

Lab1-DDPM

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Task 1 (Code)

1. TODO1-network.py:

- a. **init**: 實作模型架構, 利用 time embedded dimension 的大小來當作 MLP 的輸入與輸出 channel 數量, 接著由 dim_hids 陣列的長度看要加入多少 layer, 這ㄔ只有 [128, 128, 128], 因此加入三層 Linear+ReLU

```
1 class SimpleNet(nn.Module):
2     def __init__(
3         self, dim_in: int, dim_out: int, dim_hids: List[int], num_timesteps: int
4     ):
5         super().__init__()
6         # (TODO) Build a noise estimating network.
7         # Args:
8         #   dim_in: dimension of input
9         #   dim_out: dimension of output
10        #   dim_hids: dimensions of hidden features
11        #   num_timesteps: number of timesteps
12        ##### TODO #####
13        # DO NOT change the code outside this part.
14        time_embed_dim = dim_hids[0] # get time_emb
15        self.MLP = nn.Sequential(
16            TimeEmbedding(hidden_size=time_embed_dim),
17            nn.Linear(time_embed_dim, time_embed_dim),
18            nn.ReLU(),
19        )
20        layers = []
21        current_dim = dim_in + time_embed_dim
22        for dim_hid in dim_hids:
23            layers.append(nn.Linear(current_dim, dim_hid))
24            layers.append(nn.ReLU())
25            current_dim = dim_hid
26        layers.append(nn.Linear(current_dim, dim_out))
27        self.main_net = nn.Sequential(*layers)
28        #####
```

Listing 1: SimpleNet Class Definition

- b. **forward**: 透過 forward 呼叫 SimpleNet model, 最後在進入 main_net 前, 要先 concat x 和 time_emb

```

1 def forward(self, x: torch.Tensor, t: torch.Tensor):
2     # (TODO) Implement the forward pass. This should output
3     # the noise prediction of the noisy input x at timestep t.
4     # Args:
5     #   x: the noisy data after t period diffusion
6     #   t: the time that the forward diffusion has been running
7     ##### TODO #####
8     # DO NOT change the code outside this part.
9     # x is (batch_size, 2)
10    # t is (batch_size,)
11    time_emb = self.MLP(t)
12    x_and_t = torch.cat((x, time_emb), dim=-1)
13    x = self.main_net(x_and_t)
14    return x
15    #####

```

Listing 2: Forward Pass Implementation

2. TODO2-ddpm.py:

- a. **q_sample**: 先算出 $\bar{\alpha}_t$, 對它開根號算出 $\sqrt{\bar{\alpha}_t}$, 接著利用 $\bar{\alpha}_t$ 算出 $\sqrt{1 - \bar{\alpha}_t}$, 透過前面算出的值和 noise, 計算出 x_t

```

1 def q_sample(self, x0, t, noise=None):
2     # sample x_t from q(x_t|x_0) of DDPM.
3     if noise is None:
4         noise = torch.randn_like(x0)
5
6     alphas_prod_t = extract(self.var_scheduler.alphas_cumprod, t, x0)
7     sqrt_alphas_prod_t = torch.sqrt(alphas_prod_t)
8     sqrt_one_minus_alphas_prod_t = torch.sqrt(1.0 - alphas_prod_t)
9     xt = sqrt_alphas_prod_t * x0 + sqrt_one_minus_alphas_prod_t * noise
10    return xt

