Lektion 14

Zufallszahlen

$restart$
$rand(\,)$

$$395718860534 \quad (1.1)$$

$rand(\,)$

$$193139816415 \quad (1.2)$$

$?rand$
$randomize(\,)$

$$1422562813 \quad (1.3)$$

$rand(\,)$

$$20012777209 \quad (1.4)$$

$rand(\,)$

$$720637641484 \quad (1.5)$$

$randomize(\mathbf{(1.3)})$

$$1422562813 \quad (1.6)$$

$rand(\,)$

$$20012777209 \quad (1.7)$$

Periodenlänge 2^19937 - 1
Wenn 10^12 Elemente pro Sekunde erzeugt werden, nach welcher Zeit wiederholt sich die Folge
(1) 1 Jahr
(2) 1000 Jahre
(3) Alter des Weltalls
(4) 5972 mal das Alter des Weltalls
(5) 10^5972 mal das Alter des Weltalls
(6) 10^(10^5972) mal das Alter des Weltalls

(1.8)

$s := 10^{12} \cdot 3600 \cdot 24 \cdot 365 \cdot 13000000000$

$$409968000000000000000000000000 \quad (1.9)$$

$evalf(\log[2](s))$

$$98.37142605 \quad (1.10)$$

$19937 - evalf(\log[2](s))$

$$19838.62857 \quad (1.11)$$

$\log 10\left(2^{\mathbf{(1.11)}}\right)$

$$5972.022272 \quad (1.12)$$

$p := rand(2)$

$$\mathbf{proc}(\,)\ \mathbf{proc}(\,)\ \mathbf{option}\ builtin = RandNumberInterface;\ \ \mathbf{end\ proc}(6, 2, 1)\ \mathbf{end\ proc} \quad (1.13)$$

$seq(p(\,), n = 1..20)$

$$1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 0 \quad (1.14)$$

$with(StringTools):$
$L := Explode(\text{"Mathe ist cool"})$

$$["M", "a", "t", "h", "e", " ", "i", "s", "t", " ", "c", "o", "o", "l"] \quad (1.15)$$

$with(Statistics):$
$L1 := Shuffle(L)$

$$[" ", "t", "c", "o", "h", " ", "t", "o", "s", "a", "M", "l", "i", "e"] \quad (1.16)$$

$Implode(L1)$

" tcoh tosaMlie" **(1.17)**

$X := RandomVariable(Uniform(0, 1))$

$$\_R \quad (1.18)$$

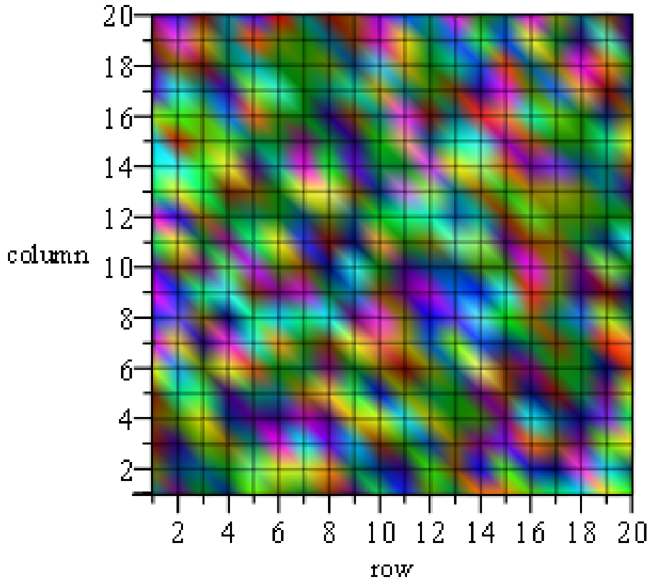
$Sample(X, 10)$

$$[0.00125735907501101, 0.707274625458114, 0.925948362800442, 0.250680877188746, 0.744418281982098, 0.784370824661492, 0.472234864437036, 0.567909752482298, 0.442033140482086, 0.349129790269057] \quad (1.19)$$

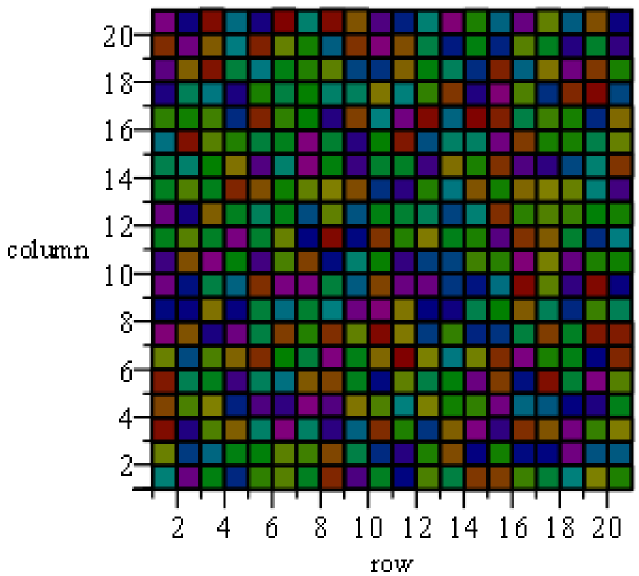
$M := Sample(X, [20, 20])$

$$\begin{bmatrix} \textit{20 x 20 Matrix} \\ \textit{Data Type: } float_8 \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \quad (1.20)$$

