Projection of Lines

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PROJECTIONS OF LINES

A straight line is the shortest distance between any two points in space. A line represents the locus of a point moving along a fixed path in space. As a line consists of a number of points, its projections are drawn by joining the projections of any two points on the line (preferably the end points) or on the extension of the line.

POSITIONS OF STRAIGHT LINES

1. Line parallel to both the RPs

    Case (i): Line away from both the RPs

    Case (ii): Line in the HP and away from the VP

    Case (iii): Line in the VP and away from the HP

    Case (iv): Line contained by both the RPs

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2. Line perpendicular to either of the RPs

   Case (i): Line perpendicular to the HP and away from the VP

   Case (ii): Line perpendicular to the HP and in the VP

   Case (iii): Line perpendicular to the VP and away from the HP

   Case (iv): Line perpendicular to the VP and in the HP

3. Line inclined to one RP and parallel to the other

   Case (i): Line inclined to the HP, parallel to the VP and away from the VP

   Case (ii): Line inclined to the HP and in the VP

   Case (iii): Line inclined to the VP, parallel to the HP and away from the HP

   Case (iv): Line inclined to the VP and in the HP

4. Line inclined to both the RPs (Oblique line)

   Case (i): *ha* less than *hb* and *da* less than *db*

   Case (ii): *ha* less than *hb* and *da* greater than *db*

   Case (iii): *ha* greater than *hb* and *da* less than *db*

   Case (iv): *ha* greater than *hb* and *da* greater than *db* (*A* and *B* being the two ends of the line.)

5. Line parallel to (or contained by) the PP

Case (i): Line parallel to the PP and the HP

Case (ii): Line parallel to the PP and the VP

Case (iii): Line parallel to the PP and inclined to the HP and the VP

TERMS USED IN PROJECTIONS OF LINES

True Length (TL) The actual length of a line is called its *true length*.

Plan Length (PL) or Top View Length The apparent length of a line seen in TV is called the *plan length* or *top view length*.

Elevation Length (EL) or Front View Length The apparent length of a line seen in FV is called the *elevation length* or *front view length*.

Side View Length (SVL) The apparent length of a line seen in SV is called its *side view length*.

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Inclination with the HP (θ) It is the true angle that a line makes with its projection on the HP. It is indicated by θ.

Inclination with the VP (Φ) It is the true angle that a line makes with its projection on the VP. It is indicated by Φ.

Apparent Angle with the HP (α) It is the angle which an oblique line seems to be making with the HP in FV. It is the angle between FV and *XY*. It is indicated by α.

Apparent Angle with the VP (β) It is the angle which an oblique line seems to be making with the VP in TV. It is the angle between TV and *XY*. It is indicated by β.

Horizontal Trace (HT) The point of intersection of the line (or its extension) with the HP is called the *horizontal trace* of the line.

Vertical Trace (VT) The point of intersection of the line (or its extension) with the VP is called the *vertical trace* of the line.

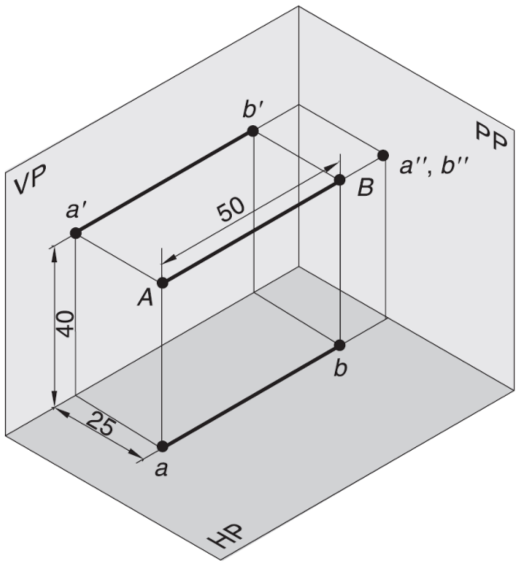
Point View of the Line The view of a line seen as a point (i.e., when the views of two ends coincide) is called the *point view*.

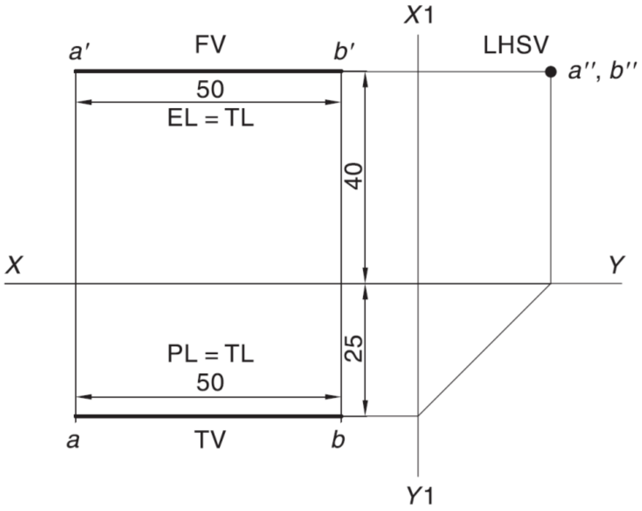
LINE PARALLEL TO BOTH THE RPs

Example 1 Draw the projections of a line AB that is 50 mm long and is parallel to both the HP and the VP. The line is 40 mm above the HP and 25 mm in front of the VP.

Solution:

The line AB is parallel to both the VP and the HP. So, its FV and TV, will be parallel to XY and both will show the TL (= 50 mm). If a line is parallel to both the HP and the VP, it must be perpendicular to the PP. Therefore, point A and point B will coincide in SV of the line. Hence, the LHSV is a point view.