```

Listing 3: q_sample Implementation

- b. **TODO3-p_sample**: 先算出需要用到參數, 接著

- i. 先使用 network.py 中實作的 $\hat{\epsilon}$ 容預測出 noise
- ii. 計算出 posterior mean
- iii. 和 iv. 一起, 若 t 大於 0, 利用 posterior variance 和 noise 計算出 x_{t-1} , 否則 $x_0 = \text{post_mean}$

```

1  @torch.no_grad()
2      def p_sample(self, xt, t):
3          """
4          One step denoising function of DDPM:  $x_t \rightarrow x_{t-1}$ .
5          Input:
6              xt (`torch.Tensor`): samples at arbitrary timestep t.
7              t (`torch.Tensor`): current timestep in a reverse process.
8          Ouput:
9              x_t_prev (`torch.Tensor`): one step denoised sample. ( $= x_{t-1}$ )
10         """
11         ##### TODO #####
12         # DO NOT change the code outside this part.
13         # compute x_t_prev.
14         if isinstance(t, int):
15             t = torch.full((xt.shape[0],), t, device=self.device, dtype=torch.long)
16         eps_factor = (1 - extract(self.var_scheduler.alphas, t, xt)) / (
17             1 - extract(self.var_scheduler.alphas_cumprod, t, xt)
18         ).sqrt()
19
20         beta_t      = extract(self.var_scheduler.betas,      t, xt)      #
21         _t          = t
22         alpha_t     = extract(self.var_scheduler.alphas,     t, xt)     #
23         _t = 1 - _t
24         alpha_bar_t = extract(self.var_scheduler.alphas_cumprod, t, xt) #
25         \bar{\alpha}_t
26         t_prev      = (t - 1).clamp(min=0)
27         alpha_bar_t_prev = extract(self.var_scheduler.alphas_cumprod, t_prev, xt) #
28         \bar{\alpha}_{t-1}
29
30         # 1. predict noise
31         predicted_noise = self.network(xt, t)
32         # 2. Posterior mean
33         post_mean = 1 / torch.sqrt(alpha_t) * (xt - eps_factor * predicted_noise)
34         # 3. Posterior variance
35         # 4. Reverse step
36         if t[0].item() > 0:
37             post_var = (1 - alpha_bar_t_prev) * beta_t / (1 - alpha_bar_t)
38             noise = torch.randn_like(xt)
39             x_t_prev = post_mean + torch.sqrt(post_var) * noise
40         else:
41             x_t_prev = post_mean
42
43         #####
44         return x_t_prev

```

Listing 4: p_sample Implementation

d. **TODO4-p_sample_loop**: iterate p_sample 來得到預測的 x_0

```

1 @torch.no_grad()
2     def p_sample_loop(self, shape):
3         """
4         The loop of the reverse process of DDPM.
5
6         Input:
7             shape (`Tuple`): The shape of output. e.g., (num particles, 2)
8         Output:
9             x0_pred (`torch.Tensor`): The final denoised output through the DDPM
10                reverse process.
11         """
12         ##### TODO #####
13         # DO NOT change the code outside this part.
14         # sample x0 based on Algorithm 2 of DDPM paper.
15         xt = torch.randn(shape).to(self.device)
16         T = self.var_scheduler.num_train_timesteps
17
18         from tqdm import tqdm
19         for i in tqdm(reversed(range(0, T))):
20             batch_size = xt.shape[0]
21             t = torch.full((batch_size,), i, device=self.device, dtype=torch.long)
22             xt = self.p_sample(xt, t)
23         x0_pred = xt
24         #####
25         return x0_pred

```

Listing 5: p_sample_loop Implementation

- e. **TODO5-compute_loss**: 先利用 x_0 和 t sample 出 x_t , 接著利用 network 預測在時間 t 時的 noise, 利用預測到的 noise, 和先前 random 的 noise 在 MSE, 算出 loss

```

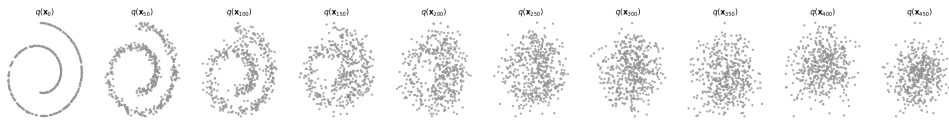
1 def compute_loss(self, x0):
2     """
3     The simplified noise matching loss corresponding Equation 14 in DDPM paper.
4     Input:
5         x0 (`torch.Tensor`): clean data
6     Output:
7         loss: the computed loss to be backpropagated.
8     """
9     ##### TODO #####
10    # DO NOT change the code outside this part.
11    # compute noise matching loss.
12    batch_size = x0.shape[0]
13
14    # 1) random choose timestep
15    t = (
16        torch.randint(0, self.var_scheduler.num_train_timesteps, size=(batch_size,))
17        )
18        .to(x0.device)
19        .long()
20
21    # 2) get GT noise, and use q_sample to get x_t
22    noise = torch.randn_like(x0)
23    x_t = self.q_sample(x0=x0, t=t, noise=noise)
24
25    # 3) predict noise
26    predicted_noise = self.network(x_t, t)
27
28    # 4) MSE loss (eps, eps_pred)
29    loss = F.mse_loss(noise, predicted_noise)
30
31    #####
32    return loss

