# Lektion 13

## Die Besselsche Differentialgleichung

```
restart
with(VectorCalculus):
SetCoordinates(polar[r, phi])
```

$$\mathrm{polar}_{r,\phi} \tag{1.1}$$

```
u := v(r)·cos(n·phi)
```

$$v(r)\cos(n\phi) \tag{1.2}$$

```
EigenwertGlg := Laplacian(u) + λ^2·u
```

$$\frac{\left(\frac{d}{dr}v(r)\right)\cos(n\phi) + r\left(\frac{d^2}{dr^2}v(r)\right)\cos(n\phi) - \frac{v(r)\cos(n\phi)\,n^2}{r}}{r} + \lambda^2 v(r)\cos(n\phi) \tag{1.3}$$

```
tmp := EigenwertGlg / cos(n·phi)
```

$$\frac{1}{\cos(n\phi)}\left(\frac{\left(\frac{d}{dr}v(r)\right)\cos(n\phi) + r\left(\frac{d^2}{dr^2}v(r)\right)\cos(n\phi) - \frac{v(r)\cos(n\phi)\,n^2}{r}}{r} + \lambda^2 v(r)\cos(n\phi)\right) \tag{1.4}$$

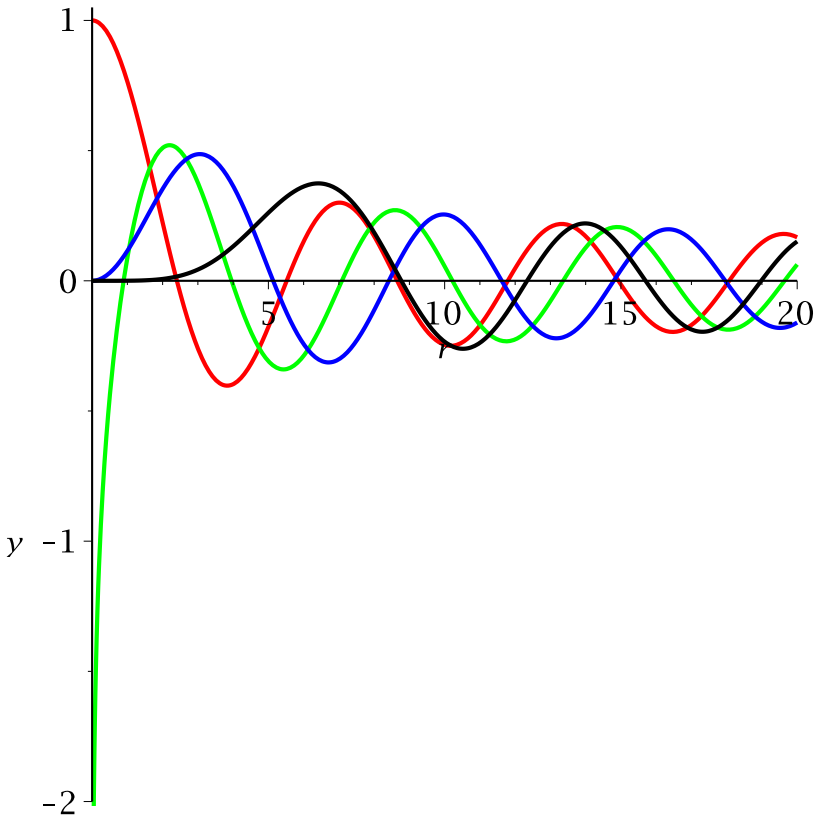
```
BesselGlg := expand(tmp) = 0
```

$$\frac{\frac{d}{dr}v(r)}{r} + \frac{d^2}{dr^2}v(r) - \frac{v(r)\,n^2}{r^2} + \lambda^2 v(r) = 0 \tag{1.5}$$

```
Lsg := dsolve(BesselGlg)
```

$$v(r) = \_C1\,\mathrm{BesselJ}(n, \lambda r) + \_C2\,\mathrm{BesselY}(n, \lambda r) \tag{1.6}$$

```
plot([BesselJ(0, r), BesselY(0, r), BesselJ(2, r), BesselJ(5, r)], r = 0..20, y = -2..1.05,
    color = [red, green, blue, black], thickness = 2)
```