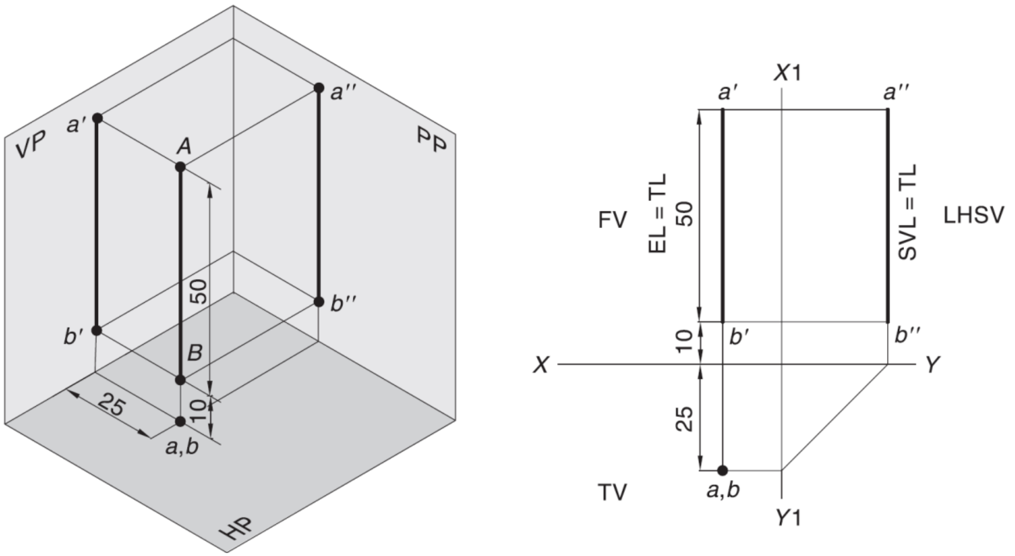


Example 2 A line AB, 50 mm long, is perpendicular to the HP and 25 mm in front of the VP. Draw its projections if the end nearest to the HP is 10 mm above the HP.

Solution:

If a line is perpendicular to the HP, it will automatically be parallel to the VP and PP. Hence, its FV will show the TL. TV will be a point view.

Wherever two (or more) points overlap, the visible point should be marked first. For example, in Fig. TV is marked as a, b. It means that a is visible and b is hidden. The hidden point(s) may be enclosed in parenthesis ( ), e.g., a(b).



PROJECTIONS OF LINES

Example 3 A line AB, 50 mm long, is inclined to the HP at 30° and parallel to the VP. The end nearest to the HP is 40 mm above it and 25 mm in front of the VP. Draw the projections.

Solution:

As the line AB is inclined to the HP and parallel to the VP, its FV will show TL.

1. Assuming that the line AB is initially parallel to both the RPs, draw its FV a'b1' and TV ab1 as shown. a'b1' is 40 mm above XY and ab1 25 mm below XY. a'b1'= ab1 = TL = 50 mm.

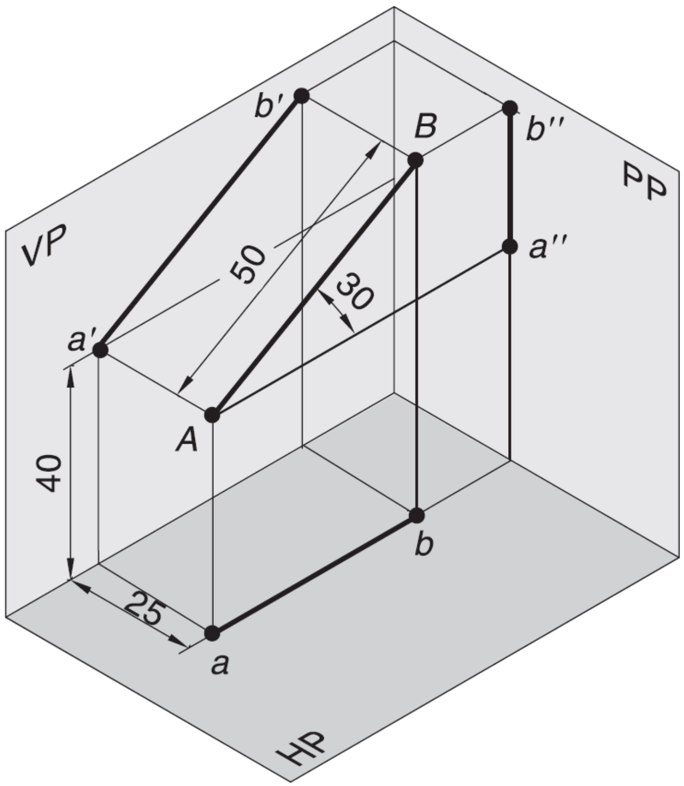
2. Rotate a'b1' about a' through 30° to get a'b'. a'b'= a'b1'= TL. The angle made by a'b' with XY represents θ.

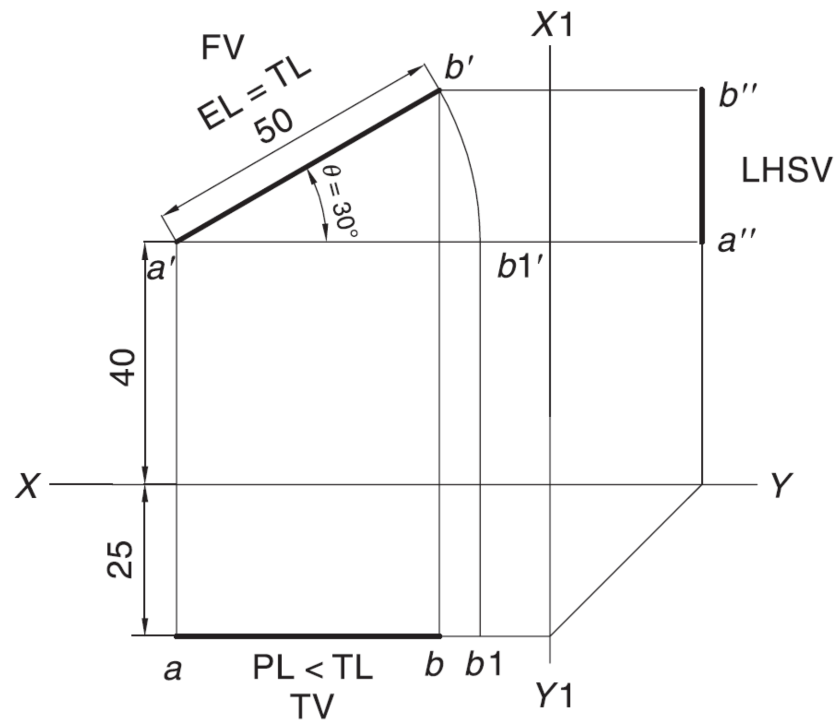
3. Project b' below XY to obtain b on ab1.

a'b' and ab represent respectively FV and TV of the line. Note that a'b' (= EL) is equal to TL and ab(= PL) is shorter than TL.