```

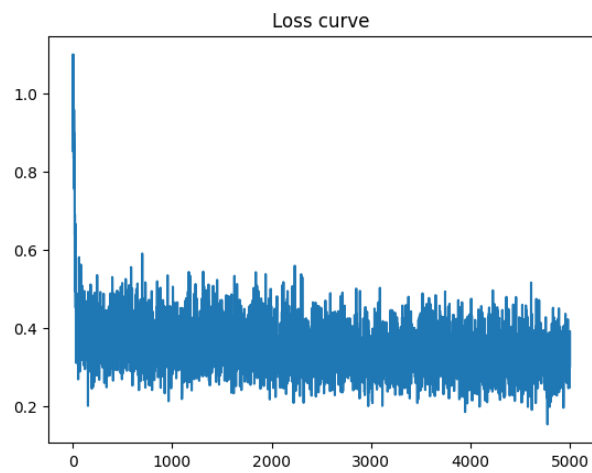
Listing 6: compute_loss Implementation

Task 1 (Result)

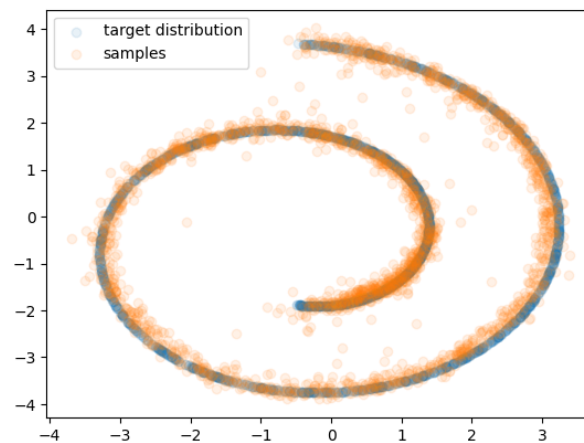
3. Loss Curve:



2. Loss Curve:



3. evaluation result:



Task 2 (Code)

1. TODO1-add_noise:

跟 task1 的 `q_sample` 一模一樣

```

1 def add_noise(
2     self,
3     x_0: torch.Tensor,
4     t: torch.IntTensor,
5     eps: Optional[torch.Tensor] = None,
6 ):
7     """
8     A forward pass of a Markov chain, i.e.,  $q(x_t | x_0)$ .
9
10    Input:
11        x_0 (`torch.Tensor [B,C,H,W]`): samples from a real data distribution  $q(x_0)$ .
12        t (`torch.IntTensor [B]`):
13        eps (`torch.Tensor [B,C,H,W]`, optional): if None, randomly sample
14            Gaussian noise in the function.
15    Output:
16        x_t (`torch.Tensor [B,C,H,W]`): noisy samples at timestep t.
17        eps (`torch.Tensor [B,C,H,W]`): injected noise.
18    """
19
20    if eps is None:
21        eps = torch.randn(x_0.shape, device='cuda')
22
23    ##### TODO #####
24    # DO NOT change the code outside this part.
25    # Assignment 1. Implement the DDPM forward step.
26    alphas_prod_t = extract(self.alphas_cumprod, t, x_0) # select the timesteps
27    # t and reshape them to match x0's dim
28    sqrt_alphas_prod_t = torch.sqrt(alphas_prod_t)
29    sqrt_one_minus_alphas_prod_t = torch.sqrt(1.0 - alphas_prod_t)
30    x_t = sqrt_alphas_prod_t * x_0 + sqrt_one_minus_alphas_prod_t * eps
31    #####
32
33    return x_t, eps

