Lektion 4

Automatisches Lösen von Gleichungen

$restart$

$Glg := (x-1)^2 = 4$

$$(x-1)^2 = 4 \tag{1.1}$$

$Lsg := solve(Glg, x)$

$$3, -1 \tag{1.2}$$

$eval(Glg, x = Lsg[1])$

$$4 = 4 \tag{1.3}$$

$is(\mathbf{(1.3)})$

$$\mathit{true} \tag{1.4}$$

$Lsg := solve(Glg, \{x\})$

$$\{x = 3\}, \{x = -1\} \tag{1.5}$$

$eval(Glg, Lsg[1])$

$$4 = 4 \tag{1.6}$$

$Gls := \{x^2 + y^2 = 1, x = y\}$

$$\{x = y, x^2 + y^2 = 1\} \tag{1.7}$$

$solve(Gls)$

$$\{x = RootOf(2\_Z^2 - 1), y = RootOf(2\_Z^2 - 1)\} \tag{1.8}$$

$Lsg := solve(Gls, \{x, y\}, Explicit)$

$$\left\{x = \frac{1}{2}\sqrt{2}, y = \frac{1}{2}\sqrt{2}\right\}, \left\{x = -\frac{1}{2}\sqrt{2}, y = -\frac{1}{2}\sqrt{2}\right\} \tag{1.9}$$

$eval(Gls, Lsg[1])$

$$\left\{1 = 1, \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}\right\} \tag{1.10}$$

for g **in (1.10) do**
  $is(g)$
end do

$$\mathit{true}$$

$$\mathit{true} \tag{1.11}$$

$map(is, \mathbf{(1.10)})$

$$\{\mathit{true}\} \tag{1.12}$$

$Glg := x^5 + x + 7$

$$x^5 + x + 7 \tag{1.13}$$

$solve(Glg, Explicit = true)$

$$RootOf(\_Z^5 + \_Z + 7, index = 1), RootOf(\_Z^5 + \_Z + 7, index = 2), RootOf(\_Z^5 + \_Z + 7, index = 3), RootOf(\_Z^5 + \_Z + 7, index = 4), RootOf(\_Z^5 + \_Z \tag{1.14}$$

$+7,\ index = 5)$

Fragen Sie Dr. Klopsch!