$with(plots):$
$matrixplot(M, shading = zhue, orientation = [-90, 0])$



$matrixplot(M, shading = zhue, orientation = [-90, 0], heights = histogram)$



Fourierreihen

$$c0 := \frac{1}{\mathrm{sqrt}(2 \cdot \mathrm{Pi})}$$

$$\frac{1}{2} \frac{\sqrt{2}}{\sqrt{\pi}} \tag{2.1}$$

$$cn := \frac{\cos(n \cdot x)}{\mathrm{sqrt}(\mathrm{Pi})}$$

$$\frac{\cos(n\, x)}{\sqrt{\pi}} \tag{2.2}$$

$$sn := \frac{\sin(n \cdot x)}{\mathrm{sqrt}(\mathrm{Pi})}$$

$$\frac{\sin(n\, x)}{\sqrt{\pi}} \tag{2.3}$$

$$skal\_prod := (f, g) \rightarrow int(f \cdot g, x = -\mathrm{Pi}..\mathrm{Pi})$$

$$(f, g) \rightarrow \int_{-\pi}^{\pi} f\, g\, \mathrm{d}x \tag{2.4}$$

$skal\_prod(c0, c0)$

$$1 \tag{2.5}$$

$skal\_prod(c0, cn)$

$$\frac{\sin(\pi\, n)\, \sqrt{2}}{\pi\, n} \tag{2.6}$$

$skal\_prod(c0, cn)$ assuming $n :: integer$

$$0 \tag{2.7}$$

$cm := eval(cn, n = m)$

$$\frac{\cos(m\, x)}{\sqrt{\pi}} \tag{2.8}$$

$skal\_prod(cn, cm)$

$$\frac{2\, (m \sin(\pi\, m)\, \cos(\pi\, n) - n \cos(\pi\, m)\, \sin(\pi\, n))}{\pi\, (m^2 - n^2)} \tag{2.9}$$

Was kommt beim Befehl **`skal_prod(cn, cm) assuming n::integer, m::integer;`** heraus?

(1) 0 und das ist auch richtig
(2) 0 und das ist auch meistens richtig
(3) 1 und das ist auch richtig
(4) 1 und das ist auch meistens richtig
(5) Error: division by zero

$skal\_prod(cn, cm)$ assuming $n :: integer, m :: integer$

$$0 \tag{2.10}$$

$skal\_prod(cn, cn)$

$$\frac{\cos(\pi\, n)\, \sin(\pi\, n) + \pi\, n}{\pi\, n} \tag{2.11}$$

$skal\_prod(cn, cn)$ assuming $n :: integer$

$$1 \tag{2.12}$$

$f := \mathrm{abs}(\cos(x))$

$$|\cos(x)| \tag{2.13}$$

$a0 := skal\_prod(f, c0)$

$$\frac{2\sqrt{2}}{\sqrt{\pi}} \tag{2.14}$$

$an := skal\_prod(f, cn)$ assuming $n :: integer$

$$-\frac{4 \cos\left(\frac{1}{2}\, \pi\, n\right)}{\sqrt{\pi}\, (n^2 - 1)} \tag{2.15}$$

$a1 := skal\_prod(f, eval(cn, n = 1))$

$$0 \tag{2.16}$$

$skal\_prod(f, sn)$

$$0 \tag{2.17}$$

$fr := a0 \cdot c0 + Sum(an \cdot cn, n = 1..10)$

$$\frac{2}{\pi} + \sum_{n=1}^{10} \left(- \frac{4 \cos\left(\frac{1}{2} \pi n\right) \cos(n x)}{\pi (n^2 - 1)} \right) \tag{2.18}$$

$L := [seq(an \cdot cn, n = 2..5)]$

$$\left[\frac{4}{3} \frac{\cos(2 x)}{\pi}, 0, -\frac{4}{15} \frac{\cos(4 x)}{\pi}, 0 \right] \tag{2.19}$$

$convert(L, `+`)$

$$\frac{4}{3} \frac{\cos(2 x)}{\pi} - \frac{4}{15} \frac{\cos(4 x)}{\pi} \tag{2.20}$$

```
for N from 2 to 6 do
  L := [seq(an·cn, n = 2..N)];
  fr[N] := a0·c0 + convert(L, `+`);
  print(N, fr[N]);
 end do:
```