4. Obtain LHSV a" b" by projecting a'b' and ab with respect to X1 Y1.

PROJECTIONS OF LINES



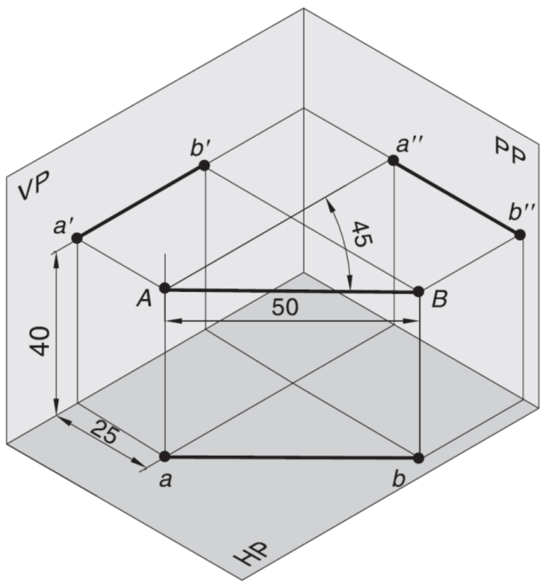


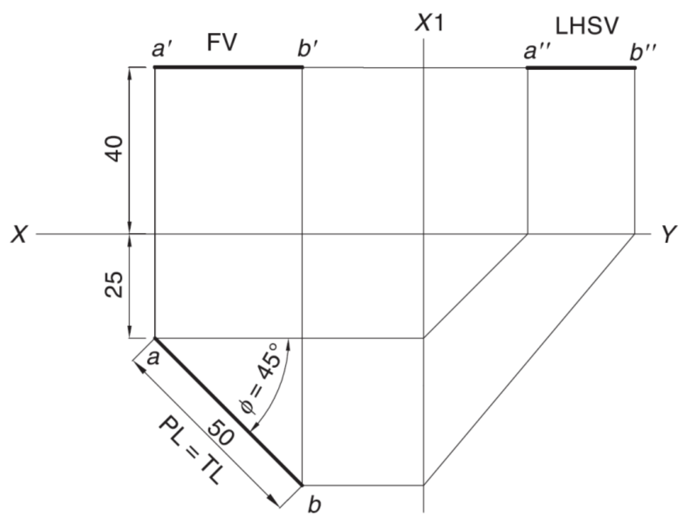
Example 4 A line AB, 50 mm long, is inclined at 45° to the VP and parallel to the HP. The nearest end of the line is 25 mm in front of the VP. Draw the projections of the line if it is 40 mm above the HP.

Solution:

1. Draw TV ab = TL, inclined at 45° to XY. a is 25 mm below XY.

2. Project ab above XY to obtain a’b’. ab and a’b’ represent respectively TV and FV of the line.





LINE INCLINED TO BOTH THE RPs (OBLIQUE LINE)

We have seen that if a line is inclined to the HP, its TV will be shorter than the TL and if a line is inclined to the VP, its FV will be shorter than the TL. Therefore, if a line is inclined to both the RPs, its TV and FV will be shorter than TL. Obviously, its true inclinations θ and ø will not be visible in FV and TV. Instead of the true inclinations, FV and TV will show apparent inclinations with the HP and the VP, i.e., α and β respectively.

The major concern in the projections of the oblique line is to obtain its apparent inclinations, α and β. This can be achieved in two stages as mentioned below.