$Glg := \exp(x) = \frac{2}{x}$

$$e^x = \frac{2}{x} \tag{1.15}$$

$solve(Glg)$

$$\mathrm{LambertW}(2) \tag{1.16}$$

$evalf(\mathrm{LambertW}(2))$

$$0.8526055020 \tag{1.17}$$

$fsolve(Glg)$

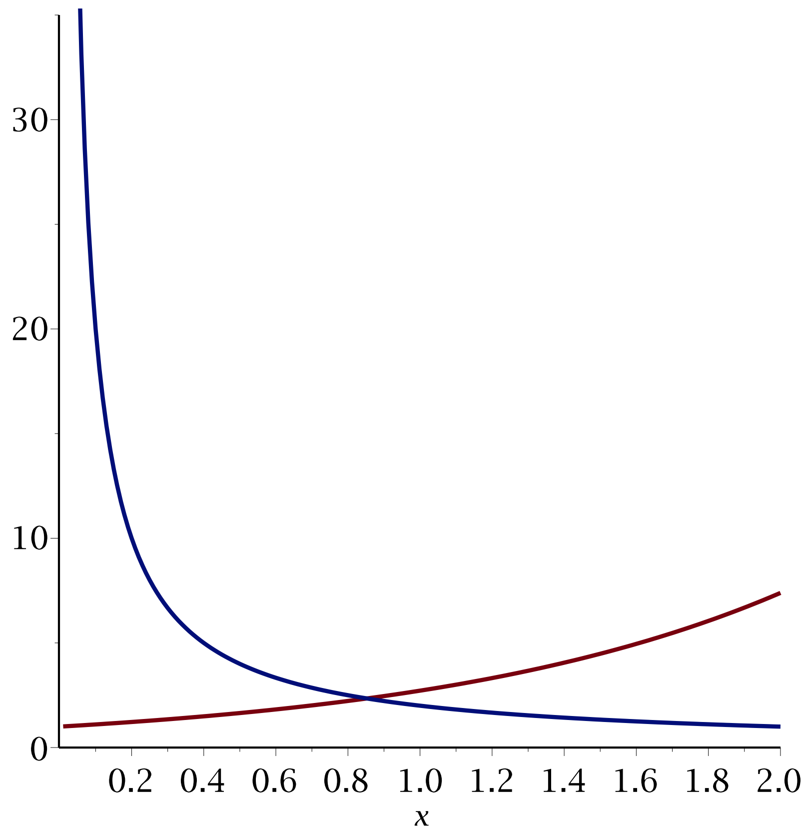
$$0.8526055020 \tag{1.18}$$

$fsolve(Glg, x, complex)$

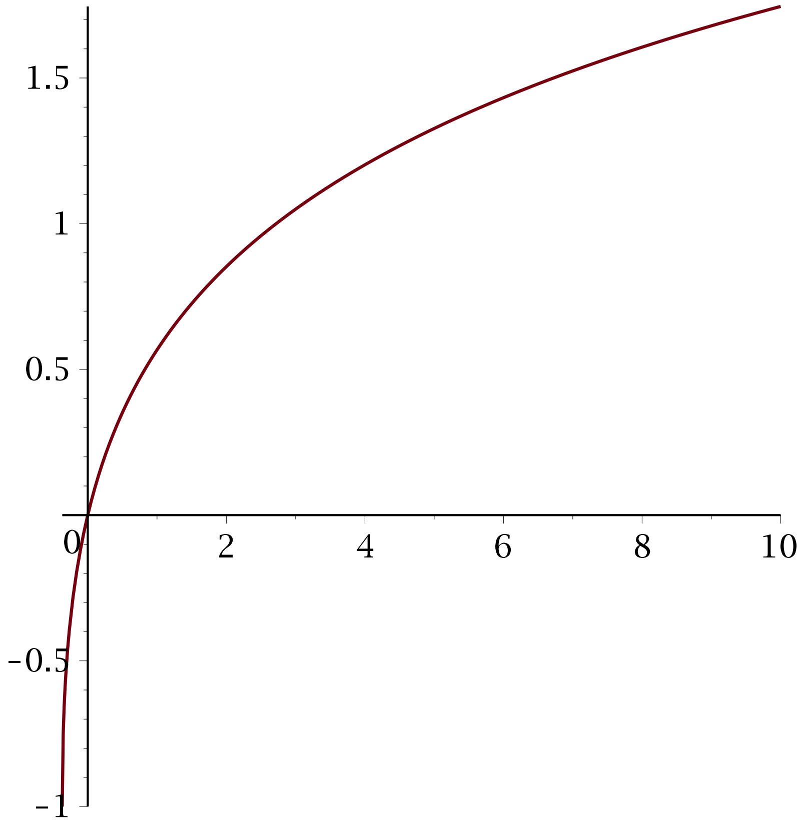
$$-0.8343103666 - 4.530265999\,\mathrm{I} \tag{1.19}$$

$solve(2 = 3)$

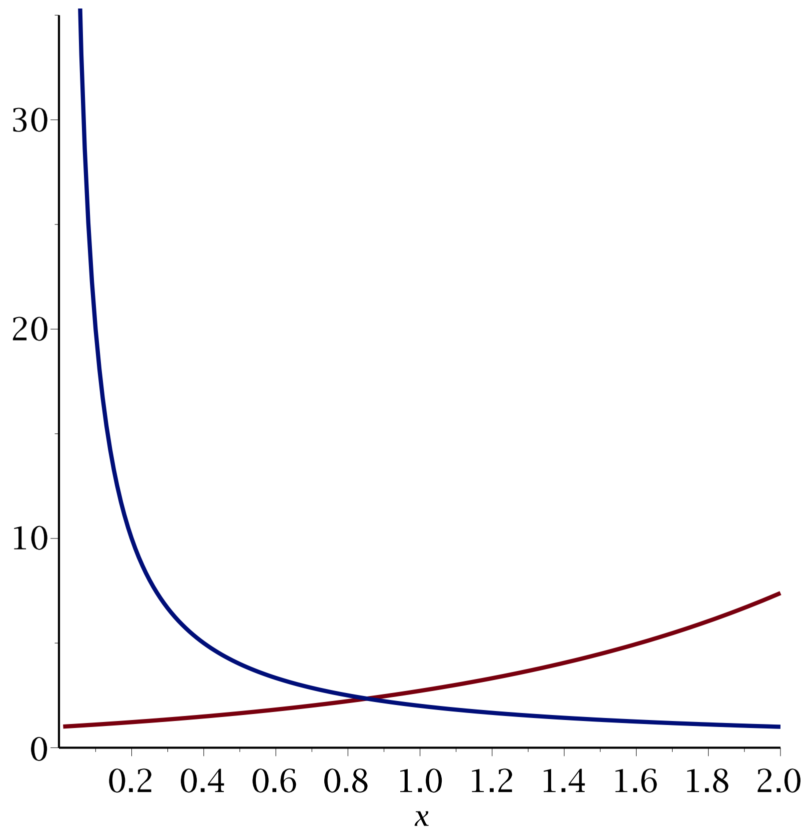
$plot([lhs(Glg), rhs(Glg)], x = 0.01 ..2, thickness = 2)$



