Optimal management of nurses' shifts

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Abstract

Healthcare facilities are struggling in fighting the spread of COVID 19. While machines needed for patients such as ventilators can be built or bought, healthcare personnel such as nurses is a very scarce resource that cannot be increased by a hospital in a short period. This article has the scope to provide healthcare facilities with some simple mathematical formulations for scheduling the nurses' shifts within a department so to improve efficiency and reduce stress for the healthcare personnel. Numerical results carried on synthetic data show the effectiveness of the formulation here introduced. Moreover, the models described here are implemented in Python and made available to all those facilities struggling for this emergence.

1 Introduction

The spread of COVID19 is requiring an unpredictable effort by healthcare facilities to assist serious patients in recovering. The healthcare system in the North of Italy is collapsing and many other centers might end up in similar conditions very soon. The steep increase of serious patients who need particular care is difficult to be satisfied by the medical staff. Resources are limited and hospitals are not ready to deal with a similar pandemic scenario. While machines such as ventilators can be purchased (assumed their availability), the resource more scarce right now is represented by highly skilled personnel such as doctors and nurses able to take care of infected patients.

In this optic, Operations Research comes to the rescue by offering automated systems to help in managing nurses' shifts. This will provide healthcare facilities with an intelligent-automated system able to minimize the effort and the stress for nursing personnel while guaranteeing the satisfaction of the management constraints in the department.

In this work, we provide some flexible formulations able to deal with different schedule settings. In section 2 we define the basic optimization model. Then in section 3 we further extend the model to consider more complex scenarios. Finally, in section 4 we analyze the performance of the model under different settings. All the formulations defined here are implemented using open-source libraries except for the optimization routine. The code is made available at the following public github repository: `https://github.com/RuggieroSeccia/Nursess-shifts-management`

2 Formulation

Let us consider a department in a hospital with a given number of nurses N. We want to organize their shifts for the next T days, e.g. $T = 7$ one week or $T = 30$ next month, and for all the followings so to minimize the effort required by the staff to satisfy the demand. By contract, each nurse i has to work H_i hours over the time horizon T (e.g. each nurse must work at least 36 hours per week, $H = 36$ and $T = 7$). If the ith nurse works for a number of hours higher than H_i, then it is counted as extra work and then paid more by the healthcare structure. Each day three shifts need to be covered by the nurses: morning, afternoon and night. Each shift s requires R_s nurses and lasts h_s hours. Each nurse cannot cover more than one shift per day. Moreover, we have the further constraint that if a nurse covers a night shift then they need to rest and cannot work the following day.

To formulate this optimization problem, let us introduce the binary variable $x_{ist} \in \{0, 1\}$ such that

$$x_{ist} = \begin{cases} 1 & \text{if nurse } i\text{th covers shift } s\text{th on day } t\text{th} \\ 0 & \text{otherwise} \end{cases}$$

Moreover, let us consider the parameter p_i which brings information about the previous period. Namely, p_i is a boolean parameter such that

$$p_i = \begin{cases} 1 & \text{if the } i\text{th nurse worked on the last day of the previous period} \\ 0 & \text{otherwise.} \end{cases}$$

We want to find the optimal schedule $x^\star$ such that the number of hours worked by nurses is minimized and all the department's constraints are satisfied.

This management problem can be formulated as a linear binary optimization problem [1]:

$$\min_{x_{ist}\in\{0,1\}} \quad \sum_{i=1}^{N}\sum_{s=1}^{3}\sum_{t=1}^{T} x_{ist}h_s \tag{1}$$

$$\text{subject to} \quad \sum_{s=1}^{3} x_{ist} \leq 1 \qquad \forall i = 1, ..., N \; t = 1, ..., T \tag{1a}$$

$$\sum_{i=1}^{N} x_{ist} \geq R_s \qquad \forall s = 1, ..., 3 \; t = 1, ..., T \tag{1b}$$

$$\sum_{s=1}^{3}\sum_{t=1}^{T} x_{ist}h_s \geq H_i \qquad \forall i = 1, ..., N \tag{1c}$$