$r20 := fsolve(\mathrm{BesselJ}(2, r), r, 0.1 .. 6)$

$$5.135622302 \qquad \textbf{(1.7)}$$

$r00 := fsolve(\mathrm{BesselJ}(0, r), r, 0..5)$

$$2.404825558 \qquad \textbf{(1.8)}$$

## Bewegte Bilder

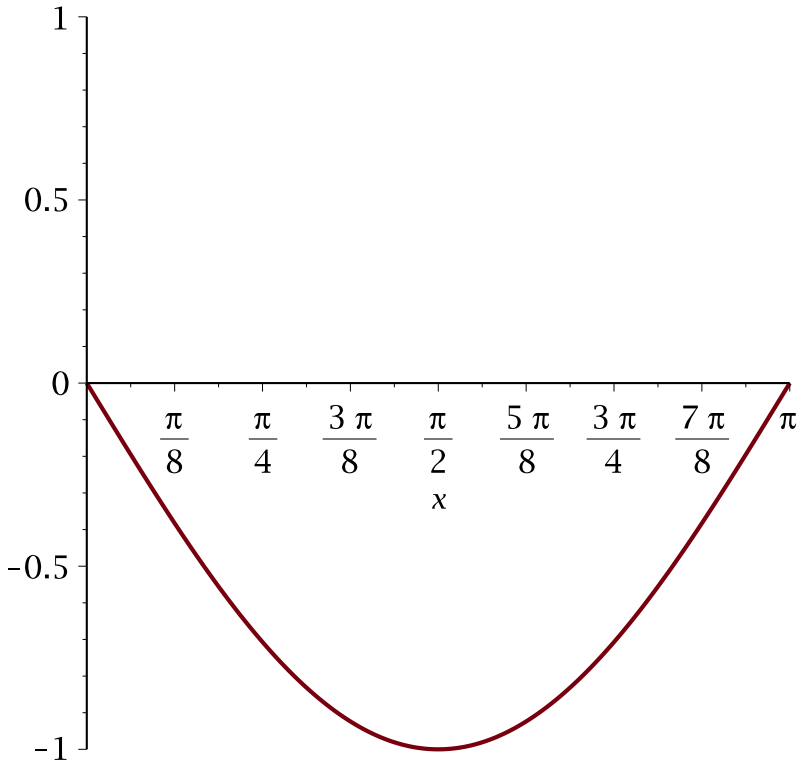
$with(plots):$

$F := \cos(t) \cdot \sin(x)$

$$\cos(t)\,\sin(x) \qquad \textbf{(2.1)}$$

$animate(plot, [F, x = 0..\mathrm{Pi}, thickness = 2], t = -\mathrm{Pi}..\mathrm{Pi}, frames = 39)$

$t = -3.1416$




$tmp := eval(v(r), Lsg)$

$$_C1\,\mathrm{BesselJ}(n, \lambda r) + _C2\,\mathrm{BesselY}(n, \lambda r) \quad \textbf{(2.2)}$$