$plot(\mathrm{LambertW}, -\exp(-1) ..10)$



$plot([lhs(Glg), rhs(Glg)], x = 0.01 ..2, thickness = 2)$



Pakete

with(*plots*)

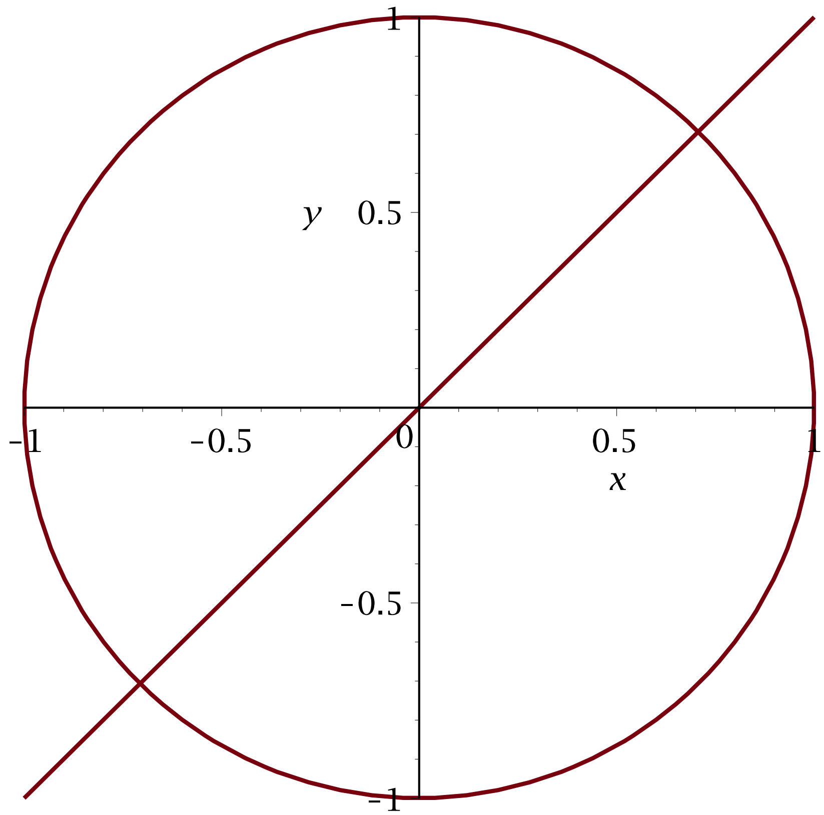
[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(2.1)**