$$2, \frac{2}{\pi} + \frac{4}{3} \frac{\cos(2 x)}{\pi}$$

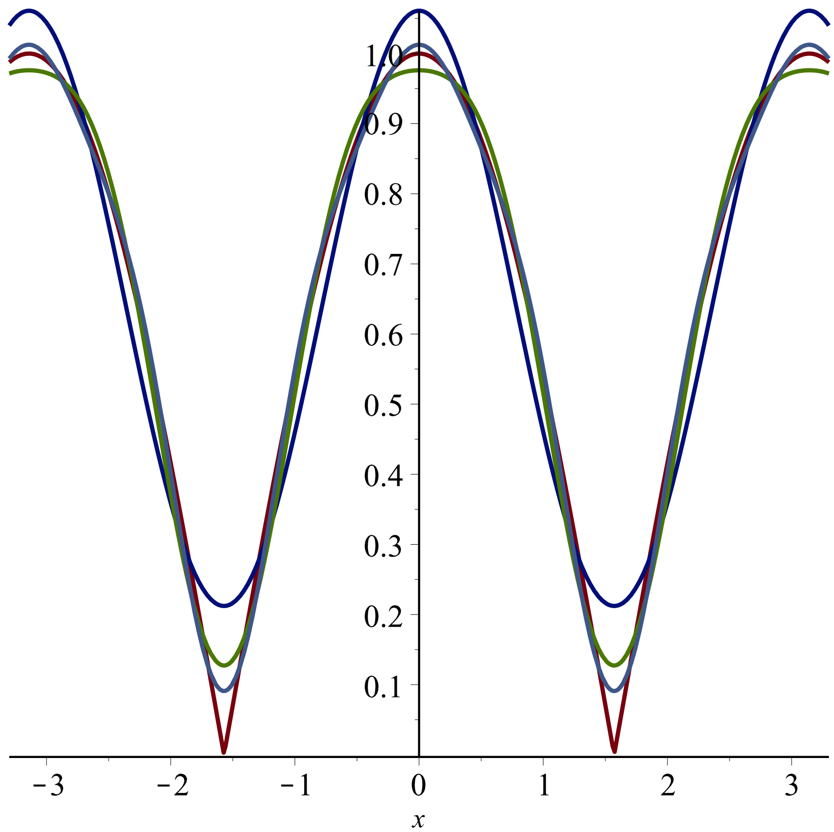
$$3, \frac{2}{\pi} + \frac{4}{3} \frac{\cos(2 x)}{\pi}$$

$$4, \frac{2}{\pi} + \frac{4}{3} \frac{\cos(2 x)}{\pi} - \frac{4}{15} \frac{\cos(4 x)}{\pi}$$

$$5, \frac{2}{\pi} + \frac{4}{3} \frac{\cos(2 x)}{\pi} - \frac{4}{15} \frac{\cos(4 x)}{\pi}$$

$$6, \frac{2}{\pi} + \frac{4}{3} \frac{\cos(2 x)}{\pi} - \frac{4}{15} \frac{\cos(4 x)}{\pi} + \frac{4}{35} \frac{\cos(6 x)}{\pi} \tag{2.21}$$

$plot([f, fr[2], fr[4], fr[6]], x = -3.3 .. 3.3, thickness = 2)$



$g := \text{Heaviside}(x)$

$$\text{Heaviside}(x) \tag{2.22}$$

$a0 := skal\_prod(g, c0)$

$$\frac{1}{2}\sqrt{2}\sqrt{\pi} \tag{2.23}$$

$skal\_prod(g, cn)$

$$\frac{\sin(\pi n)}{\sqrt{\pi}\, n} \tag{2.24}$$

$skal\_prod(g, cn)$ assuming $n :: integer$

$$0 \tag{2.25}$$

$skal\_prod(g, sn)$

$$-\frac{-1 + \cos(\pi n)}{\sqrt{\pi}\, n} \tag{2.26}$$

$bn := skal\_prod(g, sn)$ assuming $n :: integer$

$$-\frac{-1 + (-1)^n}{\sqrt{\pi}\, n} \tag{2.27}$$

```
for N from 2 to 16 do
  L := [seq(bn·sn, n = 1 ..N)];
  gr[N] := a0·c0 + convert(L, `+`);
  print(N, gr[N]);
end do:
```

$$2,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}$$

$$3,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}$$

$$4,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}$$

$$5,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}$$

$$6,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}$$

$$7,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}$$

$$8,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}$$

$$9,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}+\frac{2}{9}\,\frac{\sin(9x)}{\pi}$$

$$10,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}+\frac{2}{9}\,\frac{\sin(9x)}{\pi}$$

$$11,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}+\frac{2}{9}\,\frac{\sin(9x)}{\pi}+\frac{2}{11}\,\frac{\sin(11x)}{\pi}$$

$$12,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}+\frac{2}{9}\,\frac{\sin(9x)}{\pi}+\frac{2}{11}\,\frac{\sin(11x)}{\pi}$$

