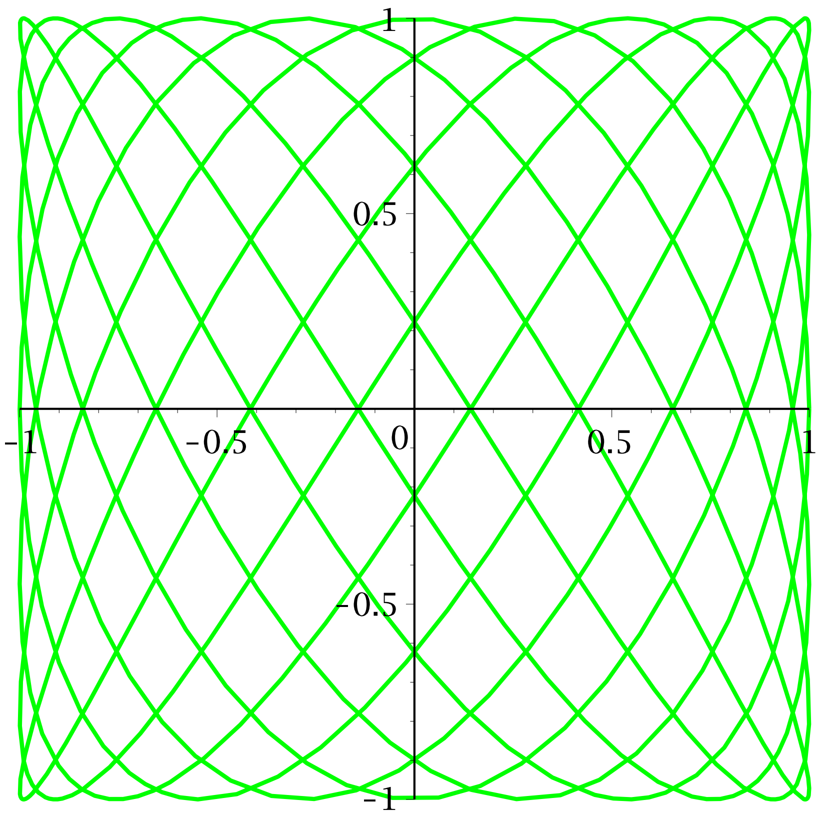
Lektion 8

ebene parametrische Plots

$plot([\cos(7\cdot t), \sin(11\cdot t), t = 0..2\cdot\mathrm{Pi}], color = green, thickness = 2)$



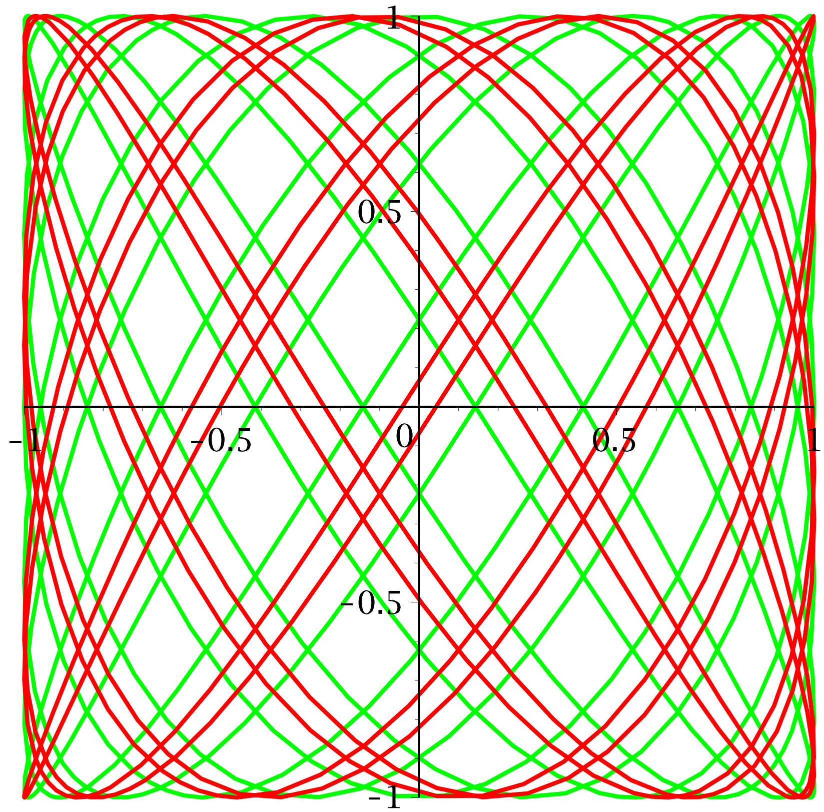
$Lissajou1 := [\cos(7\cdot t), \sin(11\cdot t), t = 0..2\cdot\mathrm{Pi}]$