Graphen von Lösungsmengen

Gls

$$\{x = y, x^2 + y^2 = 1\} \quad (3.1)$$

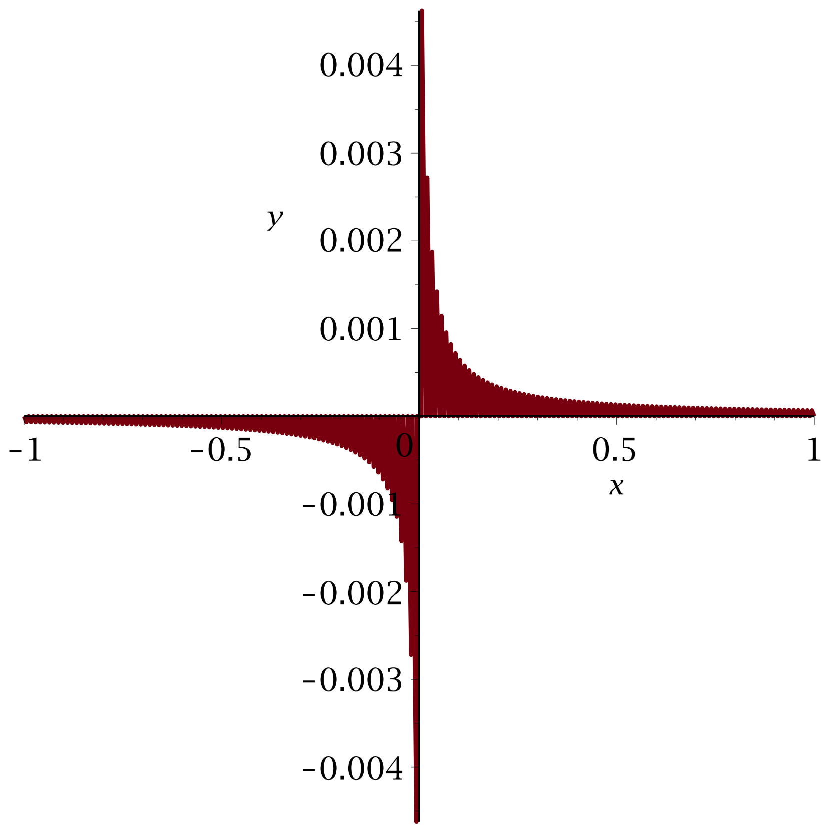
$implicitplot(Gls, x = -1 ..1, y = -1 ..1, thickness = 2)$



$Glg := x^2 \cdot y = 0$

$$x^2 y = 0 \quad (3.2)$$

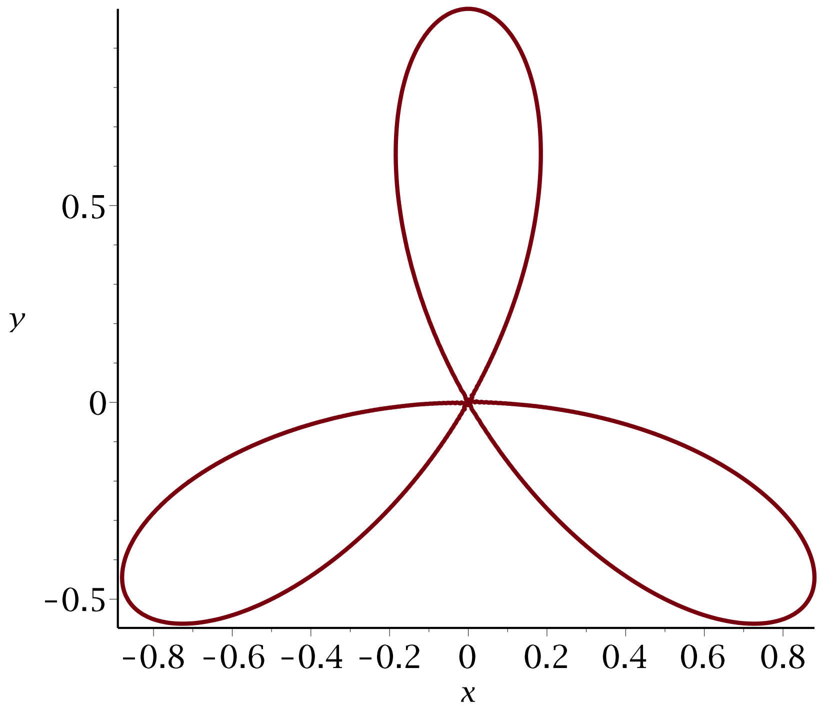
$implicitplot(Glg, x = -1 ..1, y = -1 ..1, thickness = 2, numpoints = 30000)$