$$x_{i3t} + \sum_{s=1}^{3} x_{ist+1} \leq 1 \qquad \forall i = 1, ...N \; t = 1, ..., T-1 \tag{1d}$$

$$\sum_{s=1}^{3} x_{is1} \leq (1 - p_i) \qquad \forall i = 1, ..., N. \tag{1e}$$

The objective function (1) is asking to minimize the overall number of hours worked by all nurses within the period under consideration. Note that if $\sum_{s=1}^{3}\sum_{t=1}^{T} h_s x_{ist} > H$, then the nurse ith is working $\sum_{s=1}^{3}\sum_{t=1}^{T} h_s x_{ist} - H$ extra hours. That is, by minimizing the objective function (1), we are actually minimizing the number of extra work required to each nurse.

Concerning the constraints, constraint (1a) implies that each person cannot cover more than one shift on the same day. Constraint (1b) requires that the number of personnel per each shift in each day is satisfied. Constraint (1c) requires that each nurse works at minimum the number of hours required by contract. Finally, constraint (1d) implies that if a nurse covers a night shift, then the next day they cannot work, and constraint (1e) implies that each nurse cannot work on the first day of the new period if they worked on the last day of the previous period.

Note that most of the parameters in the model can be rearranged to deal with different situations. E.g. in a chaotic scenario with many changes happening in a short amount of time, we might need to make schedule day by day. In this optic, this model can be easily rearranged by modifying the value of H_i. For instance, if we are scheduling day by day and the ith nurse has to work $\tilde{H}_i = 36$ hours per week and has already worked $w_i = 20$ hours this week, then they still need to work $H_i = \tilde{H}_i - w_i = 16$ hours.

3 Formulation extensions

The model defined so far represents a very easy representation of a scheduling problem within a department. This model can be extended in several ways. In the following we formally define some of these extensions.

3.1 Number of nurses

We can generalize the model so to include the possibility that the number of nurses N is not enough to satisfy all the constraints. Note, indeed, that if the number of nurses N is not big enough, then constraint (1b) cannot be satisfied and the problem is unfeasible. To include this possibility, we can introduce the variable $\alpha_{st} \in R$ which represents the number of nurses that are missing to satisfy the minimum demand of the sth shift on the tth department. Then we can change the objective function so to add the term $\rho \sum_{s=1}^{3} \sum_{t=1}^{T} \alpha_{st}$ with $\rho > \max_s h_s$ and modify constraint (1c) as folllows:

$$\sum_{i=1}^{N} x_{ist} + \alpha_{st} \geq R_s \qquad \forall s = 1, ..., 3\ t = 1, ..., T$$

Once the optimization problem is solved and the optimal schedule $x^\star$ is found, $\max_{st} \alpha^\star_{st}$ will give us an estimate of the number of nurses that need to be added to satisfy the demand of the department.

With this modification the problem can be formalized as follows:

$$\min_{x_{ist} \in \{0,1\ \alpha_{st} \in R} \quad \sum_{i=1}^{N} \sum_{s=1}^{3} \sum_{t=1}^{T} x_{ist} h_s + \rho \sum_{s=1}^{3} \sum_{t=1}^{T} \alpha_{st} \tag{2}$$

$$\text{subject to} \quad \sum_{s=1}^{3} x_{ist} \leq 1 \qquad \forall i = 1, ..., N\ t = 1, ..., T \tag{2a}$$

$$\sum_{i=1}^{N} x_{ist} + \alpha_{st} \geq R_s \qquad \forall s = 1, ..., 3\ t = 1, ..., T \tag{2b}$$

$$\sum_{s=1}^{3} \sum_{t=1}^{T} x_{ist} h_s \geq H_i - p_i h_3 \qquad \forall i = 1, ..., N \tag{2c}$$

$$x_{i3t} + \sum_{s=1}^{3} x_{ist+1} \leq 1 \qquad \forall i = 1, ...N\ t = 1, ..., T-1 \tag{2d}$$

$$\sum_{s=1}^{3} x_{is1} \leq (1 - p_i) \qquad \forall i = 1, ..., N \tag{2e}$$

with $\rho > \max_s h_s$. Note that, even if α represents a discrete quantity, it is modeled as a continuous variable since at the optimum it will achieve only integer values.