$$[\cos(7\,t), \sin(11\,t), t = 0..2\,\pi] \tag{1.1}$$

$Lissajou2 := [\cos(7\cdot t + 0.1), \sin(11\cdot t), t = 0..2\cdot\mathrm{Pi}]$

$$[\cos(7\,t + 0.1), \sin(11\,t), t = 0..2\,\pi] \tag{1.2}$$

$plot([Lissajou1, Lissajou2], color = [green, red], thickness = 2)$



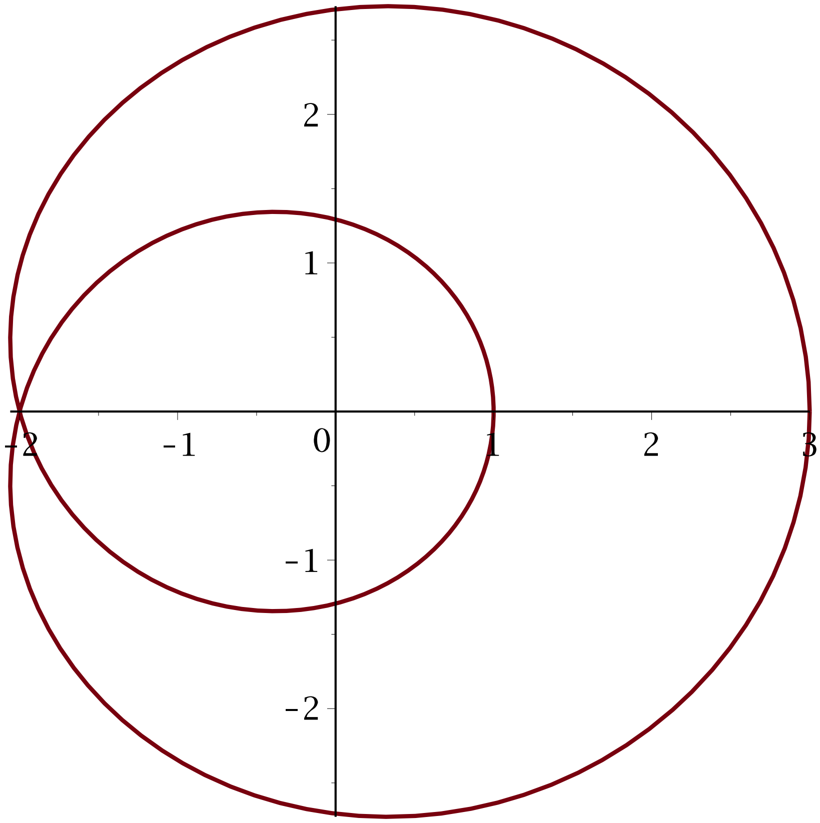
$with(plots):$
$r := 2 + \cos(t)$

$$2 + \cos(t) \tag{1.3}$$

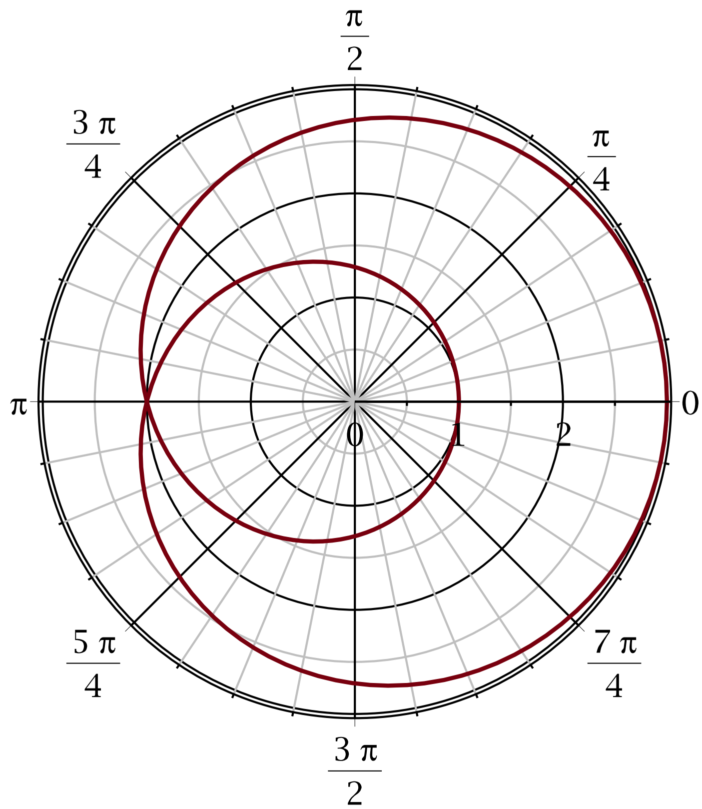
$\mathrm{phi} := 2 \cdot t$

$$2\,t \tag{1.4}$$

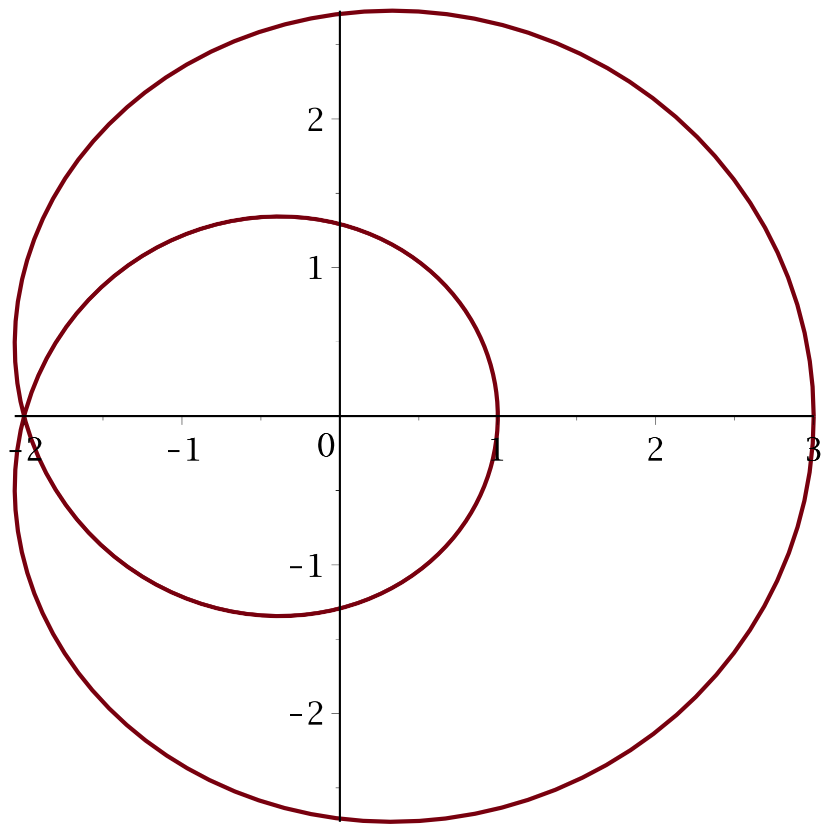
$polarplot([r, \mathrm{phi}, t = 0..2 \cdot \mathrm{Pi}], thickness = 2, axiscoordinates = cartesian)$