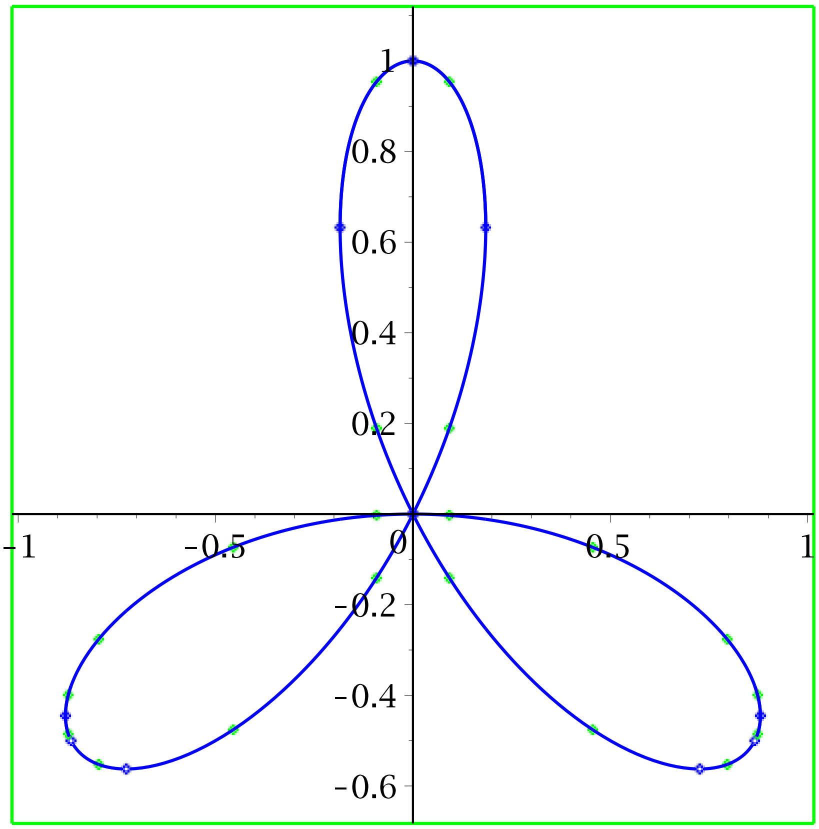
$h := (x^2 + y^2)^2 + 3 \cdot x^2 \cdot y - y^3$

$$(x^2 + y^2)^2 + 3\,x^2\,y - y^3 \tag{3.3}$$

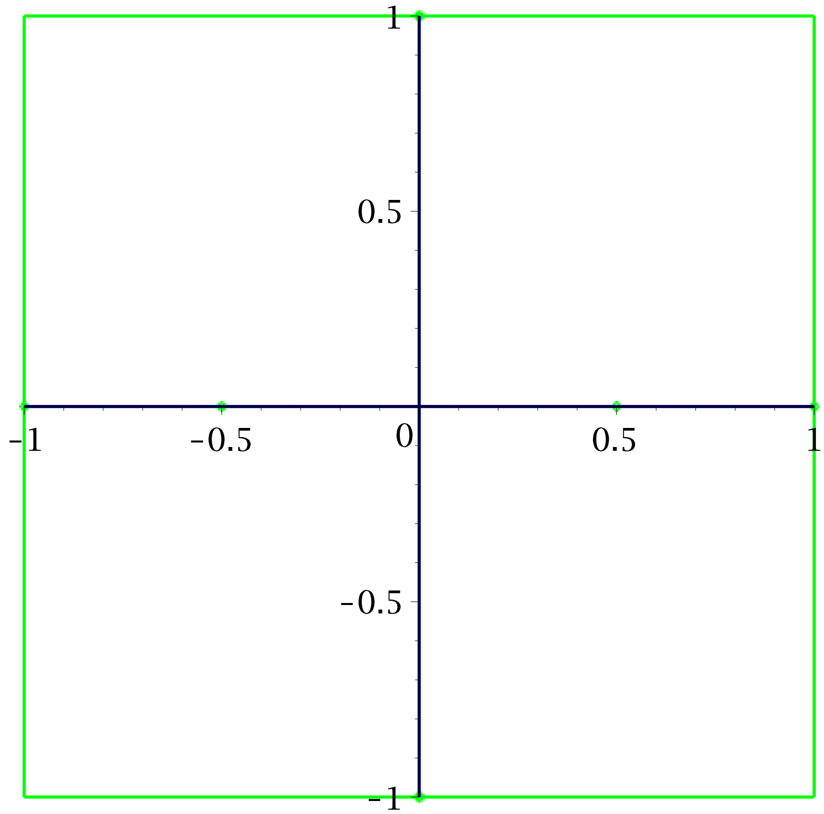
$implicitplot(h, x = -1\,..1, y = -1\,..1, scaling = constrained, axes = frame, thickness = 2, numpoints = 30000)$