3.2 Worst-case scenario

Instead of minimizing the sum of hours overall worked by all the nurses, we are interested in minimizing the worst-case scenario, i.e. minimize the maximum number of hours worked by the nurse that works the most. To do that, we can introduce the further continuous variable $y \in R$, rewrite the objective function as

$$\min_{x_{ist} \in \{0,1\}, y \in R} \quad y + \sum_{s=1}^{3} \sum_{t=1}^{T} \alpha_{st}$$

and introduce the set of constraints:

$$y \geq \sum_{s=1}^{3} \sum_{t=1}^{T} x_{ist} h_s \qquad \forall i = 1, ..., N$$

3.3 Allowing half shifts

Consider the possibility that each nurse can work more than one shift per day but up to $H^{\max}$ hours per period. This aspect can be modeled by allowing the nurse to work before or after entering the assigned shift. To this aim, we assume that a nurse can work half a shift before or after entering their proper shift. We introduce the two additional integer variables $z_{ist}, q_{ist} \in \{0, 1\}$ such that:

$$z_{ist} = \begin{cases} 1 & \text{if nurse } i\text{th works the first half of the shift } s\text{th on day } t\text{th} \\ 0 & \text{otherwise.} \end{cases}$$

$$q_{ist} = \begin{cases} 1 & \text{if nurse } i\text{th works the second half of the shift } s\text{th on day } t\text{th} \\ 0 & \text{otherwise.} \end{cases}$$

In the following, when not specified the indices i, s, t will range over all their potential values.

Assuming that the additional hours are joined to a proper shift, we must add the following constraints:

$$z_{ist} \leq x_{is-1t} \qquad q_{ist} \leq x_{is+1t} \qquad \forall i, s, t \tag{3}$$

where $x_{i0t} = x_{i3t-1}$ and $x_{i4t} = x_{i1t+1}$. Note that x_{i01} is known from the schedule of the previous week, while x_{i4T} is assumed to be zero (i.e. nobody works on the first day of the next period). Moreover, we need to consider also that nurses cannot work half of the shift sth on the day tth if they have already been assigned to that shift:

$$z_{ist} \leq 1 - x_{ist} \qquad q_{ist} \leq 1 - x_{ist}$$

Then, we can consider a maximum number of hours $H^{\max}$ that a nurse can work during the period T without burning out:

$$\sum_{s=1}^{3} \sum_{t=1}^{T} h_s \left(x_{ist} + \frac{1}{2} z_{ist} + \frac{1}{2} q_{ist} \right) \leq H^{\max} \qquad \forall\ i = 1, ..., N$$

We might further add constraints to avoid that the extra hours of the shift are in consecutive days

$$\sum_{s=1}^{3} \left(z_{ist} + z_{is(t+1)} + q_{ist} + q_{is(t+1)}\right) \leq 1 \quad \forall i = 1, ..., N \ t = 1, ..., T-1$$

Note that this constraint implies that each nurse cannot work more than one extra shift per day.

Constraint (2b) needs also to be changed and split into two constraints for the first and second half of the shift

$$\begin{aligned} &\sum_{i=1}^{N} (x_{ist} + z_{ist}) \geq R_s \qquad \forall s = 1, ..., 3 \ t = 1, ..., T \\ &\sum_{i=1}^{N} (x_{ist} + q_{ist}) \geq R_s \qquad \forall s = 1, ..., 3 \ t = 1, ..., T \end{aligned}$$

We finally add the constraint that the number of nurses covering the first and the second half of each shift, have to be the same. Indeed this allows us to define the variable α_{st} as continuous and not as discrete similarly to the previous case. In particular, we add that:

$$\sum_{i=1}^{N} z_{ist} = \sum_{i=1}^{N} q_{ist} \qquad \forall s, t \tag{4}$$

In this case, the objective function could be the proper hours worked by each nurse plus the extra shift working hours

$$\min_{x_{ist}, z_{ist} \in \{0,1\}} \quad \sum_{i=1}^{N} \sum_{s=1}^{3} \sum_{t=1}^{T} \left(h_s x_{ist} + \frac{1}{2} h_s z_{ist} + \frac{1}{2} h_s q_{ist}\right)$$