polarplot([*r*, phi, *t* = 0 ..2 · Pi], *thickness* = 2)



$plot([r\cdot\cos(\text{phi}), r\cdot\sin(\text{phi}), t = 0..2\cdot\text{Pi}], thickness = 2)$



Der Arcustangens

arctan(sqrt(3))

$$\frac{1}{3}\pi \tag{2.1}$$

$\tan\left(\frac{\mathrm{Pi}}{3}\right)$

$$\sqrt{3} \tag{2.2}$$

$\tan\left(-\frac{2}{3}\cdot\mathrm{Pi}\right)$

$$\sqrt{3} \tag{2.3}$$

arctan(sqrt(3), 1)

$$\frac{1}{3}\pi \tag{2.4}$$

$\mathrm{arctan}(-\mathrm{sqrt}(3), -1)$

$$-\frac{2}{3}\pi \tag{2.5}$$

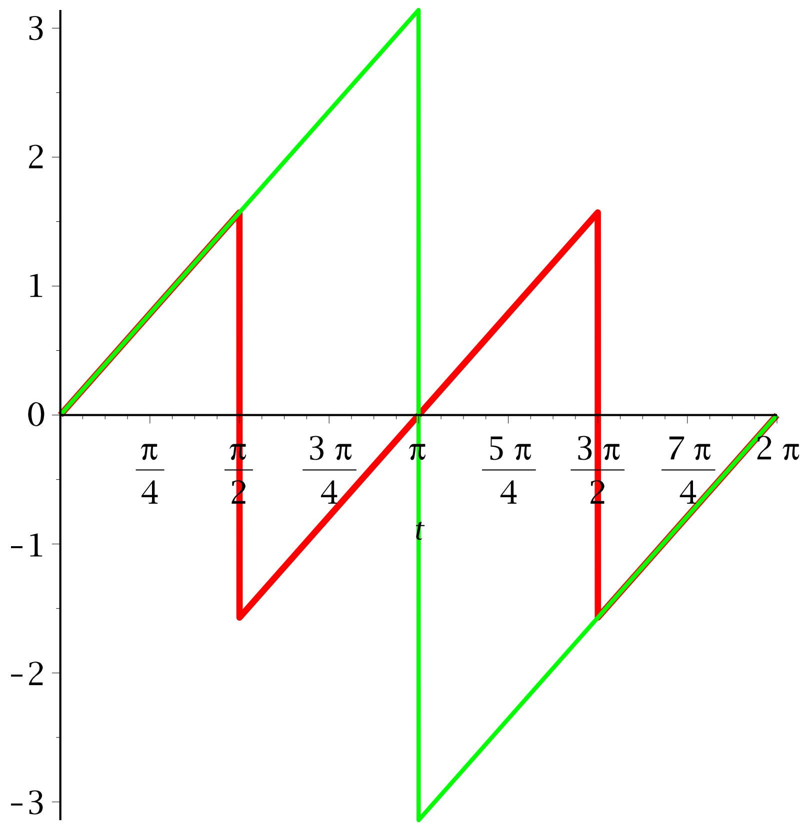
$\mathit{evalc}(\log(1+\mathrm{sqrt}(3)\cdot I))$

