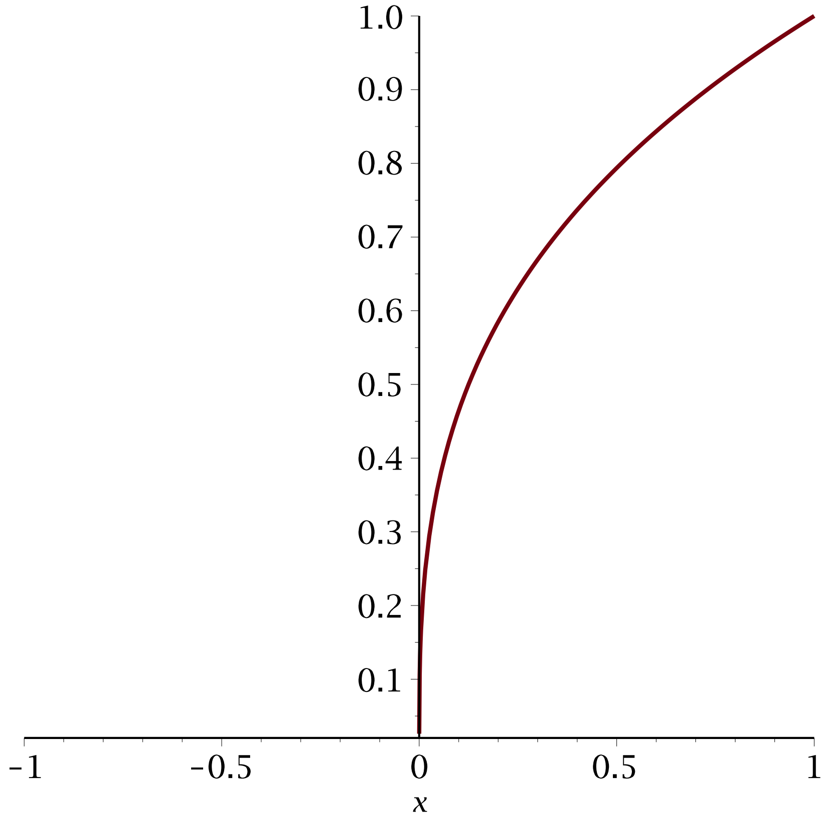
$$\frac{\cos(x-y)}{\cos(x)\sin(y)} \qquad (3.22)$$