Instead of minimizing the sum of these values, we could also minimize the worst-case scenario, namely:

$$\min_{x_{ist}, z_{ist} \in \{0,1\}} \quad \max_{i} \left\{\sum_{i=1}^{N} \sum_{s=1}^{3} \sum_{t=1}^{T} \left(h_s x_{ist} + \frac{1}{2} h_s z_{ist} + \frac{1}{2} h_s q_{ist}\right)\right\}$$

Finally, the optimization problem to solve in order to determine the optimal schedule can be written as:

$$
\begin{aligned}
\min_{x_{ist}, z_{ist}} \quad & \sum_{i=1}^{N}\sum_{s=1}^{3}\sum_{t=1}^{T}\left(h_s x_{ist} + \frac{1}{2}h_s z_{ist} + \frac{1}{2}h_s q_{ist}\right) + \rho\sum_{s=1}^{3}\sum_{t=1}^{T}\alpha_{st} && (5)\\
\text{s.t.} \quad & \sum_{s=1}^{3} x_{ist} \le 1 \qquad \forall i,t && (5a)\\
& \sum_{i=1}^{N}(x_{ist} + z_{ist}) + \alpha_{st} \ge R_s \qquad \forall s,t && (5b)\\
& \sum_{i=1}^{N}(x_{ist} + q_{ist}) + \alpha_{st} \ge R_s \qquad \forall s,t && (5c)\\
& \sum_{s=1}^{3}\sum_{t=1}^{T} h_s\left(x_{ist} + \frac{1}{2}z_{ist} + \frac{1}{2}q_{ist}\right) \le H^{\max} \qquad \forall\ i,s,t && (5d)\\
& \sum_{s=1}^{3}\sum_{t=1}^{T} x_{ist}h_s \ge H_i \qquad \forall i && (5e)\\
& x_{i3t} + \sum_{s=1}^{3} x_{ist+1} \le 1 \qquad \forall i,t && (5f)\\
& \sum_{s=1}^{3} x_{is1} \le (1-p_i) \qquad \forall i && (5g)\\
& \sum_{s=1}^{3}\left(z_{ist} + z_{is(t+1)} + q_{ist} + q_{is(t+1)}\right) \le 1 \quad \forall i\ t = 1,...,T-1 && (5h)\\
& z_{ist} \le 1 - x_{ist} \qquad \forall i,s,t && (5i)\\
& q_{ist} \le 1 - x_{ist} \qquad \forall i,s,t && (5j)\\
& \sum_{i=1}^{N} z_{ist} = \sum_{i=1}^{N} q_{ist} \qquad \forall s,t && (5k)\\
& z_{ist} \le x_{is-1t} \qquad \forall i,s,t && (5l)\\
& q_{ist} \le x_{is+1t} \qquad \forall i,s,t && (5m)\\
& x_{ist}, z_{ist} \in \{0,1\}, \alpha_{st} \in R && (5n)
\end{aligned}
$$

Note that thanks to constraint (4), we can define α as a continuous value, leaving the problem a mixed binary integer programming program. Indeed, non-integer values of α would be sub-optimal or non-feasible.

4 Implementation Details

In this section, we report and discuss the numerical results on some synthetic problems when considering formulation (5) and when changing the objective function so to minimize the worst case scenario, similarly at what done in section 3.2. The first formulation is denoted as **SUM** while the second as **W-S**. We analyze the solution found when changing the size of the department (i.e. the number of nurses N) and the time horizon T with all the other parameters fixed. In particular, we set

$$H_i = 6T \qquad H^{\max} = 10T \qquad \forall i,$$

$$x_{i4T} = 0 \qquad x_{i01} = 0 \qquad \forall i$$

$$R = [5, 4, 3] \qquad h = [7, 8, 9] \qquad p = [1, 0, ..., 0].$$

The code has been implemented in Python with Jupyter notebooks by leveraging well-known open-source libraries, like `pandas` and `numpy`. The optimization model has been developed in Python with the IBM Decision Optimization `docplex` APIs, using IBM ILOG CPLEX 12.9. The numerical results are run over a Dell XPS 15 9570 with an Intel i7-8750H at 4.41GHz and a RAM of 16 GB.