$with(algcurves):$
$plot\_real\_curve(h, x, y)$



$plot\_real\_curve(x^2 \cdot y, x, y)$

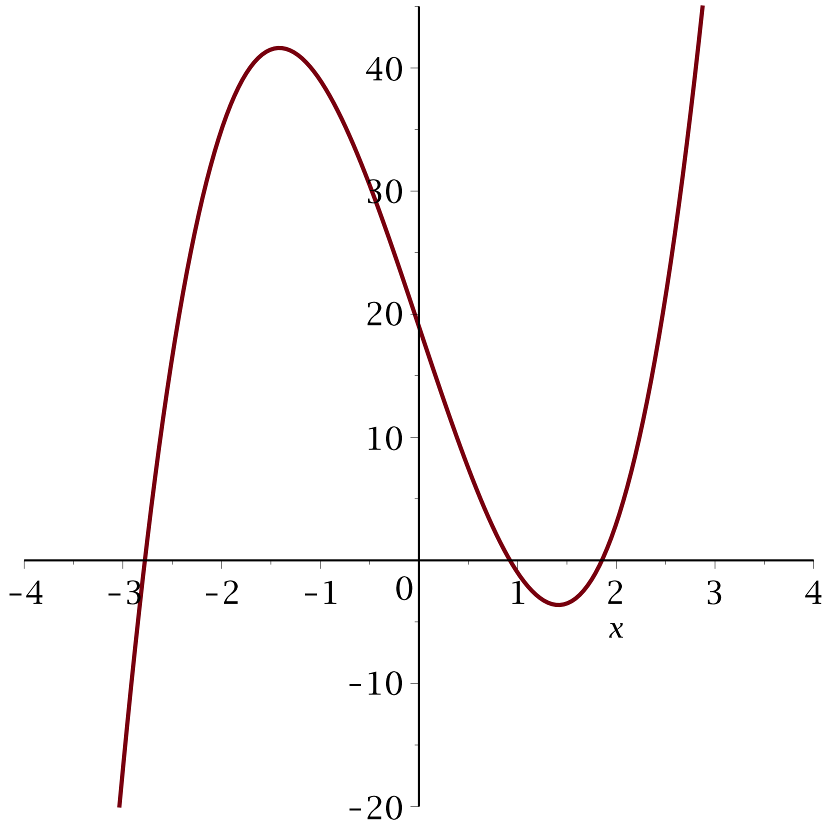


Polynomgleichungen

$f := 4 \cdot x^3 - 24 \cdot x + 19$

$$4\,x^3 - 24\,x + 19 \quad \textbf{(4.1)}$$

plot(*f*, *x* = -4 ..4, -20 ..45, *thickness* = 2)



$Lsg := solve(f = 0)$

$$\frac{1}{2}\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}+\frac{4}{\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}},\ -\frac{1}{4}\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}-\frac{2}{\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{2}\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}-\frac{4}{\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}}\right),\ -\frac{1}{4}\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}-\frac{2}{\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{2}\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}-\frac{4}{\left(-19+\mathrm{I}\sqrt{151}\right)^{1/3}}\right) \quad \textbf{(4.2)}$$

$simplify(\mathrm{Re}(Lsg[1]))$

$$2\sqrt{2}\,\sin\left(\frac{1}{3}\arctan\left(\frac{1}{19}\sqrt{151}\right)+\frac{1}{6}\pi\right) \quad \textbf{(4.3)}$$