Potenzfunktionen

$f := x^{\frac{1}{3}}$

$$x^{1/3} \qquad (4.1)$$

$plot(f, x = -1..1, thickness = 2)$



$eval(f, x = -1)$

$$(-1)^{1/3} \quad \textbf{(4.2)}$$

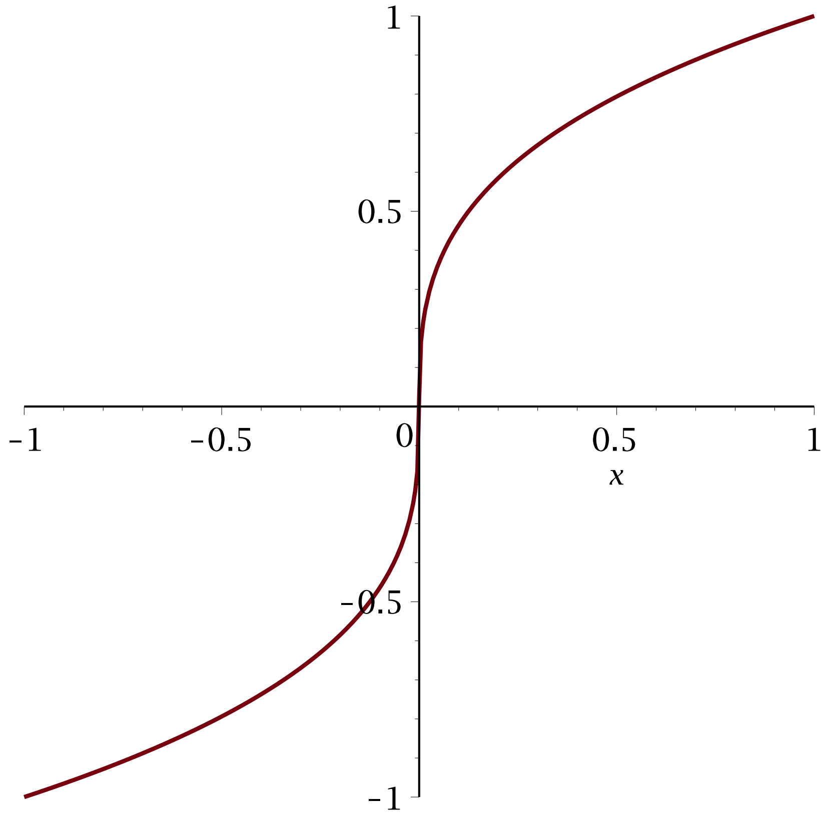
$evalc(eval(f, x = -1))$

$$\frac{1}{2} + \frac{1}{2}\,\mathrm{I}\sqrt{3} \quad \textbf{(4.3)}$$

$g := \mathrm{surd}(x, 3)$

$$\sqrt[3]{x} \quad \textbf{(4.4)}$$

$plot(g, x = -1..1, thickness = 2)$



Integration

restart

$A := Int(x^n, x)$

$$\int x^n \, dx \tag{5.1}$$

$A = value(A)$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \tag{5.2}$$

$A1 := Int\left(\frac{1}{x}, x\right)$

$$\int \frac{1}{x} \, dx \tag{5.3}$$

$A1 = value(A1)$

$$\int \frac{1}{x}\,dx = \ln(x) \tag{5.4}$$

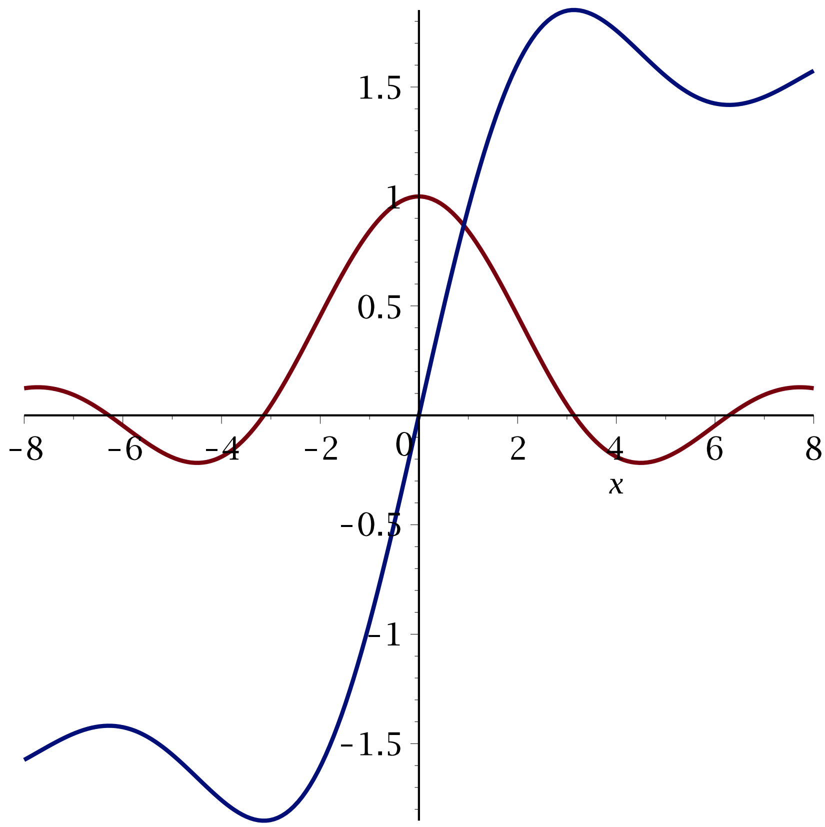
$C := Int\left(\frac{\sin(x)}{x}, x\right)$

$$\int \frac{\sin(x)}{x}\,dx \tag{5.5}$$

$C = value(C)$

$$\int \frac{\sin(x)}{x}\,dx = \mathrm{Si}(x) \tag{5.6}$$

$plot\left(\left[\frac{\sin(x)}{x}, \mathrm{Si}(x)\right], x = -8..8, thickness = 2\right)$