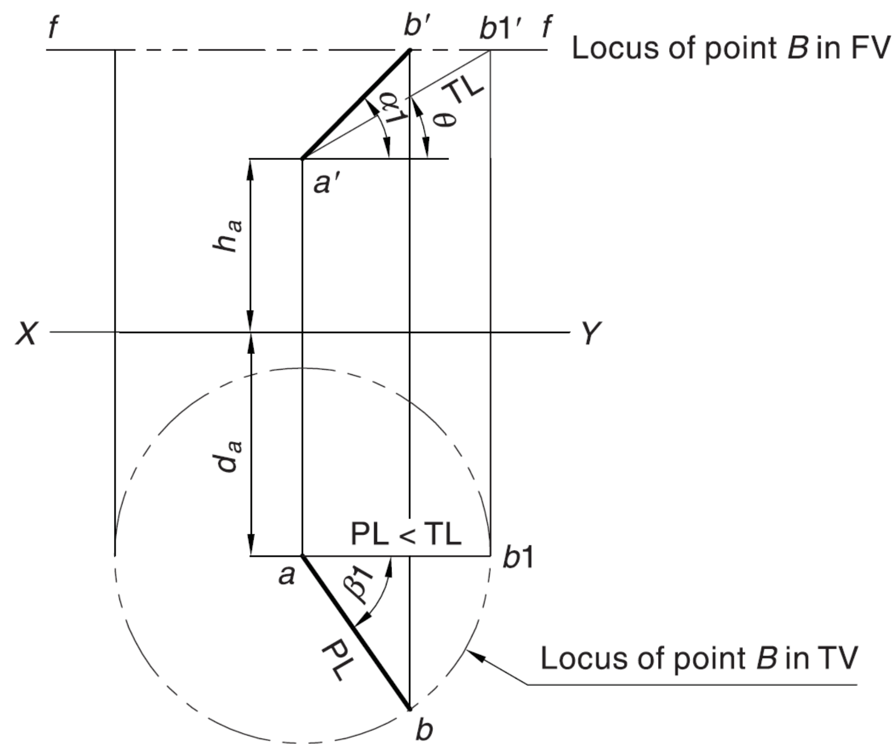
Stage 1

Assuming that a line (say *AB*1 = TL) is inclined to the HP at $\theta°$ and parallel to the VP, draw its projections *a'*–*b*1' and *ab*1. *ab*1 will show PL. If the line is made inclined to the VP at any angle (keeping θ unchanged) then TV will no longer remain parallel to *XY*. It will get tilted through a specific angle. In such a case, if the TV is tilted about one of its endpoints (say *a*), its other end (i.e., *b*1) will trace a circle. This circle represents the locus of that point (i.e., *B*1) in TV. Note that, in FV, this circle is seen as a line parallel to *XY*. This line represents the locus of that point (i.e., *B*1) in FV. It is shown by *f*–*f*.

Now, suppose *ab*1 is tilted about *a* through, say *β*1°, in such a way that *b*1 occupies the new position *b* on the circle. *ab* now represents the TV of the line inclined to both the RPs. *β*1 will give the apparent angle between the line and the VP. Let the real angle corresponding to *β*1 be ø1.

As soon as *b*1 moves to *b*, *b*1' moves to *b'* along *f*–*f*. This is so because the circular path of *b*1 in TV is represented by linear path *f*–*f* in FV. a'*b'* now represents the FV of the line inclined to both the RPs. Let the angle made by *a'b'* with *XY* is α1°. α1° represents the apparent angle between the line and the HP.

Note that *ab* and *a'b'* represent the final TV and final FV respectively of a line *AB* which is inclined to both the HP and the VP. To obtain these views, we must know TL, *θ* and ø. However, in this stage, ø was unknown.



Stage 1

Stage 2

Assuming that a line (say *AB*2 = TL = *AB*1 in Stage 1) is inclined to the VP at ø° and parallel to the HP, draw its projections *ab*2 and *a'–b*2'. a'–*b*2' will show EL. If the line is made inclined to the HP at any angle (keeping ø unchanged) then FV will get tilted through a specific angle. In such a case, if the FV is tilted about one of its endpoints (say *a'*), its other end (i.e., *b*2') will trace a circle. This circle represents the locus of *B*2 in FV. In TV, this circle is seen as a line parallel to *XY*. This line represents the locus of *B*2 in TV. It is shown by *t–t*.

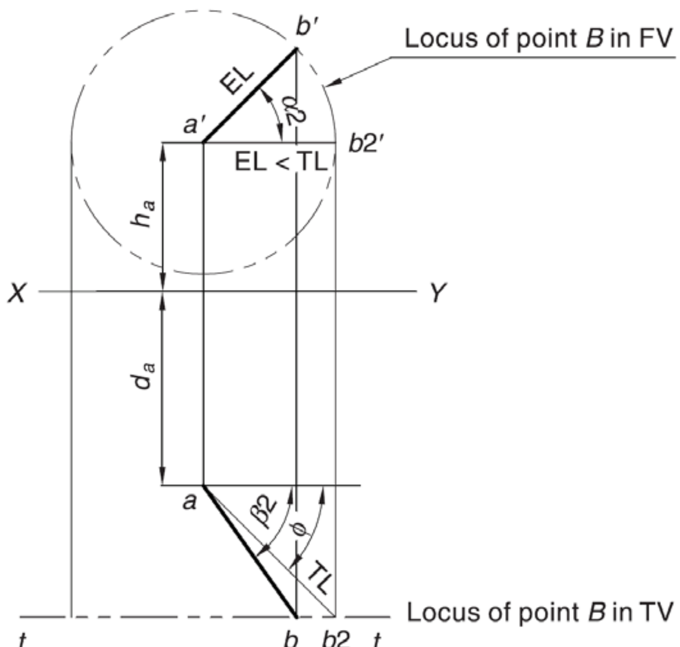
Now, suppose *a'b*2' is tilted about *a'* through, say α2°, in such a way that *b*2' occupies new position *b*' on the circle. a*'b'* now represents the FV of the line inclined to both the RPs. α2 will give the apparent angle between the line and the HP. Let the real angle corresponding to α2 be *θ*2.

As soon as *b*2' moves to *b'*, *b*2 moves to *b* along *t–t. ab* now represents the TV of the line inclined to both the RPs. Let the angle made by *ab* with *XY* be *β*2°. *β*2° will represent the apparent angle between the line and the VP.

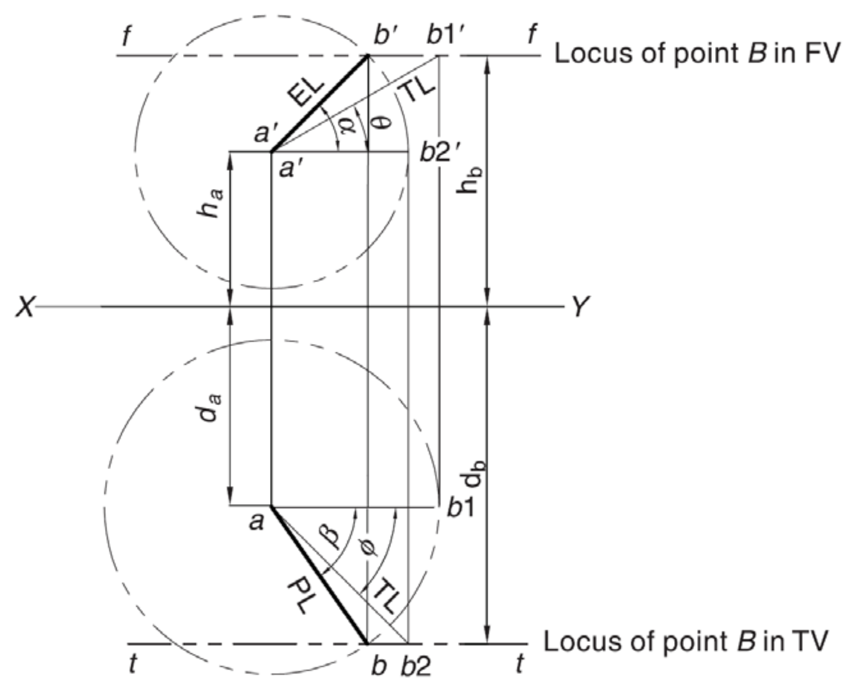
a'b' and *ab* represent the final FV and final TV respectively of a line *AB* which is inclined to both the HP and the VP. To obtain these views, we must know TL, ø and *θ*. However, in this stage, *θ* was unknown.