$evalf(\textbf{(4.3)})$

$$1.854281936 \quad \textbf{(4.4)}$$

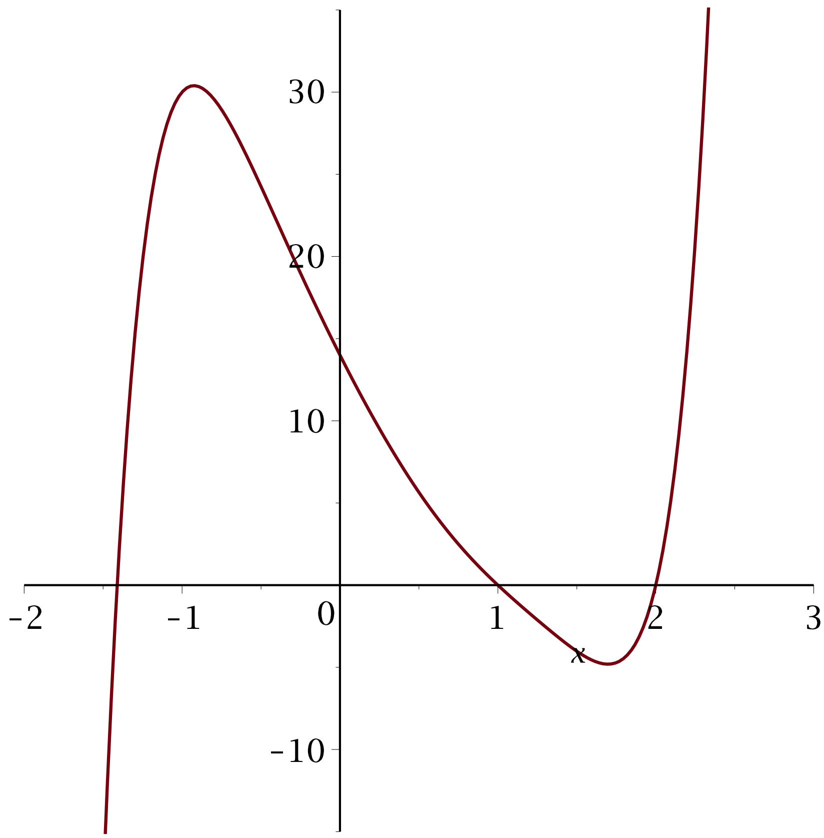
$map(simplify@\mathrm{Re}, [Lsg])$

$$\left[2\sqrt{2}\,\sin\left(\frac{1}{3}\arctan\left(\frac{1}{19}\sqrt{151}\right)+\frac{1}{6}\pi\right), -\sqrt{2}\left(\sin\left(-\frac{1}{3}\arctan\left(\frac{1}{19}\sqrt{151}\right)+\frac{1}{3}\pi\right)\sqrt{3}+\sin\left(\frac{1}{3}\arctan\left(\frac{1}{19}\sqrt{151}\right)+\frac{1}{6}\pi\right)\right), \sqrt{2}\left(\sin\left(-\frac{1}{3}\arctan\left(\frac{1}{19}\sqrt{151}\right)+\frac{1}{3}\pi\right)\sqrt{3}-\sin\left(\frac{1}{3}\arctan\left(\frac{1}{19}\sqrt{151}\right)+\frac{1}{6}\pi\right)\right)\right] \quad \textbf{(4.5)}$$

$g := x^7 - 3\cdot x^6 + 2\cdot x^5 + x^3 + 4\cdot x^2 - 19\cdot x + 14$

$$x^7 - 3\,x^6 + 2\,x^5 + x^3 + 4\,x^2 - 19\,x + 14 \quad \textbf{(4.6)}$$

$plot(g, x = -2..3, -15..35)$



$solve(g = 0)$

$$2, 1, RootOf(\_Z^5 + \_Z + 7, index = 1), RootOf(\_Z^5 + \_Z + 7, index = 2), RootOf(\_Z^5 + \_Z + 7, index = 3), RootOf(\_Z^5 + \_Z + 7, index = 4), RootOf(\_Z^5 + \_Z + 7, index = 5) \quad \textbf{(4.7)}$$

$fsolve(g = 0, x, complex)$

$$-1.41081385105958, -0.508469408973023 - 1.36861648832990\,I, -0.508469408973023 + 1.36861648832990\,I, 1., 1.21387633450281 \quad \textbf{(4.8)}$$