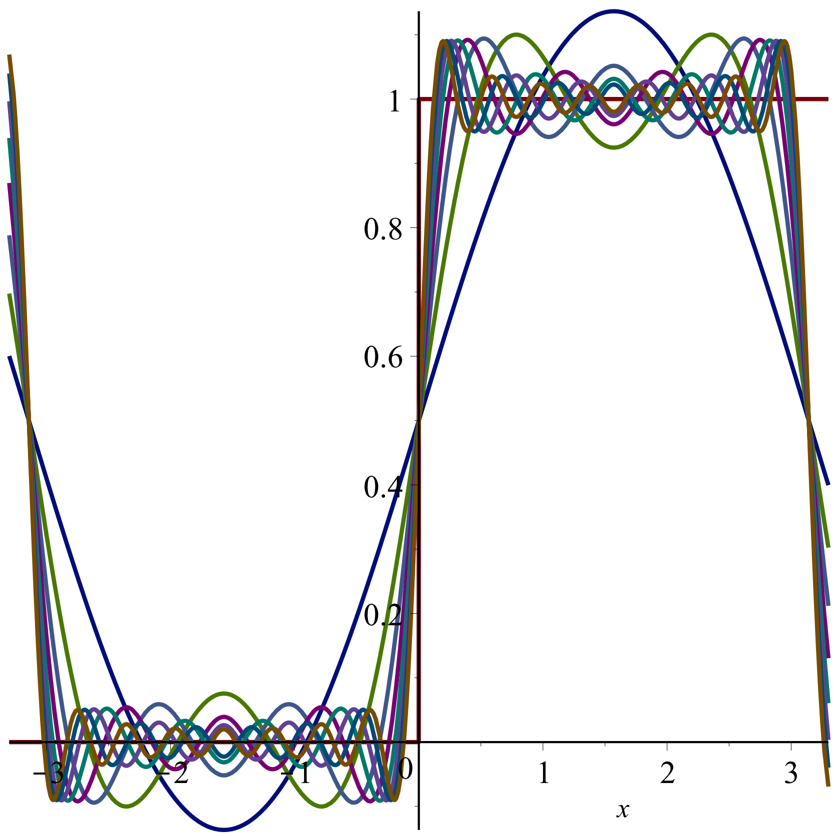
$$13,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}+\frac{2}{9}\,\frac{\sin(9x)}{\pi}+\frac{2}{11}\,\frac{\sin(11x)}{\pi}+\frac{2}{13}\,\frac{\sin(13x)}{\pi}$$

$$14,\ \frac{1}{2}+\frac{2\sin(x)}{\pi}+\frac{2}{3}\,\frac{\sin(3x)}{\pi}+\frac{2}{5}\,\frac{\sin(5x)}{\pi}+\frac{2}{7}\,\frac{\sin(7x)}{\pi}+\frac{2}{9}\,\frac{\sin(9x)}{\pi}+\frac{2}{11}\,\frac{\sin(11x)}{\pi}+\frac{2}{13}\,\frac{\sin(13x)}{\pi}$$

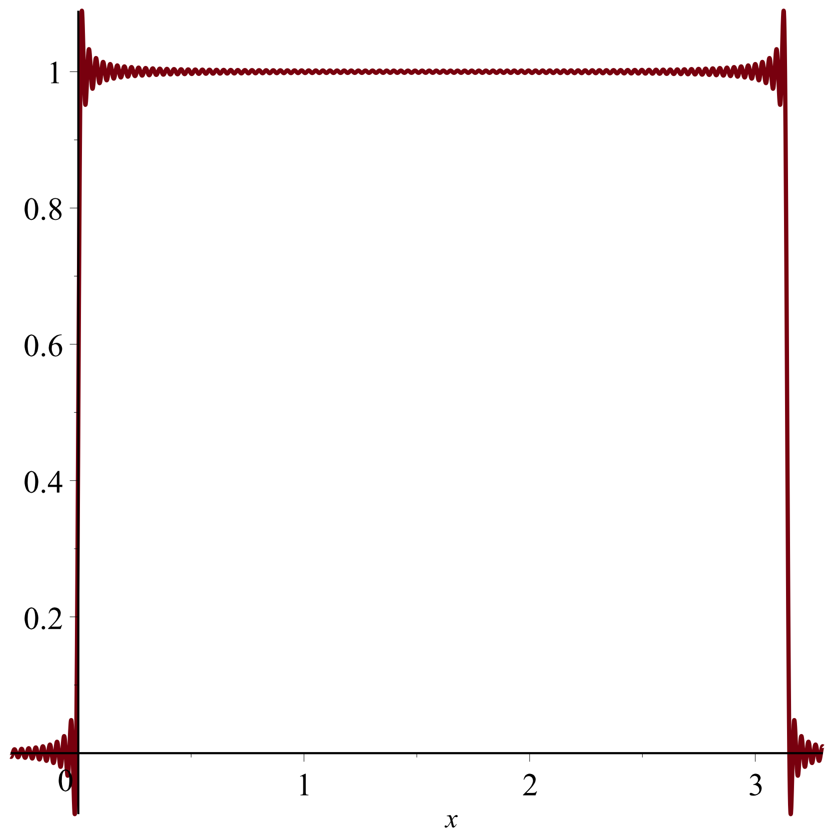
$$15, \frac{1}{2} + \frac{2\sin(x)}{\pi} + \frac{2}{3}\frac{\sin(3x)}{\pi} + \frac{2}{5}\frac{\sin(5x)}{\pi} + \frac{2}{7}\frac{\sin(7x)}{\pi} + \frac{2}{9}\frac{\sin(9x)}{\pi} + \frac{2}{11}\frac{\sin(11x)}{\pi} + \frac{2}{13}\frac{\sin(13x)}{\pi} + \frac{2}{15}\frac{\sin(15x)}{\pi}$$