$\mathrm{Si}(6.) - \mathrm{Si}(0)$

$$1.424687551 \tag{5.7}$$

$evalf\left(Int\left(\frac{\sin(x)}{x}, x = 0..6\right)\right)$

$$1.424687551 \tag{5.8}$$

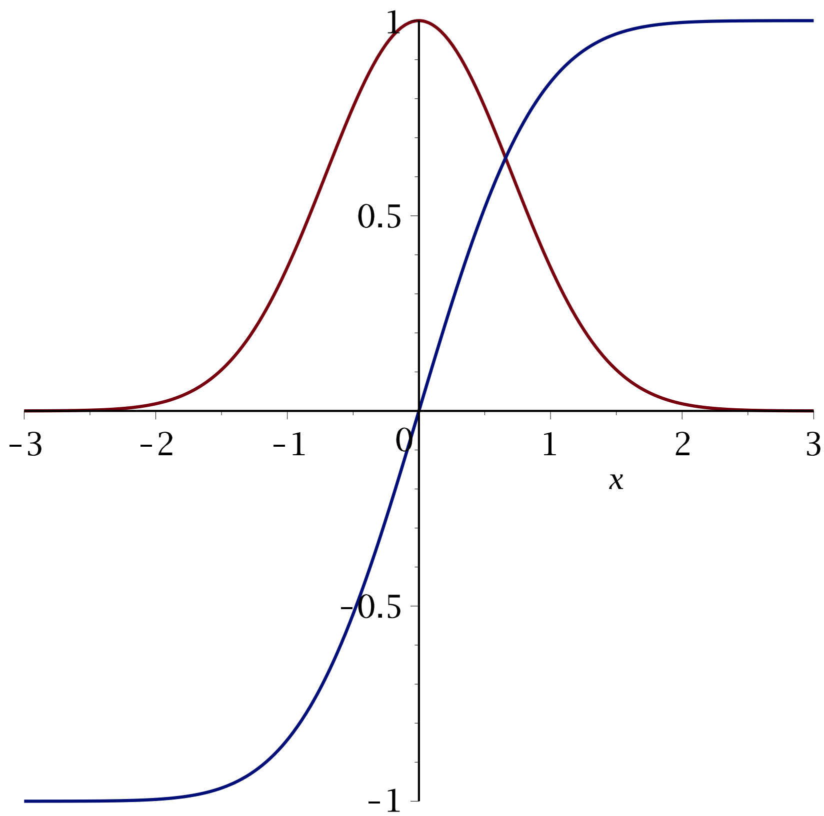
$E := Int(\exp(-x^2), x)$

$$\int e^{-x^2}\,dx \qquad (5.9)$$

$E = value(E)$

$$\int e^{-x^2}\,dx = \frac{1}{2}\sqrt{\pi}\,\mathrm{erf}(x) \qquad (5.10)$$

$plot([\exp(-x^2), \mathrm{erf}(x)], x = -3..3)$



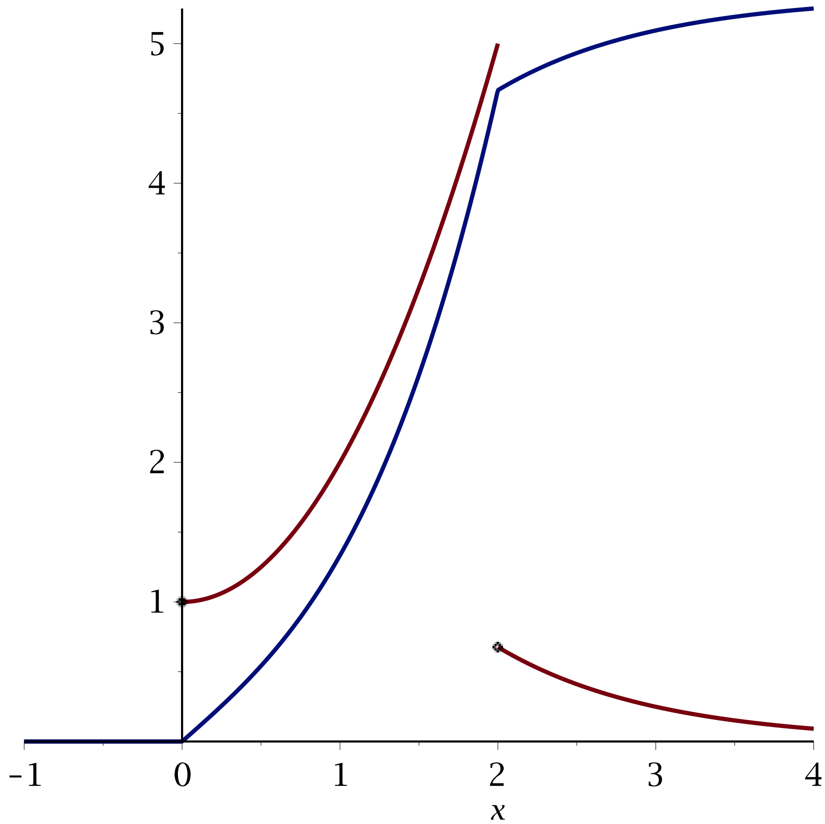
$f := piecewise(x \geq 0 \textbf{ and } x < 2, 1 + x^2, x \geq 2, 5 \cdot \exp(-x))$

$$\begin{cases} x^2 + 1 & 0 \leq x \textbf{ and } x < 2 \\ 5\,e^{-x} & 2 \leq x \end{cases} \qquad (5.11)$$

$F := Int(f, x) :$
$F = value(F)$

$$\int \begin{cases} x^2+1 & 0 \le x \text{ and } x < 2 \\ 5\,e^{-x} & 2 \le x \end{cases} dx = \begin{cases} 0 & x \le 0 \\ \frac{1}{3}x^3 + x & x \le 2 \\ -5\,e^{-x} + \frac{14}{3} + 5\,e^{-2} & 2 < x \end{cases} \quad (5.12)$$

plot([*f*, *value*(*F*)], $x = -1..4$, *thickness* = 2, *discont* = *true*)



Gesteuerte Integration