PROJECTIONS OF LINES

In Stage 1, we have obtained the FV and TV of line *AB* inclined at *θ*° to the HP and ø1° to the VP. Similarly, in Stage 2, we have obtained the FV and TV of line *AB* inclined at *θ*2° to the HP and ø° to the VP. Note that, in both the stages, the line *AB* has same TL. We knew *θ* in Stage 1 and ø in Stage 2. If $\theta = \theta 2$ and $ø1 = ø$, then $\alpha 1 = \alpha 2$ and $\beta 1 = \beta 2$. If end *A* is at same distances from the HP and the VP in both the stages the end *B* will assume same positions in both the stages. As all the parameters, namely, TL, *θ*, ø, h_a, d_a, h_b and d_b are same in Fig. (Stage 1) and Fig. (Stage 2), we can overlap these figures as shown in Fig. (Stage 3).



Stage 2



Stage 3

<u>Example 5</u> A line AB, 50 mm long, is inclined to the HP at 30° and to the VP at 45°. The point A is 20 mm above the HP and 35 mm in front of the VP. Draw the projections of the line. Assume that the end A is nearer to both the RPs than end B.

Given: TL = 50 $\theta = 30°$ $ø = 45°$ $h_a = +20$ $d_a = +35$

1. Draw the initial FV a'b1' and initial TV ab1 of the line assuming that it is inclined to the HP at 30° and parallel to the VP.

a' is 20 mm above XY, a'b1'= 50 mm and $\theta = 30°$.

a is 35 mm below XY and ab1 parallel to XY.

2. Draw the initial TV ab2 and initial FV a'b2' assuming that the line is inclined to the VP at 45° and parallel to the HP.

ab2 = 50 mm and ø = 45°.

a'b2' is parallel to XY.

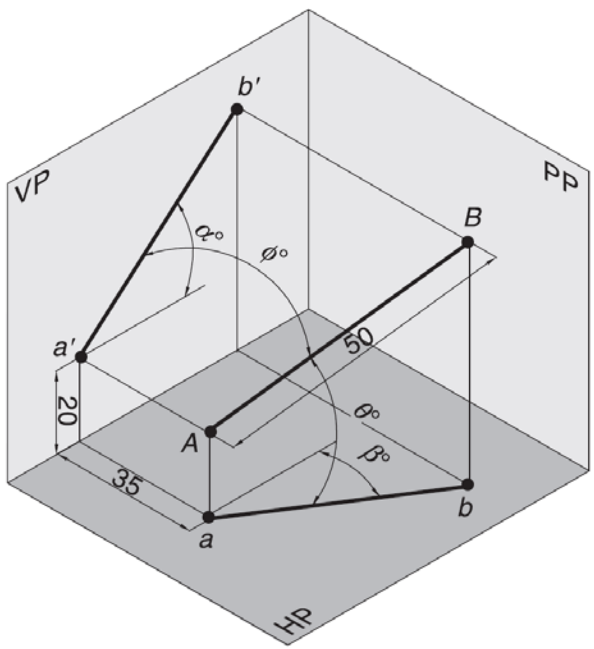
3. Draw f–f passing through b1' and parallel to XY.

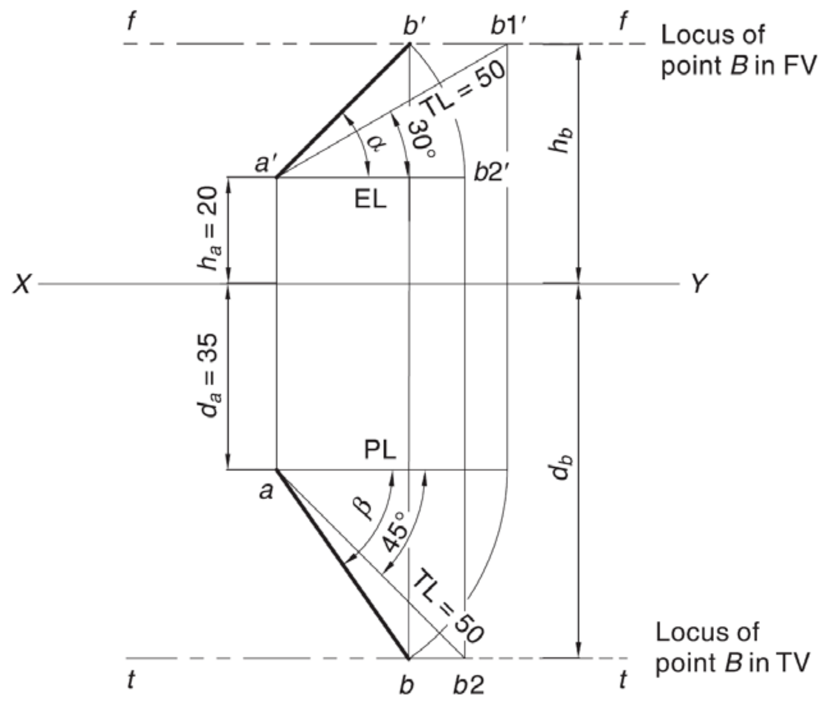
4. Draw t–t passing through b2 and parallel to XY.