$tmp := eval(tmp, \{_C1 = 1, _C2 = 0\})$

$$\mathrm{BesselJ}(n, \lambda r) \quad \textbf{(2.3)}$$

$f := eval(tmp, \{n = 0,\ \mathrm{lambda} = r00\})$

$$\mathrm{BesselJ}(0, 2.404825558\, r) \quad \textbf{(2.4)}$$

$x := r \cdot \cos(\mathrm{phi})$

$$r\cos(\phi) \quad \textbf{(2.5)}$$

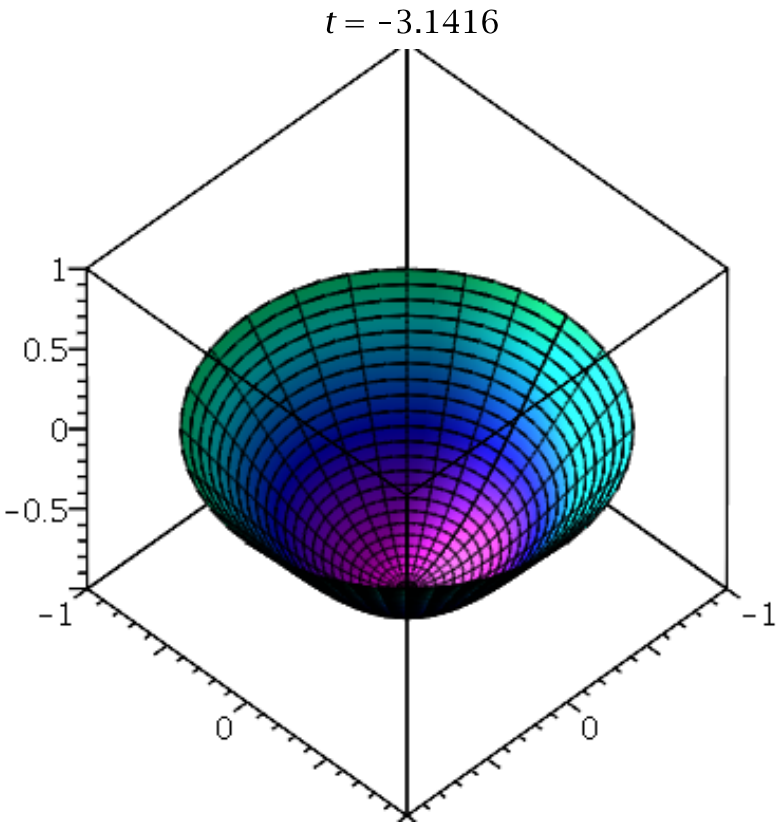
$y := r \cdot \sin(\mathrm{phi})$

$$r\sin(\phi) \quad \textbf{(2.6)}$$

$F := f \cdot \cos(t)$

$$\mathrm{BesselJ}(0, 2.404825558\, r)\cos(t) \quad \textbf{(2.7)}$$

$animate(plot3d, [[x, y, F], r = 0..1, \mathrm{phi} = -\mathrm{Pi}..\mathrm{Pi}], t = -\mathrm{Pi}..\mathrm{Pi}, shading = zhue, scaling = constrained)$




$f2 := eval(tmp, \{n = 2, \text{lambda} = r20\}) \cdot \cos(2 \cdot \text{phi})$

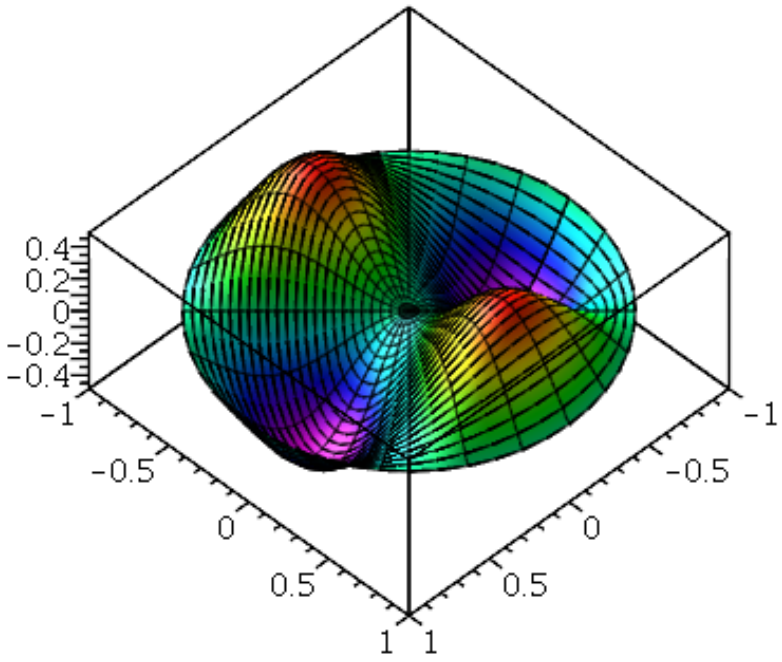
$$\text{BesselJ}(2, 5.135622302\, r)\cos(2\,\phi) \qquad \textbf{(2.8)}$$

$F2 := f2 \cdot \cos(t)$

$$\text{BesselJ}(2, 5.135622302\, r)\cos(2\,\phi)\cos(t) \qquad \textbf{(2.9)}$$

$animate(plot3d, [[x, y, F2], r = 0..1, \text{phi} = -\text{Pi}..\text{Pi}], t = -\text{Pi}..\text{Pi}, shading = zhue, scaling = constrained)$