$$16, \frac{1}{2} + \frac{2\sin(x)}{\pi} + \frac{2}{3}\frac{\sin(3x)}{\pi} + \frac{2}{5}\frac{\sin(5x)}{\pi} + \frac{2}{7}\frac{\sin(7x)}{\pi} + \frac{2}{9}\frac{\sin(9x)}{\pi} + \frac{2}{11}\frac{\sin(11x)}{\pi} + \frac{2}{13}\frac{\sin(13x)}{\pi} + \frac{2}{15}\frac{\sin(15x)}{\pi} \quad \textbf{(2.28)}$$

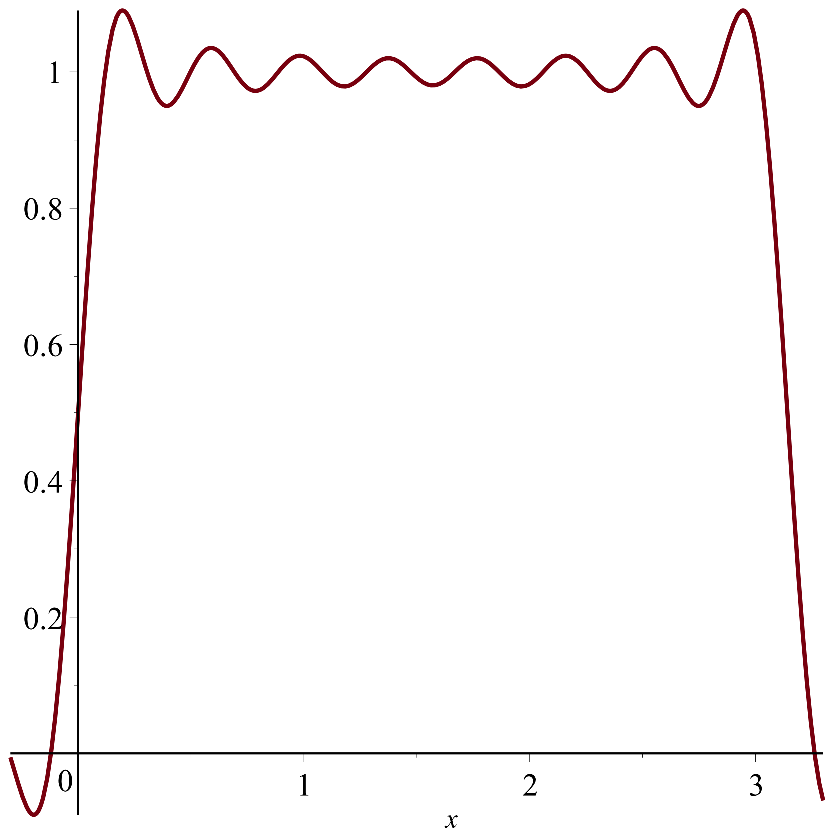
$plot([g, seq(gr[2\cdot k], k=1..8)], x=-3.3..3.3, thickness=2)$



$L := [seq(bn\cdot sn, n=1..200)]:$
$gr[200] := a0\cdot c0 + convert(L, `+`):$
$plot(gr[200], x=-0.3..3.3, thickness=2, numpoints=3000)$



$plot(gr[16], x = -0.3\ ..\ 3.3, thickness = 2)$



$dg16 := diff(gr[16], x)$

$$\frac{2\cos(x)}{\pi} + \frac{2\cos(3x)}{\pi} + \frac{2\cos(5x)}{\pi} + \frac{2\cos(7x)}{\pi} + \frac{2\cos(9x)}{\pi} + \frac{2\cos(11x)}{\pi} + \frac{2\cos(13x)}{\pi} + \frac{2\cos(15x)}{\pi} \quad \textbf{(2.29)}$$

$Lsg := \{solve(\{dg16\})\}$

$$\left\{\left\{x=\frac{1}{2}\pi\right\}, \left\{x=\frac{1}{4}\pi\right\}, \left\{x=\frac{1}{8}\pi\right\}, \left\{x=\frac{3}{4}\pi\right\}, \left\{x=\frac{3}{8}\pi\right\}, \left\{x=\frac{5}{8}\pi\right\}, \left\{x=\frac{7}{8}\pi\right\}, \left\{x=\pi-\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right\}, \left\{x=\pi-\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right\}, \left\{x=\pi-\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right\}, \left\{x=\pi-\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right\}, \left\{x=\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right\}, \left\{x=\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right\}, \left\{x=\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right\}, \left\{x=\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right\}\right\} \quad \textbf{(2.30)}$$