```

Listing 7: add_noise Implementation

2. TODO4-beta scheduling:

實作 Cosine Scheduler

i. 先依照公式算出 $\bar{\alpha}_t$, s 設成 0.008

ii. $\beta_t = 1 - \bar{\alpha}_t / \alpha_{t-1}$

iii. 回傳 betas

```

1  def __init__(
2      self, num_train_timesteps: int, beta_1: float, beta_T: float, mode="linear"
3  ):
4      super().__init__()
5      self.num_train_timesteps = num_train_timesteps
6      self.num_inference_timesteps = num_train_timesteps
7      self.timesteps = torch.from_numpy(
8          np.arange(0, self.num_train_timesteps)[::-1].copy().astype(np.int64)
9      )
10
11     if mode == "linear":
12         betas = torch.linspace(beta_1, beta_T, steps=num_train_timesteps)
13     elif mode == "quad":
14         betas = (
15             torch.linspace(beta_1**0.5, beta_T**0.5, num_train_timesteps) ** 2
16         )
17     elif mode == "cosine":
18         ##### TODO #####
19         # Implement the cosine beta schedule (Nichol & Dhariwal, 2021).
20         # Hint:
21         # 1. Define  $\alpha_t = f(t/T)$  where  $f$  is a cosine schedule:
22         #      $\alpha_t = \cos^2((t/T + s) / (1+s)) * (\pi/2)$ 
23         #     with  $s = 0.008$  (a small constant for stability).
24         # 2. Convert  $\alpha_t$  into betas using:
25         #      $\beta_t = 1 - \alpha_t / \alpha_{t-1}$ 
26         # 3. Return betas as a tensor of shape [num_train_timesteps].
27         s = 0.008
28         t = torch.linspace(0, 1, steps=num_train_timesteps+1)
29         f_t = torch.cos(((t / num_train_timesteps + s) / (1 + s)) * (torch.pi /
30             2)) ** 2
31         alpha_cumprod_t = f_t / f_t[0]
32         betas = 1 - alpha_cumprod_t[1:] / alpha_cumprod_t[:-1] # [1:T] / [1:T
33             -1] Noted:  $\alpha_{t=0} = 1$ 
34         betas = torch.clip(betas, 0.0001, 0.9999)
35     else:
36         raise NotImplementedError(f"{mode} is not implemented.")
37
38     alphas = 1 - betas
39     alphas_cumprod = torch.cumprod(alphas, dim=0)
40
41     self.register_buffer("betas", betas)

```



```

40     self.register_buffer("alphas", alphas)
41     self.register_buffer("alphas_cumprod", alphas_cumprod)

```

Listing 8: Cosine Beta Schedule

2. TODO5-predictor:

a. step_predict_noise:

i. 一樣先 extract 出需要用到的參數

ii. 利用參數計算出 mean

iii. 和 iv. 一起，如果 t 大於 0，利用 posterior variance 計算出 x_{t-1}

```

1  def step_predict_noise(self, x_t: torch.Tensor, t: int, eps_theta: torch.
    Tensor):
2      """
3      Noise prediction version (the standard DDPM formulation).
4
5      Input:
6          x_t: noisy image at timestep t
7          t: current timestep
8          eps_theta: predicted noise  $\hat{\epsilon}_\theta(x_t, t)$ 
9      Output:
10         sample_prev: denoised image sample at timestep t-1
11     """
12     ##### TODO #####
13     # 1. Extract beta_t, alpha_t, and alpha_bar_t from the scheduler.
14     # 2. Compute the predicted mean  $\bar{x}_t = 1/\sqrt{\alpha_t} * (x_t - (\epsilon_t/\sqrt{(1-\alpha_t)})) * \sqrt{\alpha_t}$ .
15     # 3. Compute the posterior variance  $\tilde{\alpha}_t = ((1-\alpha_{t-1})/(1-\alpha_t)) * \alpha_t$ .
16     # 4. Add Gaussian noise scaled by  $\sqrt{(\tilde{\alpha}_t)}$  unless  $t == 0$ .
17     # 5. Return the final sample at t-1.
18     if isinstance(t, int):
19         t = torch.full((x_t.shape[0],), t, device=self.device, dtype=torch.
            long)
20     eps_factor = (1 - extract(self.alphas, t, x_t)) / (
21         1 - extract(self.alphas_cumprod, t, x_t)
22     ).sqrt()
23
24     beta_t = extract(self.betas, t, x_t) #  $\beta_t$ 
25     alpha_t = extract(self.alphas, t, x_t) #  $\alpha_t = 1 - \beta_t$ 
26     alpha_bar_t = extract(self.alphas_cumprod, t, x_t) #  $\bar{\alpha}_t$ 
27     t_prev = (t - 1).clamp(min=0)
28     alpha_bar_t_prev = extract(self.alphas_cumprod, t_prev, x_t) #  $\bar{\alpha}_{t-1}$ 
29
30     # 1. predict noise

```

```

31     predicted_noise = eps_theta
32     # 2. Posterior mean
33     post_mean = 1 / torch.sqrt(alpha_t) * (x_t - eps_factor * predicted_noise)
34     # 3. Posterior variance
35     # 4. Reverse step
36     if t.item() > 0:
37         post_var = (1 - alpha_bar_t_prev) * beta_t / (1 - alpha_bar_t)
38         noise = torch.randn_like(x_t)
39         sample_prev = post_mean + torch.sqrt(post_var) * noise
40     else:
41         sample_prev = post_mean
42     #####
43     return sample_prev

```

Listing 9: Noise Predictor

b. step_predict_x0:

- i. 利用預測出來的 x_{0_pred} 來計算出 posterior mean
- ii. 若 t 大於 0，則使用 posterior variance 和 noise 計算出 x_{t-1} ，否則 $x_{t-1} = \text{post_mean}$

```

1  def step_predict_x0(self, x_t: torch.Tensor, t: int, x0_pred: torch.Tensor):
2      """
3      x0 prediction version (alternative DDPM objective).
4
5      Input:
6          x_t: noisy image at timestep t
7          t: current timestep
8          x0_pred: predicted clean image  $\hat{x}$  (x_t, t)
9      Output:
10         sample_prev: denoised image sample at timestep t-1
11     """
12     ##### TODO #####
13     if isinstance(t, int):
14         t = torch.full((x_t.shape[0],), t, device=self.device, dtype=torch.
15                        long)
16
17     beta_t      = extract(self.betas,      t, x_t)      # _t
18     alpha_t     = extract(self.alphas,     t, x_t)     # _t = 1 -
19         _t
20     alpha_bar_t = extract(self.alphas_cumprod, t, x_t)  # \bar{ }_t
21     t_prev      = (t - 1).clamp(min=0)
22     alpha_bar_t_prev = extract(self.alphas_cumprod, t_prev, x_t) # \bar{ }_{t
23         -1}
24
25     # 1. posterior mean
26     x0_coef = torch.sqrt(alpha_bar_t_prev) * beta_t / (1 - alpha_bar_t)
27     x_t_coef = torch.sqrt(alpha_t) * (1 - alpha_bar_t_prev) / (1 - alpha_bar_t
28         )
29     post_mean = x0_coef * x0_pred + x_t_coef * x_t

```

```

26     # 2. Reverse step
27     if t.item() > 0:
28         # 3. Posterior variance
29         post_var = (1 - alpha_bar_t_prev) * beta_t / (1 - alpha_bar_t)
30         noise = torch.randn_like(x_t)
31         sample_prev = post_mean + torch.sqrt(post_var) * noise
32     else:
33         sample_prev = post_mean
34     #####
35     return sample_prev