$-0.924188110922052\,I,\ 1.21387633450281+0.924188110922052\,I,\ 2.$

Transzendente Gleichungen

$solve(\sin(x)=\cos(x))$

$$\frac{1}{4}\pi \qquad (5.1)$$

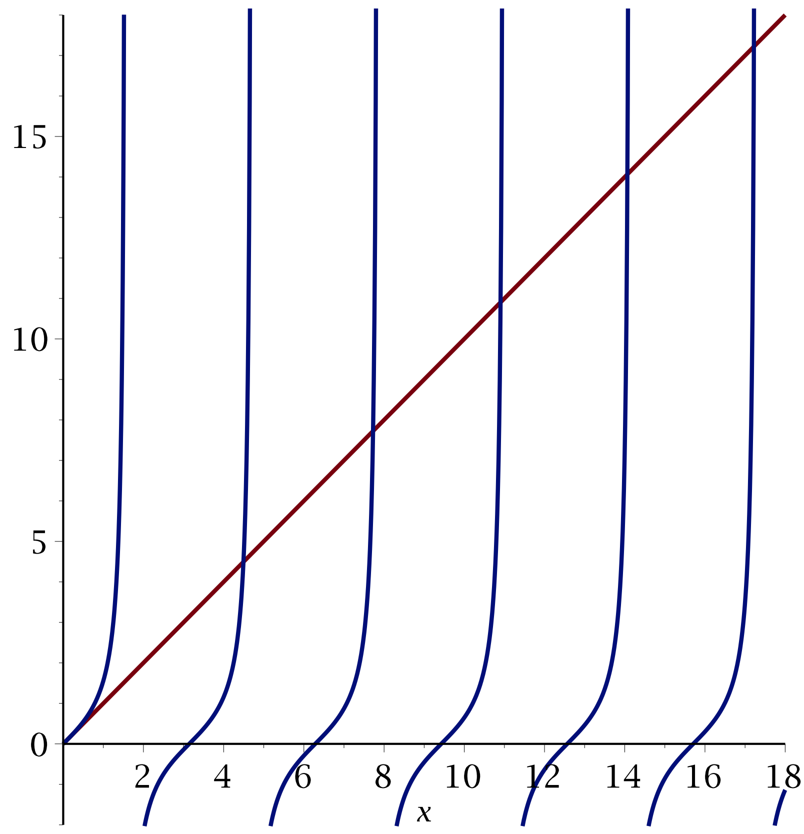
$solve(\sin(x)=\cos(x), allsolutions)$

$$\frac{1}{4}\pi+\pi\_Z2\sim \qquad (5.2)$$

$about(\_Z1)$

Originally _Z1, renamed _Z1~:
  is assumed to be: integer

$plot([x, \tan(x)], x=0..18, -2..18, discont=true, thickness=2)$



$Glg := \tan(x) = x$

$$\tan(x) = x \tag{5.3}$$

$solve(Glg)$

$$RootOf(-\tan(\_Z) + \_Z) \tag{5.4}$$

$fsolve(Glg, x = 3..5)$

$$4.493409458 \tag{5.5}$$

$f := \exp(-a \cdot x)$

$$e^{-a x} \tag{5.6}$$

$g := 3 \cdot x^a$

$$3\, x^a \tag{5.7}$$

$Lsg := solve(f = g, x)$

$$e^{-\frac{\mathrm{LambertW}\left(e^{-\frac{\ln(3)}{a}}\right) a + \ln(3)}{a}} \tag{5.8}$$

$evalf(eval(Lsg, a = 5))$

$$0.4911938878 \tag{5.9}$$

$series(\sin(x), x = 0)$

$$x - \frac{1}{6}\, x^3 + \frac{1}{120}\, x^5 + \mathrm{O}(x^7) \tag{5.10}$$

$series(Lsg, a = infinity, 3)$

$$e^{-\mathrm{LambertW}(1)} - \frac{e^{-\mathrm{LambertW}(1)} \ln(3)}{(1 + \mathrm{LambertW}(1))\, a} + \frac{1}{2}\, \frac{e^{-\mathrm{LambertW}(1)} \ln(3)^2}{(1 + \mathrm{LambertW}(1))^3\, a^2} + \mathrm{O}\left(\frac{1}{a^3}\right) \tag{5.11}$$

$simplify(diff(Lsg, a))$

$$\frac{e^{-\mathrm{LambertW}\left(3^{-\frac{1}{a}}\right)}\, 3^{-\frac{1}{a}} \ln(3)}{a^2 \left(1 + \mathrm{LambertW}\left(3^{-\frac{1}{a}}\right)\right)} \tag{5.12}$$

$evalf(eval(\mathbf{(5.12)}, a = 5))$

$$0.01447515700 \tag{5.13}$$