5. With a’ as a centre and radius = a’b2’, draw an arc cutting f–f at b’. a’b’ is the final FV. a’b’ makes α° with XY.

6. With a as a centre and radius = ab1, draw an arc cutting t–t at b. ab is the final TV. ab makes β° with XY.

Note that b’ and b lie on the same projector. Hence, after Step 5, b can be obtained directly by projecting b’ on t–t.





LINE PARALLEL TO (OR CONTAINED BY) THE PP

If the sum of the inclinations of a line with the HP and the VP is equal to 90°, i.e., $\theta + ø = 90°$, then the line is parallel to the PP. In this case, TV and FV will be perpendicular to *XY*. The SV will give TL and true inclinations.

Example 6 A line AB, 50 mm long, is inclined at 30° to the HP and 60° to the VP. Its end A is 25 mm above the HP and 20 mm in front of the VP. Draw its projections.

Given: TL = 50 $\theta = 30°$ $ø = 60°$ $h_a = +25$ $d_a = +20$

Solution As $\theta + ø = 90°$, the line is parallel to (or in) the PP.

Method 1: Refer Fig. (a).

1. Assuming the line inclined at 30° to the HP and parallel to the VP, draw its FV a'b1' and TV ab1 as shown.

2. Assuming the line inclined at 60° to the VP and parallel to the HP, draw its TV ab2 and FV a'b2' as shown.

3. Draw f–f and t–t.

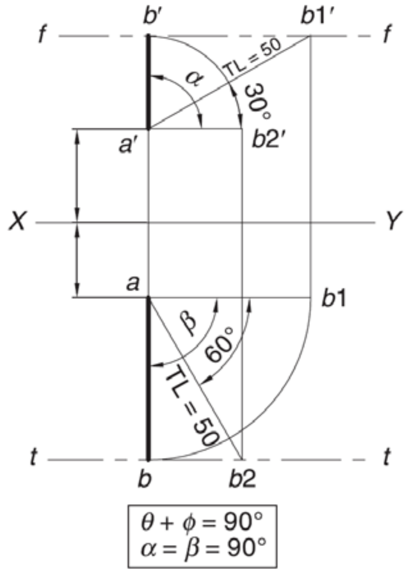
4. With a' as a centre and radius = a'b2', draw an arc meeting f–f at b'. Join a'b' for the final FV.

5. With a as a centre and radius = ab1, draw an arc meeting t–t at b. Join ab for the final TV.

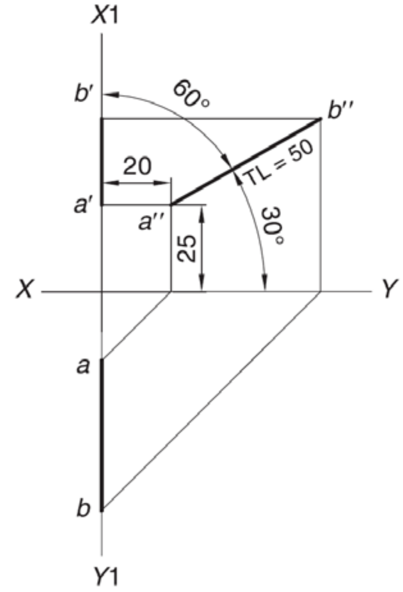
Note that the arcs b2'b' and b1 b are tangent to f–f and t–t respectively. Hence α = β = 90°

Method 2: Refer Fig. (b).

As the line is in the PP, its SV will give TL and true inclinations. Hence, first draw SV a" b" as shown. Then obtain its FV a'b' and TV ab by projecting the SV on X1 Y1.



(a)



(b)

TRACES OF A LINE

A trace is a point at which the line or its extension meets the HP or the VP. A line will show an HT if it is inclined to the HP. Similarly, a line will show a VT if it is inclined to the VP. A line parallel to both the RPs will have no trace.

The HT is always seen on TV or extension of TV, whereas the VT is always seen on FV or extension of FV. The projection of the HT on the *XY* line is indicated by *h*. The projection of the VT on the *XY* line is indicated by *v*. The terms *h*-HT and *v*-VT may be used to indicate the distances of the HT and the VT respectively from the RPs.

Procedure to Locate the Traces

To Locate HT

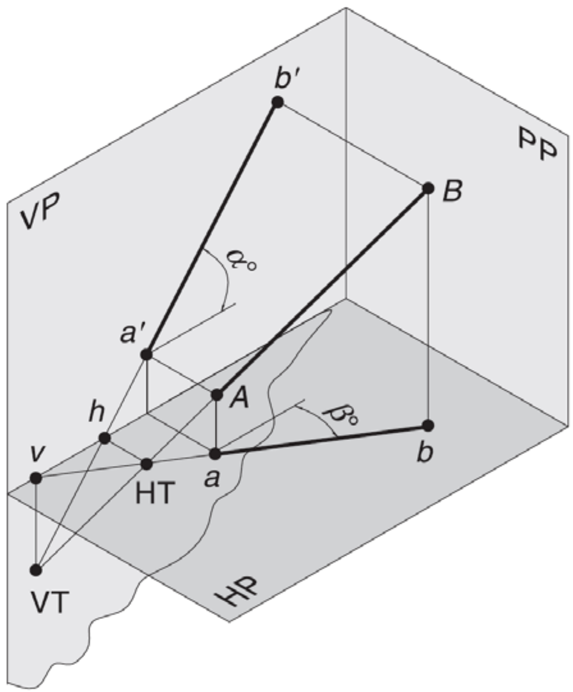
1. Produce, if necessary, the FV to meet the *XY*. Mark the point of intersection of FV (produced) with *XY* as *h*.