```

Listing 10: x_0 Predictor

c. step_predict_mean

i. t 大於 0 的話, 使用 posterior mean, mean_θ 和 noise 來計算 x_{t-1} , 否則 $x_{t-1} = \text{mean}_\theta$

```

1  def step_predict_mean(self, x_t: torch.Tensor, t: int, mean_theta: torch.
    Tensor):
2      """
3      Mean prediction version (directly outputting the posterior mean).
4
5      Input:
6          x_t: noisy image at timestep t
7          t: current timestep
8          mean_theta: network-predicted posterior mean  $\hat{\cdot}$  (x_t, t)
9      Output:
10         sample_prev: denoised image sample at timestep t-1
11     """
12     ##### TODO #####
13     if isinstance(t, int):
14         t = torch.full((x_t.shape[0],), t, device=self.device, dtype=torch.
            long)
15
16     beta_t      = extract(self.betas,          t, x_t)          # _t
17     alpha_t     = extract(self.alphas,         t, x_t)          # _t = 1 -
        _t
18     alpha_bar_t = extract(self.alphas_cumprod, t, x_t)          # \bar{ }_t
19     t_prev      = (t - 1).clamp(min=0)
20     alpha_bar_t_prev = extract(self.alphas_cumprod, t_prev, x_t) # \bar{ }_{t
        -1}
21
22     if t.item() > 0:
23         post_var = (1 - alpha_bar_t_prev) * beta_t / (1 - alpha_bar_t)
24         noise = torch.randn_like(x_t)
25         sample_prev = mean_theta + torch.sqrt(post_var) * noise
26     else:
27         sample_prev = mean_theta
28     #####
29     return sample_prev

```

Listing 11: Mean Predictor

- d. `get_loss_x0` sample a t , 然後使用這個 t 在 x_0 上加上 noise 得到 x_t , 利用 x_t 和 t 透過 network 預測出 x_0_pred , 接著使用 x_0_pred 和 x_0 算出 loss

```

1 def get_loss_x0(self, x0, class_label=None, noise=None):
2     ##### TODO #####
3     # Here we implement the "predict x0" version.
4     # 1. Sample a timestep and add noise to get (x_t, noise).
5     # 2. Pass (x_t, timestep) into self.network, where the output should
        represent the clean sample x0_pred.
6     # 3. Compute the loss as MSE(predicted x0_pred, ground-truth x0).
7     #####
8     B = x0.shape[0]
9     t = self.var_scheduler.uniform_sample_t(B, x0.device)
10    x_t, eps = self.var_scheduler.add_noise(x0, t, eps=noise)
11    x0_pred = self.network(x_t, t, class_label) if class_label is not None
        else self.network(x_t, t)
12    loss = F.mse_loss(x0_pred, x0)
13    return loss

```

Listing 12: Loss Function of x0

- e. `get_loss_mean`

- i. sample a t , 得到加噪過的 x_t , 使用 network 預測出 `mean_pred`
- ii. extract 出要使用的參數
- iii. 利用參數算出真實的 mean
- iv. 使用 `mean_pred` 和真實的 mean 計算出 loss

```

1 def get_loss_mean(self, x0, class_label=None, noise=None):
2     ##### TODO #####
3     # Here we implement the "predict mean" version.
4     # 1. Sample a timestep and add noise to get (x_t, noise).
5     # 2. Pass (x_t, timestep) into self.network, where the output should
        represent the posterior mean (x_t, t).
6     # 3. Compute the *true* posterior mean from the closed-form DDPM formula (
        using x0, x_t, noise, and scheduler terms).
7     # 4. Compute the loss as MSE(predicted mean, true mean).
8     #####
9     B = x0.shape[0]
10    t = self.var_scheduler.uniform_sample_t(B, x0.device)
11    x_t, eps = self.var_scheduler.add_noise(x0, t, eps=noise)
12    mean_pred = self.network(x_t, t, class_label) if class_label is not None
        else self.network(x_t, t)
13
14    beta_t = extract(self.var_scheduler.betas, t, x0)
15    alpha_t = extract(self.var_scheduler.alphas, t, x0)