$$\ln(2)+\frac{1}{3}\mathrm{I}\pi \tag{2.6}$$

$\mathit{evalc}(\log(-1-\mathrm{sqrt}(3)\cdot I))$

$$\ln(2)-\frac{2}{3}\mathrm{I}\pi \tag{2.7}$$

$\mathit{plot}\left(\left[\mathrm{arctan}\left(\frac{\sin(t)}{\cos(t)}\right), \mathrm{arctan}(\sin(t), \cos(t))\right], t = 0..2\cdot\mathrm{Pi}, \mathit{color} = [\mathit{red}, \mathit{green}],\right.$
$\left.\mathit{thickness} = [3, 2]\right)$

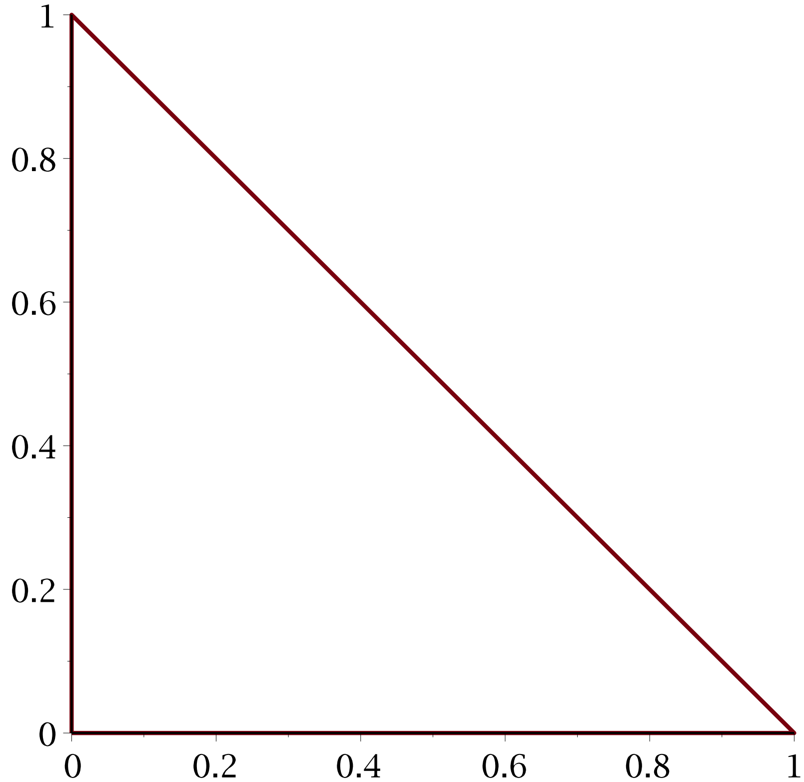


Polygonzüge

$dreieck := [[0, 0], [1, 0], [0, 1], [0, 0]]$

$$[[0, 0], [1, 0], [0, 1], [0, 0]] \tag{3.1}$$

$plot(dreieck, thickness = 2)$



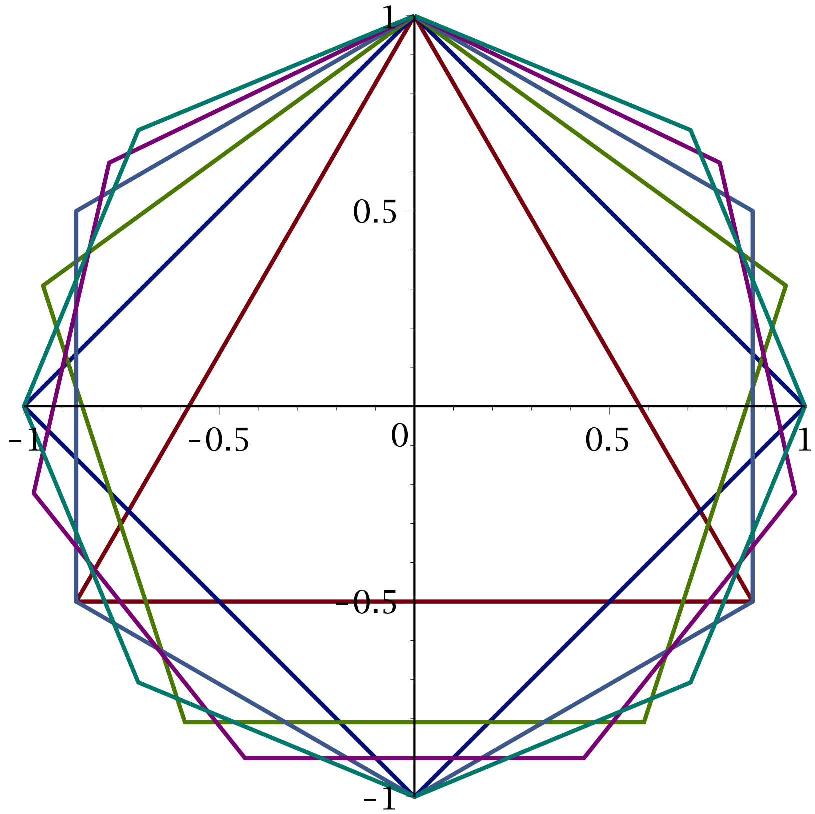
$n\_eck := n \rightarrow \left[seq\left(\left[\sin\left(\frac{2 \cdot \mathrm{Pi} \cdot j}{n} \right), \cos\left(\frac{2 \cdot \mathrm{Pi} \cdot j}{n} \right) \right], j = 0 .. n \right) \right]$

$$n \rightarrow \left[seq\left(\left[\sin\left(\frac{2 \pi j}{n} \right), \cos\left(\frac{2 \pi j}{n} \right) \right], j = 0 .. n \right) \right] \tag{3.2}$$

$n\_eck(3)$

$$\left[[0, 1], \left[\frac{1}{2} \sqrt{3}, -\frac{1}{2} \right], \left[-\frac{1}{2} \sqrt{3}, -\frac{1}{2} \right], [0, 1] \right] \tag{3.3}$$