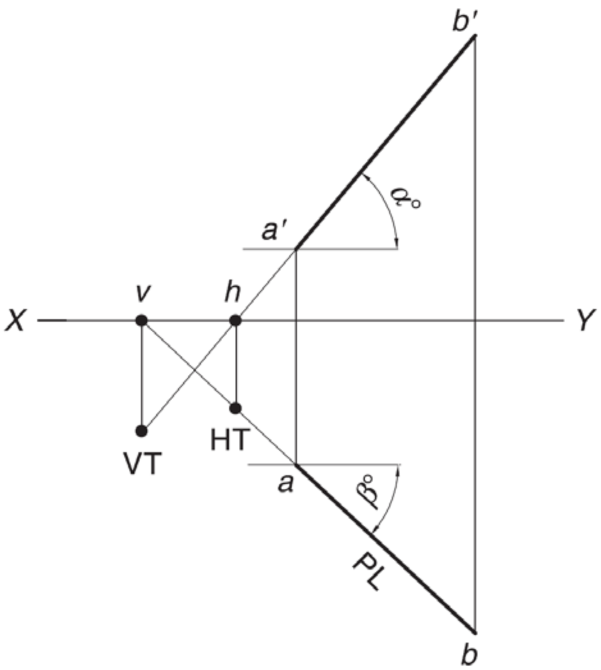
2. Draw a projector through *h*. The point at which this projector meets the TV (produced if necessary) is the HT of the line.

To Locate VT

1. Produce, if necessary, the TV to meet the *XY*. Mark the point of intersection of TV (produced) with *XY* as *v*.

2. Draw a projector through *v*. The point at which this projector meets the FV (produced if necessary) is the VT of the line.





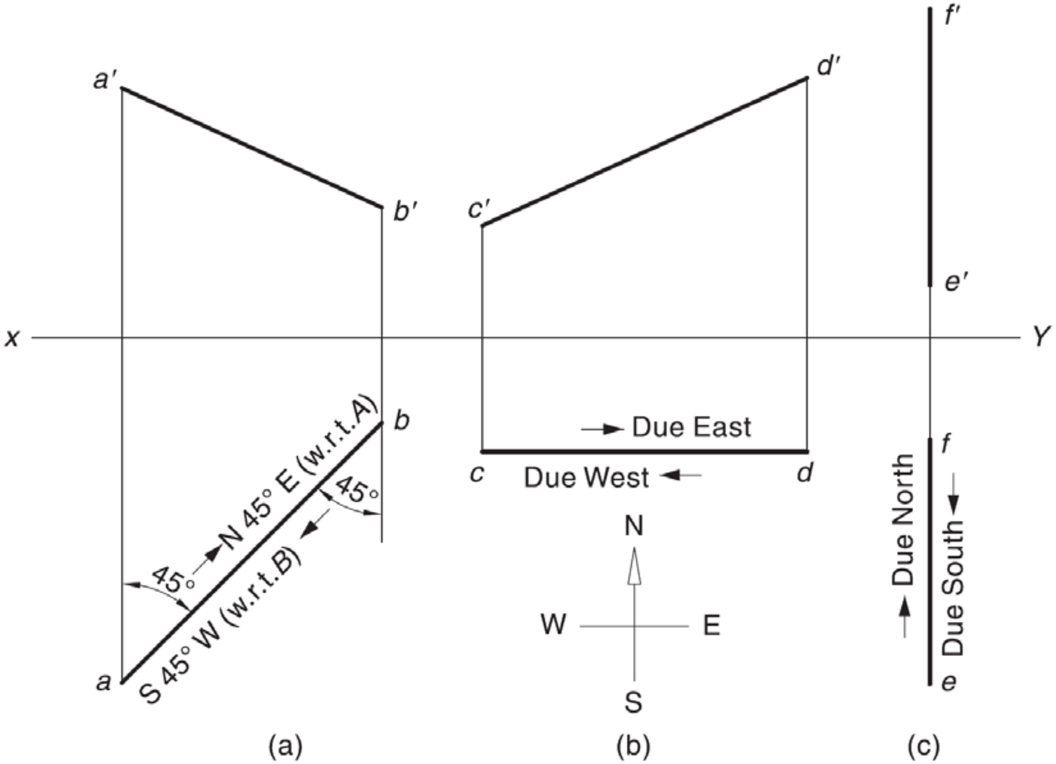
Bearing of a line is the acute angle made by its TV with the north or south direction. It is customary to assume north vertically upward. A bearing indicates the direction of one end of the line with respect to the other.

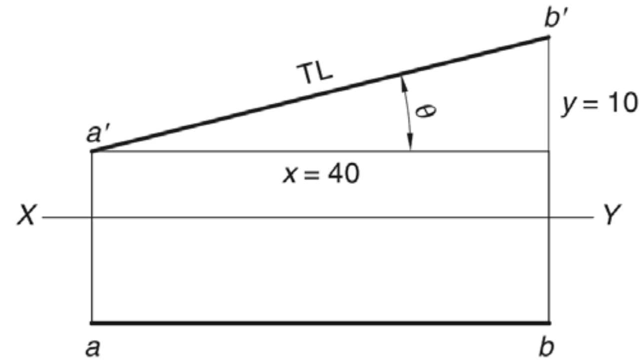
The bearings of the lines are as follows:

		Bearings	*Meaning*
(a)	Line *AB*	N 45° E (w.r.t. *A*)	*B* is due 45° East of North of *A*
		S 45° W (w.r.t. *B*)	*A* is due 45° West of South of *B*
(b)	Line *CD*	Due East (w.r.t. *C*)	*D* is due East of *C*
		Due West (w.r.t. *D*)	*C* is due West of *D*
(c)	Line *EF*	Due North (w.r.t. *E*)	*F* is due North of *E*
		Due South (w.r.t. *F*)	*E* is due South of *F*

Grade (or gradient) of a line gives its inclination with the HP. It is a vertical rise (or fall) of the line per unit horizontal advancement, expressed as percentage, i.e. $(y/x)*100$, Fig. However, in many cases, gradient is expressed as '*y* in *x*' or '*y* : *x*'. *Slope* is synonymous with grade.