```

```

16     alpha_bar_t      = extract(self.var_scheduler.alphas_cumprod, t, x0)
17     t_prev          = (t - 1).clamp(min=0)
18     alpha_bar_t_prev = extract(self.var_scheduler.alphas_cumprod, t_prev, x0)
19
20     x0_coef = torch.sqrt(alpha_bar_t_prev) * beta_t / (1 - alpha_bar_t)
21     x_t_coef = torch.sqrt(alpha_t) * (1 - alpha_bar_t_prev) / (1 - alpha_bar_t
22     )
23     mean = x0_coef * x0 + x_t_coef * x_t
24     loss = F.mse_loss(mean_pred, mean)
25     return loss

```

Listing 13: Loss Function of x0

3. Training:

我修改`-train_num_steps=50000`, `-log_interval=2500`, 來 reduce 訓練的時間

```

1 parser.add_argument(
2     "--train_num_steps",
3     type=int,
4     default=50000, #50000, #100000 # 100000 -> 50000
5     help="the number of model training steps.",
6 )
7
8 parser.add_argument("--log_interval", type=int, default=2500) # 200 -> 2500

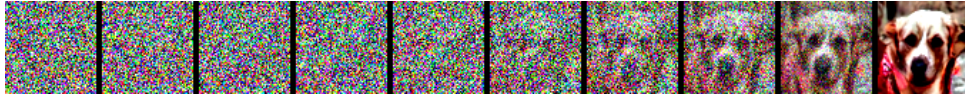
```

Listing 14: Training Arguments

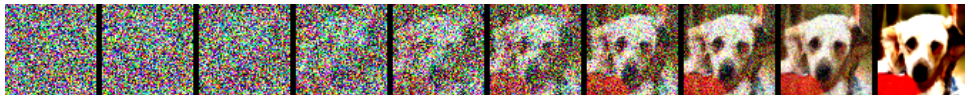
Task 2 (Result)

1. trajectory fig

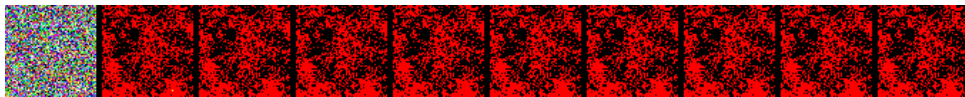
i. `-mode = linear, -predictor = noise`



ii. `-mode = quad, -predictor = noise`

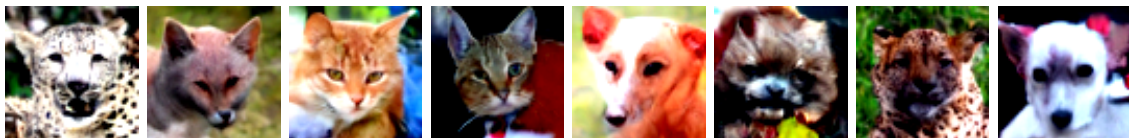


iii. `-mode = cosine, -predictor = noise`

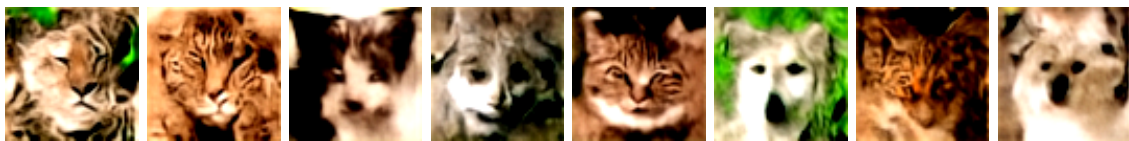


2. results fig of different predictor

i. `-mode = linear, -predictor = noise`



ii. `-mode = linear, -predictor = x0`



iii. `-mode = linear, -predictor = mean`