$plot([seq(n\_eck(n), n = 3 .. 8)], thickness = 2, scaling = constrained)$

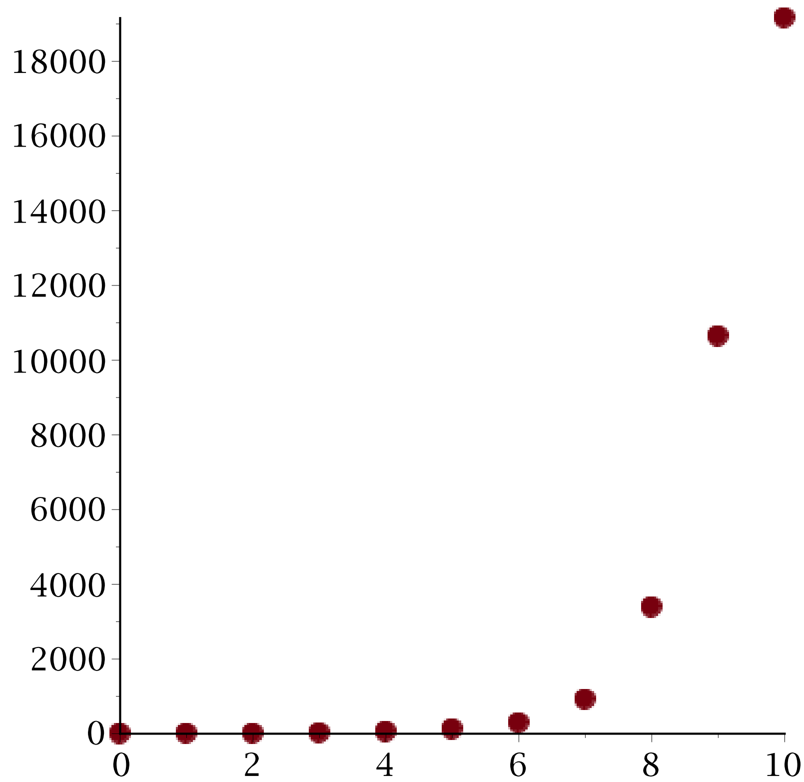


Datenimport

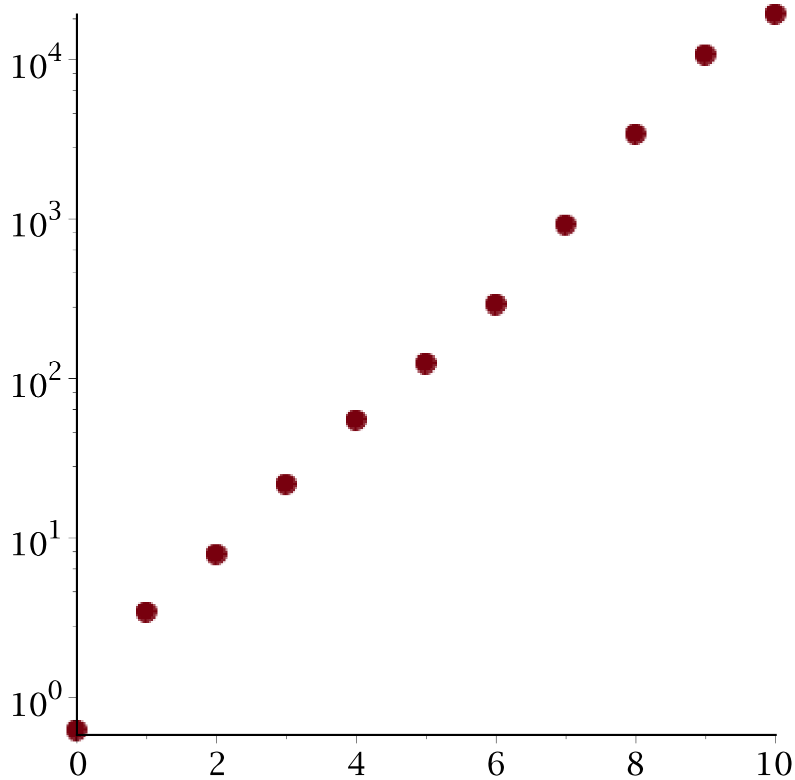
read("beispiel.data")

[[0., 0.618844], [1.000000, 3.411437], [2.000000, 7.840819], [3.000000, 21.566996], [4.000000, 54.610662], [5.000000, 123.532079], [6.000000, 289.389952], [7.000000, 917.005121], [8.000000, 3385.918836], [9.000000, 10641.210488], [10.000000, 19156.574429]] **(4.1)**

plot(*daten*, *style* = *point*, *symbol* = *solidcircle*, *symbolsize* = 20)



$logplot(daten, style = point, symbol = solidcircle, symbolsize = 20)$



$tmp := NULL;$
for $paar$ **in** $daten$ **do**
$x := paar[1];$
$y := paar[2];$
$tmp := tmp, [x, \log(y)]$
end do :

(4.2)