$extremwerte := map(l \rightarrow eval(gr[16], l), Lsg)$

$$\left\{\frac{1}{2} + \frac{67952}{45045\,\pi}, \frac{1}{2} + \frac{47248}{45045}\,\frac{\sqrt{2}}{\pi}, \frac{1}{2} + \frac{608}{315}\,\frac{\sin\left(\frac{1}{8}\,\pi\right)}{\pi} + \frac{1568}{2145}\,\frac{\sin\left(\frac{3}{8}\,\pi\right)}{\pi}, \frac{1}{2} + \frac{608}{315}\,\frac{\sin\left(\frac{3}{8}\,\pi\right)}{\pi} - \frac{1568}{2145}\,\frac{\sin\left(\frac{1}{8}\,\pi\right)}{\pi}, \frac{1}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{\pi} + \frac{2}{3}\,\frac{\sin\left(3\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{5}\,\frac{\sin\left(5\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{7}\,\frac{\sin\left(7\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{9}\,\frac{\sin\left(9\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{11}\,\frac{\sin\left(11\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{13}\,\frac{\sin\left(13\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{15}\,\frac{\sin\left(15\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\right)}{\pi}, \frac{1}{2} + \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{\pi} + \frac{2}{3}\,\frac{\sin\left(3\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{5}\,\frac{\sin\left(5\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} + \frac{2}{7}\,\frac{\sin\left(7\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right)}{\pi} \tag{2.31}$$

$$+\frac{2}{9}\,\frac{\sin\left(9\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{11}\,\frac{\sin\left(11\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{13}\,\frac{\sin\left(13\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{15}\,\frac{\sin\left(15\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\right)}{\pi},\ \frac{1}{2}+\frac{\sqrt{2-\sqrt{2+\sqrt{2}}}}{\pi}$$

$$+\frac{2}{3}\,\frac{\sin\left(3\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{5}\,\frac{\sin\left(5\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{7}\,\frac{\sin\left(7\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{9}\,\frac{\sin\left(9\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{11}\,\frac{\sin\left(11\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{13}\,\frac{\sin\left(13\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{15}\,\frac{\sin\left(15\arccos\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)\right)}{\pi},\ \frac{1}{2}+\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{\pi}$$

$$+\frac{2}{3}\,\frac{\sin\left(3\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{5}\,\frac{\sin\left(5\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{7}\,\frac{\sin\left(7\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{9}\,\frac{\sin\left(9\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{11}\,\frac{\sin\left(11\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{13}\,\frac{\sin\left(13\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}$$

$$+\frac{2}{15}\,\frac{\sin\left(15\arccos\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)\right)}{\pi}\Bigg\}$$

$map(evalf, extremwerte)$

$$\{0.9500894977, 0.9721738848, 0.9785762607, 0.9801818934, 1.020200420, 1.023757159, 1.035122437, 1.090142065\} \quad \textbf{(2.32)}$$

Die Elemente der Fourierreihe schießen im ca 9% der Sprunghöhe über den wahren Wert hinaus. Das ist das Gibbssche Phänomen.

Eine Bernoullische Differentialgleichung

$Dgl := y'(x) = 2\cdot y(x) - 4\cdot y(x)^{\frac{3}{4}}$

$$\frac{d}{dx}\,y(x) = 2\,y(x) - 4\,y(x)^{3/4} \quad \textbf{(3.1)}$$

$Lsg1 := dsolve(\{Dgl, y(0) = 1\})$

$$y(x) = \left(e^{\frac{1}{2}x}\right)^4 - 8\left(e^{\frac{1}{2}x}\right)^3 + 24\left(e^{\frac{1}{2}x}\right)^2 - 32\,e^{\frac{1}{2}x} + 16 \quad \textbf{(3.2)}$$

$f1 := rhs(Lsg1)$

$$\left(e^{\frac{1}{2}x}\right)^4 - 8\left(e^{\frac{1}{2}x}\right)^3 + 24\left(e^{\frac{1}{2}x}\right)^2 - 32\,e^{\frac{1}{2}x} + 16 \quad \textbf{(3.3)}$$