PROJECTIONS OF LINES





Grade of line AB (w.r.t.A) $= \frac{+10}{40} \times 100 = +25\%$

Grade of line AB (w.r.t.B) $= \frac{-10}{40} \times 100 = +25\%$

ANGLES OF DEPRESSION AND ELEVATION

If an observer is looking toward the object situated below his eye level, the angle made by his rays of sight with the horizontal is called the *angle of depression*. If the object is situated above the eye level of the observer, the similar angle is called the *angle of elevation*.

Example 7 Two straight roads AB and AC are 2 km and 1.4 km long respectively. AB bears N 40° E on a downward slope of 30°. AC bears S 35° E on a downward grade of 15°. Draw the projections. Find TL, bearing and grade of the new road joining B to C.

Given: AB: TL = 2 km, Bearing = N 40° E, Grade = –30°

AC: TL = 1.4 km, Bearing = S 35° E, Grade = –15°

Solution:

1. Mark a and a', at suitable distances, below and above XY respectively.

2. Draw a'b1'= 2 km and a'c1'= 1.4 km making 30^o and 15^o respectively to XY. Note that AB and AC has downward grade with respect to A.

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3. Project b1' and c1' below XY to obtain ab1 and ac1 parallel to XY. ab1 and ac1 give PLs of AB and AC respectively.

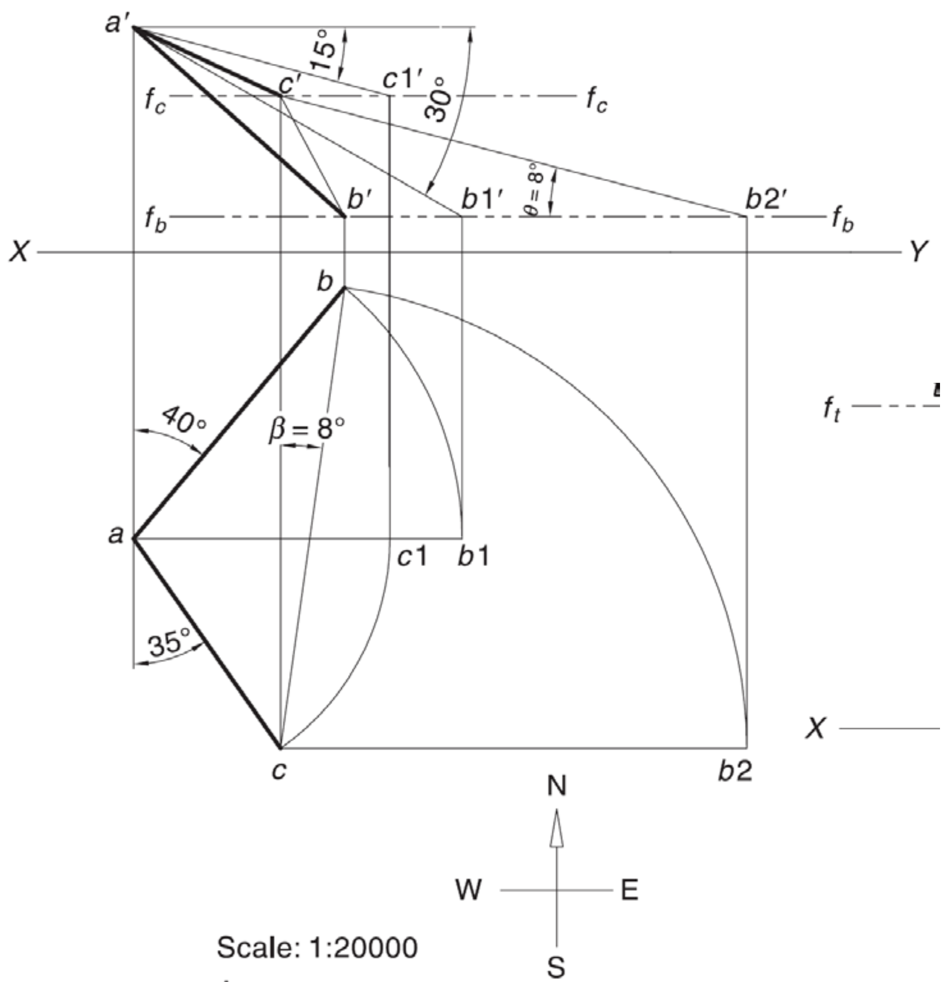
4. Rotate ab1 to ab to make 40^o with the vertical. Similarly, rotate ac1 to ac to make 35^o with the vertical. ab and ac gives required TVs.

5. Project b and c on f_b– f_b and f_c– f_c respectively to obtain desired FVs— a'b' and a'c'.

6. Join c'b' and cb. Rotate cb to obtain cb2 parallel to XY and then project b2 to b2' on f_b–f_b. Join c'b2'. Measure θ. c'b2' and θ gives TL and grade of CB.

7. Measure β, i.e., angle made by cb with the vertical. It gives the bearing of BC, i.e., N β^o E.

PROJECTIONS OF LINES



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Ans:

TL of $BC = c'b2' = 127$ mm = 2.54 km

Bearing of BC = N 8° E

Grade of BC = +15° (w.r.t. B)

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Example 8 The TV of a line CD measures 80 mm and makes an angle of 55° with XY. End C is in the VP and the HT of the line is 25 mm above XY. The line is inclined at 30° to the HP. Draw the projections of line CD. Determine its TL, true inclination with the VP, and the VT.

Given: PL = 80 β = 55° d_c = 0 h-HT = +25 θ = 30°

Solution:

1. Mark c on XY and draw cd = PL = 80 mm inclined at 55° to XY. Extend dc and locate HT 25 mm above XY on it.

2. Draw HT- d1 = HT- d parallel to XY.

3. Obtain h and draw h– d1' inclined at 30° to XY meeting D1 at d1'.

4. Draw f– f and obtain d' on it by projecting d. Join hd'.

5. Project c on hd' to locate c'. c'd' represents the final FV.

6. To find TL and ø, obtain cd2 as shown. As c is on XY, c' gives VT.

PROJECTIONS OF LINES