with(*IntegrationTools*) :

$$b := \frac{\sin(\ln(x))}{x}$$

$$\frac{\sin(\ln(x))}{x} \quad (6.1)$$

$B := Int(b, x)$

$$\int \frac{\sin(\ln(x))}{x}\, dx \qquad (6.2)$$

$ers1 := y = \ln(x)$

$$y = \ln(x) \qquad (6.3)$$

$B1 := Change(B, ers1)$

$$\int \sin(y)\, dy \qquad (6.4)$$

$B2 := value(B1)$

$$-\cos(y) \qquad (6.5)$$

$B3 := eval(B2, ers1)$

$$-\cos(\ln(x)) \qquad (6.6)$$

$diff(B3, x) = b$

$$\frac{\sin(\ln(x))}{x} = \frac{\sin(\ln(x))}{x} \qquad (6.7)$$

$is(\mathbf{(6.7)})$

$$true \qquad (6.8)$$

$c := \frac{1}{(1-x^2)^{\frac{3}{2}}}$

$$\frac{1}{(-x^2+1)^{3/2}} \qquad (6.9)$$

$C := Int(c, x)$

$$\int \frac{1}{(-x^2+1)^{3/2}}\, dx \qquad (6.10)$$

$ers1 := x = \sin(u)$

$$x = \sin(u) \qquad (6.11)$$

$C1 := Change(C, ers1)$

$$\int \frac{\sqrt{-\sin(u)^2+1}\,\cos(u)}{(\sin(u)^2-1)^2}\, du \qquad (6.12)$$

$simplify(C1)$

$$\int \frac{\mathrm{csgn}(\cos(u))}{\cos(u)^2}\, du \qquad (6.13)$$

$C2 := simplify(C1)$ assuming $u > -\frac{\mathrm{Pi}}{2}, u < \frac{\mathrm{Pi}}{2}$