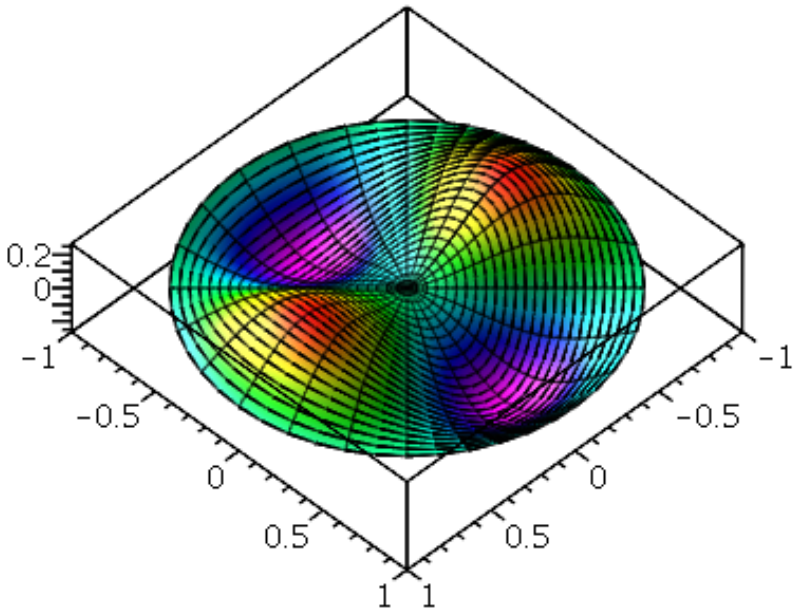
$t = -3.1416$




## Ausdrucke

*pl1* := *plot3d*([*x*, *y*, *eval*(*F2*, *t* = 1)], *r* = 0..1, phi = -Pi..Pi, *shading* = *zhue*, *scaling* = *constrained*) :

*pl1*




*plotsetup*(*jpeg*, *plotoutput* = "besselplot.jpg")
*pl1*
*plotsetup*(*window*)
*pl1*
*plotsetup*(*inline*)

## Prozeduren und Verzweigungen

*restart*
*Myfactorial* := **proc**(*n*);
  **if** $n \le 1$ **then return** 1;
  **else return** $n \cdot$ *Myfactorial*$(n-1)$;
  **end if**;
 **end proc**
**proc**(*n*) **(4.1)**
    **if** *n* <= 1 **then return** 1 **else return** *n** *Myfactorial*(*n* + ( − 1)) **end if**

```
end proc
```

*Myfactorial*(5)

120 **(4.2)**

*Myfactorial*(70) - factorial(70)

0 **(4.3)**

```
My2 := proc(n)
  options remember;
  if n ≤ 1 then return 1;
  else return n·My2(n−1);
  end if;
 end proc
```

```
proc(n)                                                    (4.4)
    option remember;
    if n <= 1 then return 1 else return n*My2(n+(−1)) end if
end proc
```

*My2*(5)

120 **(4.5)**

*showtime*( )

4 **mod** 3

1 **(4.6)**

```
Collatz := proc(n)
  options remember;
  if n = 1 then return 1;
  elif n mod 2 = 0 then return n, Collatz(n/2);
  else return n, Collatz(3·n+1);
  end if;
 end proc
```

```
proc(n)                                                    (4.7)
    option remember;
    if n = 1 then
        return 1
    elif n mod 2 = 0 then
        return n, Collatz(1*1 / 2*n)
    else
        return n, Collatz(3*n+1)
    end if
end proc
```

*Collatz*(1)

1 **(4.8)**

*Collatz*(2)

$$2, 1 \tag{4.9}$$

$Collatz(3)$

$$3, 10, 5, 16, 8, 4, 2, 1 \tag{4.10}$$

$Collatz(6)$

$$6, 3, 10, 5, 16, 8, 4, 2, 1 \tag{4.11}$$

$Collatz(7)$

$$7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1 \tag{4.12}$$

$Collatz(1609)$

$$1609, 4828, 2414, 1207, 3622, 1811, 5434, 2717, 8152, 4076, 2038, 1019, 3058, 1529, 4588, 2294, 1147, 3442, 1721, 5164, 2582, 1291, 3874, 1937, 5812, 2906, 1453, 4360, 2180, 1090, 545, 1636, 818, 409, 1228, 614, 307, 922, 461, 1384, 692, 346, 173, 520, 260, 130, 65, 196, 98, 49, 148, 74, 37, 112, 56, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1 \tag{4.13}$$

## Primzahlzwillinge

```
pzw := proc(n)
  local j;
  for j from n do
   if isprime(j) and isprime(j + 2) then return j, j + 2;
   end if;
  end do;
 end proc
```

```
proc(n)
    local j;
    for j from n do
        if isprime(j) and isprime(j + 2) then return j, j + 2 end if
    end do
end proc
```

(5.1)

$pzw(2)$

$$3, 5 \tag{5.2}$$

$pzw(100)$

$$101, 103 \tag{5.3}$$

$pzw(1000)$

$$1019, 1021 \tag{5.4}$$

$pzw(10000000)$

$$10000139, 10000141 \tag{5.5}$$

Achtung: isprime ist ein probabilistischer Test