$logdaten := [tmp]$

[[0., -0.4799020574], [1.000000, 1.227133610], [2.000000, 2.059343293], [3.000000, 3.071164183], [4.000000, 4.000229138], [5.000000, 4.816500871], [6.000000, 5.667775093], [7.000000, 6.821113057], [8.000000, 8.127380592], [9.000000, 9.272489524], [10.000000, 9.860401248]] **(4.3)**

save(*logdaten*, "logbsp.data")

Raumkurven

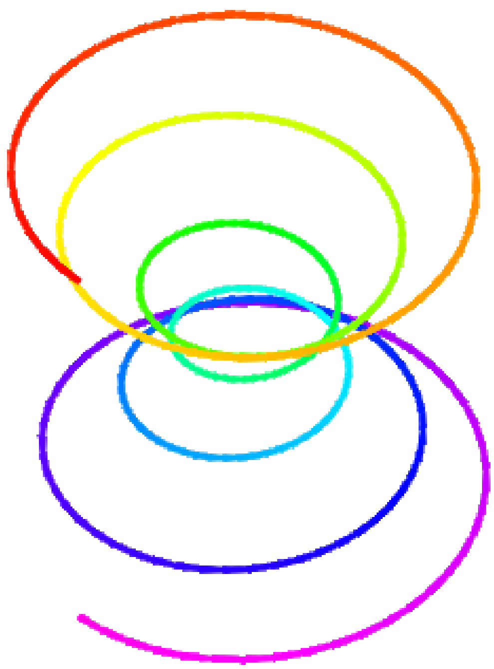
restart

with(*plots*) :

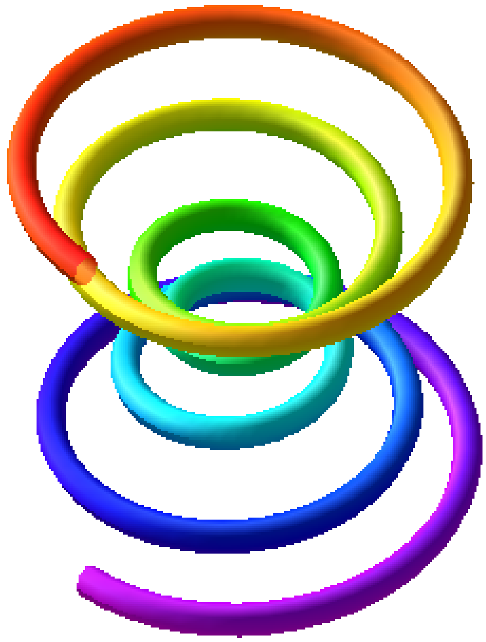
$kurve := \left(2-\cos\left(\frac{t}{6}\right)\right)\cdot\cos(t), \left(2-\cos\left(\frac{t}{6}\right)\right)\cdot\sin(t), \frac{t}{8}$

$$\left(2-\cos\left(\frac{1}{6}\,t\right)\right)\cos(t), \left(2-\cos\left(\frac{1}{6}\,t\right)\right)\sin(t), \frac{1}{8}\,t \qquad \textbf{(5.1)}$$