$$\int \frac{1}{\cos(u)^2}\, du \qquad (6.14)$$

$C3 := value(C2)$

$$\frac{\sin(u)}{\cos(u)} \qquad (6.15)$$

$ers2 := u = solve(ers1, u)$

$$u = \arcsin(x) \tag{6.16}$$

$C4 := eval(C3, ers2)$

$$\frac{x}{\sqrt{-x^2+1}} \tag{6.17}$$

$diff(C4, x) = c$

$$\frac{x^2}{(-x^2+1)^{3/2}} + \frac{1}{\sqrt{-x^2+1}} = \frac{1}{(-x^2+1)^{3/2}} \tag{6.18}$$

$simplify(\%)$

$$\frac{1}{(-x^2+1)^{3/2}} = \frac{1}{(-x^2+1)^{3/2}} \tag{6.19}$$

$e := \sin(x)\cdot\exp(-x\cdot y)$

$$\sin(x)\,\mathrm{e}^{-xy} \tag{6.20}$$

$E := Int(e, x)$

$$\int \sin(x)\,\mathrm{e}^{-xy}\,\mathrm{d}x \tag{6.21}$$

$E1 := Parts(E, \sin(x))$

$$-\frac{\mathrm{e}^{-xy}\sin(x)}{y} - \left(\int \left(-\frac{\mathrm{e}^{-xy}\cos(x)}{y}\right)\mathrm{d}x\right) \tag{6.22}$$

$E2 := Parts(E1, \cos(x))$

$$-\frac{\mathrm{e}^{-xy}\sin(x)}{y} - \frac{\mathrm{e}^{-xy}\cos(x)}{y^2} + \int \left(-\frac{\mathrm{e}^{-xy}\sin(x)}{y^2}\right)\mathrm{d}x \tag{6.23}$$

$Glg1 := E = E2$

$$\int \sin(x)\,\mathrm{e}^{-xy}\,\mathrm{d}x = -\frac{\mathrm{e}^{-xy}\sin(x)}{y} - \frac{\mathrm{e}^{-xy}\cos(x)}{y^2} + \int \left(-\frac{\mathrm{e}^{-xy}\sin(x)}{y^2}\right)\mathrm{d}x \tag{6.24}$$

$solve(Glg1, E)$

$$\frac{\left(\int \left(-\frac{\mathrm{e}^{-xy}\sin(x)}{y^2}\right)\mathrm{d}x\right)y^2 - \mathrm{e}^{-xy}\sin(x)\,y - \mathrm{e}^{-xy}\cos(x)}{y^2} \tag{6.25}$$

$Glg2 := expand(Glg1)$

$$\int \frac{\sin(x)}{\mathrm{e}^{xy}}\,\mathrm{d}x = -\frac{\sin(x)}{y\,\mathrm{e}^{xy}} - \frac{\cos(x)}{y^2\,\mathrm{e}^{xy}} - \frac{\int \frac{\sin(x)}{\mathrm{e}^{xy}}\,\mathrm{d}x}{y^2} \tag{6.26}$$

$E3 := solve(Glg2, expand(E))$

$$-\frac{\sin(x)\,y + \cos(x)}{\mathrm{e}^{xy}\,(y^2+1)} \tag{6.27}$$

$diff(E3, x) = e$

$$-\frac{\cos(x)\,y-\sin(x)}{e^{xy}\,(y^2+1)}+\frac{(\sin(x)\,y+\cos(x))\,y}{e^{xy}\,(y^2+1)}=\sin(x)\,e^{-xy} \qquad \textbf{(6.28)}$$

simplify(**(6.28)**)

$$\sin(x)\,e^{-xy}=\sin(x)\,e^{-xy} \qquad \textbf{(6.29)}$$