In Table 1 for each experiment we report: the running time in seconds; a boolean which specify whether the algorithm finds the integer optimum or not; x_M which represents the maximum number of hours worked by a nurse considering the regular shifts ($\max_i \sum_{s,t} x_{ist} h_s$); z_q_M which represents the maximum number of extra hours worked by a nurse ($\max_i \sum_{s,t} \frac{1}{2} h_s(z_{ist} + q_{ist})$); x_z_q which represents the maximum number of hours worked overall by a nurse ($\max_i \sum_{s,t} x_{ist} h_s \frac{1}{2} h_s(z_{ist} + q_{ist})$); the maximum number of nurses missing to satisfy all the constraints ($\alpha_M = \max_{st} \alpha$).

When considering small departments, $N \leq 15$, formulation **SUM** is always able to find optimal solutions within 2 seconds of computation while **W-S** in some cases stops without finding an optimal solution (maximum running time has been set to 240 seconds). However, the solutions returned by **W-S** are slightly better considering the worst case scenario. On the other hand, for bigger departments, $N \geq 20$, solutions found with the two formulations are the same, with **W-S** able to find solutions in slightly smaller running times.

5 Conclusion

In this work we have introduced some simple scheduling formulations that can be easily implemented to improve the efficiency and reduce stress for nurses working within a department. All the formulations proposed are described and implemented. The resulting frameworks are made available on github to all those interested in this topic. Further modifications to the problem can be included so to consider more complicated shift rules and settings. Numerical results show the effectiveness on the frameworks and their ability to solve in very short amount of time the scheduling problem under consideration.

Table 1: Results comparing formulation (5) and its worst-scenario modification when changing the size of the department and the time horizon.

N	T	Prob	Cpu Time (s)	Optimal	x_M	z_q_M	x_z_q	α_M
5	7	**SUM**	**0.42**	1	52	18	67	4
		W-S	0.45	1	**51**	18	**64**	4
5	14	**SUM**	**0.30**	1	**102**	32	134	4
		W-S	2.49	1	103	**27**	**125**	4
5	30	**SUM**	**1.07**	1	220	63	283	4
		W-S	3.33	1	**219**	**59**	**269**	4
10	7	**SUM**	**0.26**	1	54	18	70	0
		W-S	0.97	1	**53**	18	**67**	0
10	14	**SUM**	**0.26**	1	109	32	140	0
		W-S	2.09	1	**101**	32	**132**	0
10	30	**SUM**	**1.28**	**1**	228	68	295	0
		W-S	240.11	0	**217**	68	**284**	0
15	7	**SUM**	**2.47**	**1**	47	0	47	0
		W-S	240.06	0	**45**	0	**45**	0
15	14	**SUM**	**0.55**	1	89	9	98	0
		W-S	89.72	1	**88**	**4**	**88**	0
15	30	**SUM**	**0.96**	1	192	18	199	0
		W-S	111.27	1	**181**	**8**	**188**	0
20	7	**SUM**	1.98	1	42	0	42	0
		W-S	**0.13**	1	42	0	42	0
20	14	**SUM**	0.40	1	84	0	84	0
		W-S	**0.20**	1	84	0	84	0
20	30	**SUM**	1.41	1	180	0	180	0
		W-S	**0.45**	1	180	0	180	0
25	7	**SUM**	1.81	1	42	0	42	0
		W-S	**0.15**	1	42	0	42	0
25	14	**SUM**	0.58	1	84	0	84	0
		W-S	**0.25**	1	84	0	84	0
25	30	**SUM**	1.31	1	180	0	180	0
		W-S	**0.50**	1	180	0	180	0

References

[1] Michele Conforti, Gérard Cornuéjols, Giacomo Zambelli, et al. *Integer programming*, volume 271. Springer, 2014.