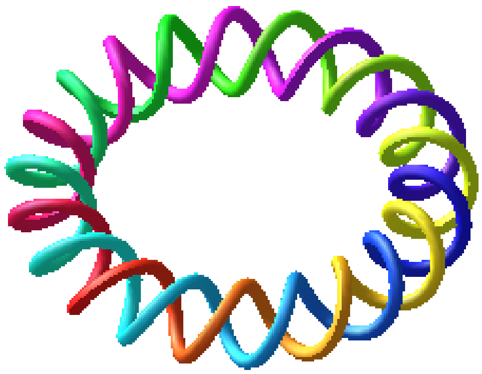
spacecurve([*kurve*, *t* = -6·Pi..6·Pi], *thickness* = 4, *shading* = *zhue*, *numpoints* = 300, *axes* = *none*)



tubeplot([*kurve*, *t* = -6·Pi..6·Pi], *thickness* = 4, *shading* = *zhue*, *numpoints* = 300, *axes* = *none*, *style* = *patchnogrid*, *radius* = 0.2)



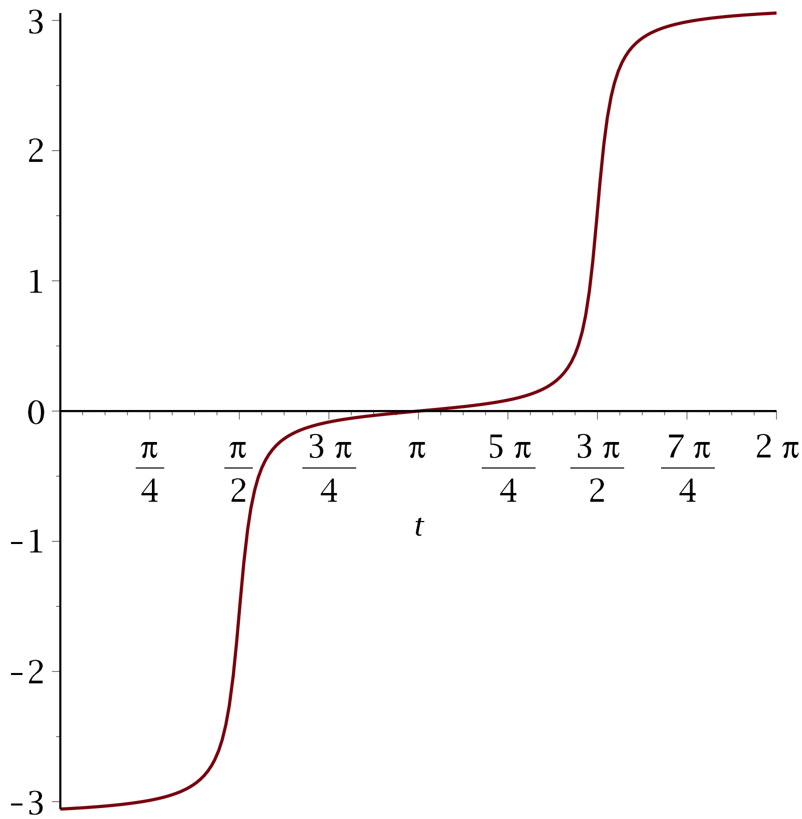
$kurve := (5+\cos(21\cdot t))\cdot\cos(2\cdot t), (5+\cos(21\cdot t))\cdot\sin(2\cdot t), \sin(21\cdot t) :$
$tubeplot([kurve, t=0..2\cdot\mathrm{Pi}], radius=0.2, numpoints=500, style=patchnogrid,$
$scaling=constrained, axes=none, color=t)$



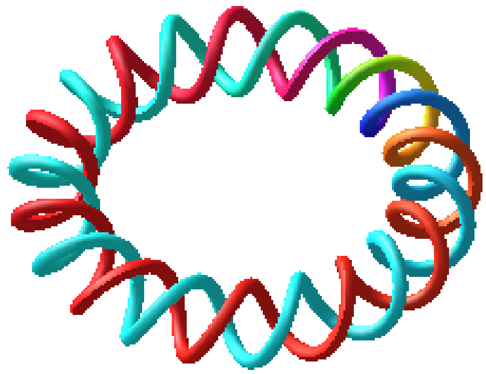
$$w := \arctan\left(10\left(t - \frac{3\cdot\text{Pi}}{2}\right)\right) + \arctan\left(10\cdot\left(t - \frac{\text{Pi}}{2}\right)\right)$$

$$\arctan(10\,t - 15\,\pi) + \arctan(10\,t - 5\,\pi) \tag{5.2}$$

$plot(w, t = 0..2\cdot\text{Pi})$



$tubeplot([kurve, t = 0..2 \cdot \mathrm{Pi}], radius = 0.2, numpoints = 500, style = patchnogrid, scaling = constrained, axes = none, color = w)$