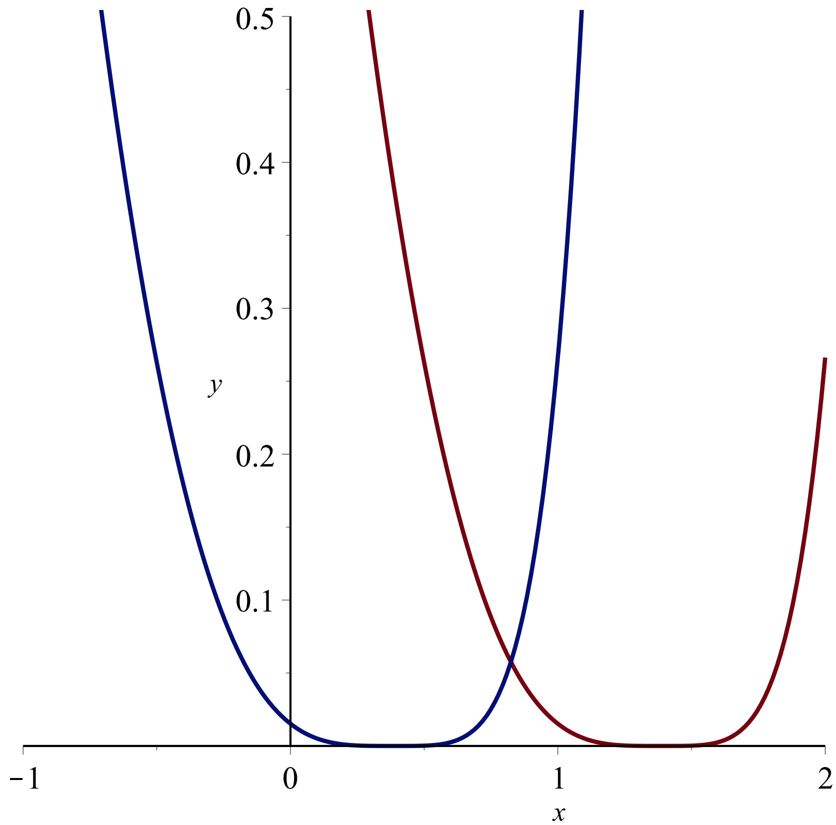
$Lsg2 := dsolve(\{Dgl, y(-1) = 1\})$

$$y(x) = \left(e^{\frac{1}{2}x}\,e^{\frac{1}{2}} - 2\right)^4 \quad \textbf{(3.4)}$$

$f2 := rhs(Lsg2)$

$$\left(e^{\frac{1}{2}x} e^{\frac{1}{2}} - 2\right)^4 \quad \textbf{(3.5)}$$

$pl1 := plot([f1, f2], x = -1 ..2, y = 0 .. 0.5, thickness = 2) :$
$pl1$

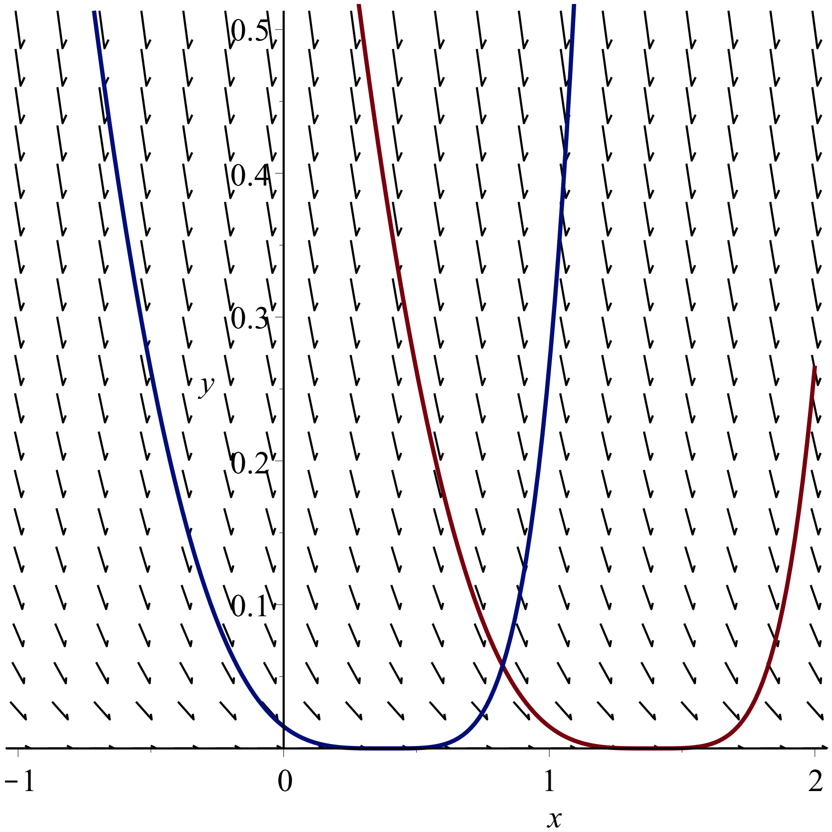


Der Satz von Picard-Lindelöf sagt für diese Dgl
(1) dass sich Lösungen nie schneiden können
(2) dass sich Lösungen überall schneiden können
(3) dass sich Lösungen nur in y=0 schneiden können
(4) dass sich Lösungen nur in x = 1/e schneiden können

$v := [1, eval(rhs(Dgl), y(x) = y)]$

$$\left[1, 2\,y - 4\,y^{3/4}\right] \quad \textbf{(3.6)}$$

$with(plots) :$
$pl2 := fieldplot(v, x = -1 ..2, y = 0 ..0.5) :$
$display(\{pl1, pl2\})$



Welche Lösungskurve ist richtig?
(1) die rote
(2) die blaue
(3) die rote und die blaue jeweils links von ihren Nullstellen
(4) die rote und die blaue jeweils rechts von ihren Nullstellen

$eval\left(Dgl, y(x) = z(x)^4\right)$

$$4\, z(x)^3 \left(\frac{\mathrm{d}}{\mathrm{d}x} z(x)\right) = 2\, z(x)^4 - 4\left(z(x)^4\right)^{3/4} \tag{3.7}$$

