

山崎先生のご質問に答えられなかったので、回答を用意しました。

Q : Ridge 回帰がガウス事前分布ベイズ線形回帰なのは分かったが、Lasso は？

A : Lasso の原論文 [Tibshirani \(1996\)](#) の Chapter 5 に書いてありました。

導出は定義から明らかなので省略しますが、ラプラス分布です。

正規分布と比べて、裾が長めで鋭いピークを持つ分布です。

5. LASSO AS BAYES ESTIMATE

The lasso constraint $\Sigma |\beta_j| \leq t$ is equivalent to the addition of a penalty term $\lambda \Sigma |\beta_j|$ to the residual sum of squares (see Murray *et al.* (1981), chapter 5). Now $|\beta_j|$ is proportional to the (minus) log-density of the double-exponential distribution. As a result we can derive the lasso estimate as the Bayes posterior mode under independent double-exponential priors for the β_j s,

$$f(\beta_j) = \frac{1}{2\tau} \exp\left(-\frac{|\beta_j|}{\tau}\right)$$

with $\tau = 1/\lambda$.

Fig. 7 shows the double-exponential density (full curve) and the normal density (broken curve); the latter is the implicit prior used by ridge regression. Notice how the double-exponential density puts more mass near 0 and in the tails. This reflects the greater tendency of the lasso to produce estimates that are either large or 0.

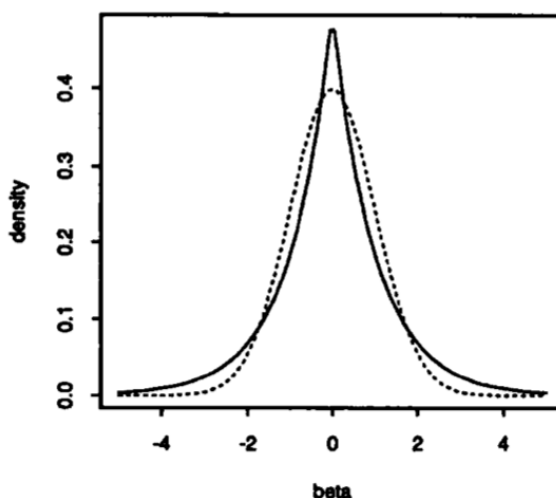


Fig. 7. Double-exponential density (—) and normal density (- - -): the former is the implicit prior used by the lasso; the latter by ridge regression