Flächen im Raum

$profil := \frac{1}{2} - \frac{t^2}{2}$

$$\frac{1}{2} - \frac{1}{2} t^2 \tag{6.1}$$

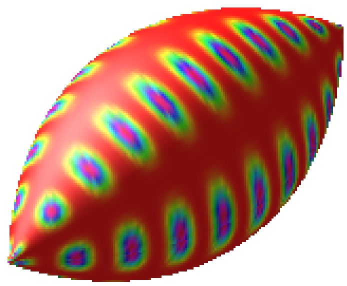
$flaeche := [t, \cos(2 \cdot \mathrm{Pi} \cdot s) \cdot profil, \sin(2 \cdot \mathrm{Pi} \cdot s) \cdot profil]$

$$\left[t, \cos(2 \pi s) \left(\frac{1}{2} - \frac{1}{2} t^2\right), \sin(2 \pi s) \left(\frac{1}{2} - \frac{1}{2} t^2\right)\right] \tag{6.2}$$

$w := (\cos(16 \cdot s) \cdot \cos(16 \cdot t))^4$

$$\cos(16 s)^4 \cos(16 t)^4 \tag{6.3}$$

plot3d(*flaeche*, $s = -1 ..1$, $t = -1 ..1$, *scaling* = *constrained*, *axes* = *none*, *style* = *patchnogrid*, *numpoints* = 14000 , *color* = *w*)



Partielle Ableitungen

restart

$g := \exp(a \cdot x + b \cdot y + c \cdot z)$

$$e^{a x + b y + c z} \tag{7.1}$$

$diff(g, y)$

$$b\, e^{a x + b y + c z} \tag{7.2}$$

$d9g := Diff(g, x, y, y, z\$6):$

$d9g = value(d9g)$

$$\frac{\partial^9}{\partial z^6\, \partial y^2\, \partial x} e^{a x + b y + c z} = a\, b^2\, c^6\, e^{a x + b y + c z} \tag{7.3}$$

Ableitungen von Vektorfunktionen

$v := \langle t, t^2, t^3 \rangle$

$$\begin{bmatrix} t \\ t^2 \\ t^3 \end{bmatrix} \qquad (8.1)$$

$map(diff, v, t)$

$$\begin{bmatrix} 1 \\ 2\,t \\ 3\,t^2 \end{bmatrix} \qquad (8.2)$$

$diff(v, t)$

Error, non-algebraic expressions cannot be differentiated

$with(VectorCalculus) :$

$diff(v, t)$

$$e_x + 2\,t e_y + 3\,t^2 e_z \qquad (8.3)$$

$BasisFormat(false)$

$$true \qquad (8.4)$$

$diff(v, t)$

$$\begin{bmatrix} 1 \\ 2\,t \\ 3\,t^2 \end{bmatrix} \qquad (8.5)$$

Gradienten und der Laplace-Operator

$Gradient(g, [x, y, z])$

$$\begin{bmatrix} a\,\mathrm{e}^{a x + b y + c z} \\ b\,\mathrm{e}^{a x + b y + c z} \\ c\,\mathrm{e}^{a x + b y + c z} \end{bmatrix} \qquad (9.1)$$

$Laplacian(g, [x, y, z])$

$$a^2\,\mathrm{e}^{a x + b y + c z} + b^2\,\mathrm{e}^{a x + b y + c z} + c^2\,\mathrm{e}^{a x + b y + c z} \qquad (9.2)$$

$h := r \cdot \cos(\mathrm{phi}) \cdot \sin(\mathrm{phi})$

$$r \cos(\phi) \sin(\phi) \qquad (9.3)$$

$Laplacian(h, \mathrm{polar}[r, \mathrm{phi}])$

$$-\frac{3 \cos(\phi) \sin(\phi)}{r} \qquad (9.4)$$

$$tmp := eval\left(h, \left\{\cos(\mathrm{phi}) = \frac{x}{r}, \sin(\mathrm{phi}) = \frac{y}{r}\right\}\right)$$

$$\frac{x\,y}{r} \tag{9.5}$$

$$H := eval(tmp, r = \mathrm{sqrt}(x^2 + y^2))$$

$$\frac{x\,y}{\sqrt{x^2 + y^2}} \tag{9.6}$$

$$Laplacian(H, [x, y])$$

$$\frac{3\,x^3\,y}{(x^2 + y^2)^{5/2}} - \frac{6\,x\,y}{(x^2 + y^2)^{3/2}} + \frac{3\,x\,y^3}{(x^2 + y^2)^{5/2}} \tag{9.7}$$

$$simplify(\textbf{(9.7)})$$

$$-\frac{3\,x\,y}{(x^2 + y^2)^{3/2}} \tag{9.8}$$