$tmp := simplify(\mathbf{(3.7)})$ assuming $z(x) :: positive$

$$4\, z(x)^3 \left(\frac{\mathrm{d}}{\mathrm{d}x} z(x)\right) = 2\, z(x)^4 - 4\, z(x)^3 \tag{3.8}$$

$Dglz := \dfrac{factor(tmp)}{z(x)^3}$

$$4 \left(\frac{\mathrm{d}}{\mathrm{d}x} z(x)\right) = 2\, z(x) - 4 \tag{3.9}$$

$Lsg3 := dsolve(\{Dglz, z(-1) = 1\})$

$$z(x) = 2 - \frac{e^{\frac{1}{2}x}}{e^{-\frac{1}{2}}} \tag{3.10}$$

$solve(rhs(Lsg3) > 0)$

$$RealRange(-\infty, Open(2\ln(2) - 1)) \tag{3.11}$$

$x0 := 2 \cdot \ln(2) - 1$

$$2\ln(2) - 1 \tag{3.12}$$

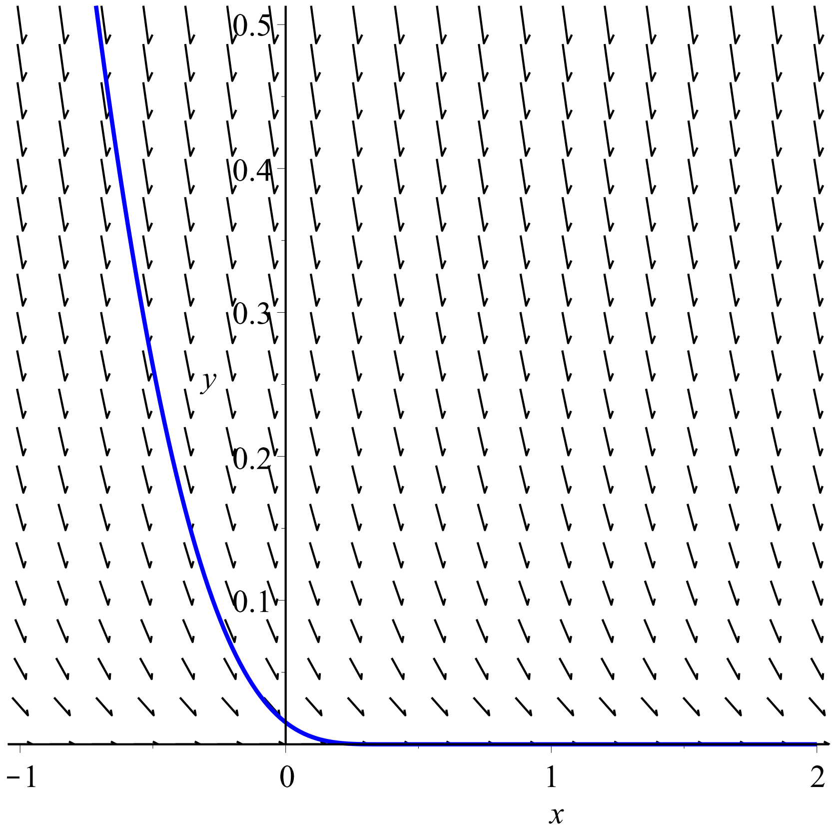
$evalf(x0)$

$$0.386294361 \tag{3.13}$$

$f3 := piecewise\left(x < x0, rhs(Lsg3)^4, x \geq x0, 0\right)$

$$\begin{cases} \left(2 - \frac{e^{\frac{1}{2}x}}{e^{-\frac{1}{2}}}\right)^4 & x < 2\ln(2) - 1 \\ 0 & 2\ln(2) - 1 \leq x \end{cases} \tag{3.14}$$

$pl3 := plot(f3, x = -1..2, y = 0..0.5, thickness = 2, color = blue):$
$display(\{pl3, pl2\})$



$eval(Dgl, y(x) = f3)$

$$\begin{cases} -\dfrac{2\left(2-\dfrac{e^{\frac{1}{2}x}}{e^{-\frac{1}{2}}}\right)^{3} e^{\frac{1}{2}x}}{e^{-\frac{1}{2}}} & x < 2\ln(2)-1 \\ 0 & 2\ln(2)-1 \le x \end{cases} = 2\left(\begin{cases} \left(2-\dfrac{e^{\frac{1}{2}x}}{e^{-\frac{1}{2}}}\right)^{4} & x < 2\ln(2)-1 \\ 0 & 2\ln(2)-1 \le x \end{cases}\right) - 4\left(\begin{cases} \left(2-\dfrac{e^{\frac{1}{2}x}}{e^{-\frac{1}{2}}}\right)^{4} & x < 2\ln(2)-1 \\ 0 & 2\ln(2)-1 \le x \end{cases}\right)^{3/4} \tag{3.15}$$

$simplify(lhs(\mathbf{(3.15)}) - rhs(\mathbf{(3.15)}))$

$$0 \tag{3.16}$$