16 Astronauts on the 1971 Apollo 14 mission to the Moon brought back many rock samples. It is now believed that one of these contains a piece of rock that originated on Earth about 4 billion years (4×10^9 years) ago.

The piece of rock is believed to have been launched into space when an asteroid struck the Earth.

(a) The rock sample contains uranium. The radioactive decay of uranium allows it to be used to determine the time since the rock was formed on the Earth.

(i) The uranium isotope $^{238}_{92}U$ becomes the lead isotope $^{206}_{82}Pb$ through a series of radioactive decays.

Calculate the number of α particles and the number of β particles emitted for one nucleus of $^{238}_{92}U$ to decay to become a nucleus of $^{206}_{82}Pb$.

(2)

Number of α particles = ...

Number of β particles = ...

(ii) The half-life of $^{238}_{92}U$ is 4.47×10^9 years.

The half-lives of the other stages in the decay to $^{206}_{82}Pb$ are relatively so short that they can be ignored.

There was no lead in the rock when it formed, so all the $^{206}_{82}Pb$ in the sample is a product of $^{238}_{92}U$ decay. In the sample, for every 103 uranium nuclei present at the start, 50 are now lead nuclei.

Show that the age of the sample is about 4×10^9 years.

(3)

(b) The gravitational potential between the Earth and the Moon due to the combined effect of their gravitational fields increases to a maximum value of $-1.28\,MJ\,kg^{-1}$ at a point between them.

Calculate the minimum speed at which a rock would have to leave the Earth in order to reach the Moon.

In your calculation, you may assume the rock has zero kinetic energy when it has maximum potential energy.

mass of Earth = 5.97×10^{24} kg
radius of Earth = 6370 km

(4)

Minimum speed = ...

(c) Four billion years ago, the Moon had a different orbital period, because it was closer to the Earth than it is today.

Calculate the period of the Moon's orbit four billion years ago, when the radius of its orbit was 1.34×10^8 m.

mass of Earth = 5.97×10^{24} kg

(3)

Period = ...

(Total for Question 16 = 12 marks)