Linear Convergence of Natural Policy Gradient Methods with Log-Linear Policies

Rui Yuan[1,4], Simon S. Du[2], Robert M. Gower[3], Alessandro Lazaric[1], Lin Xiao[1]

[1]Meta AI, [2]University of Washington, [3]Flatiron Institute, [4]Télécom Paris

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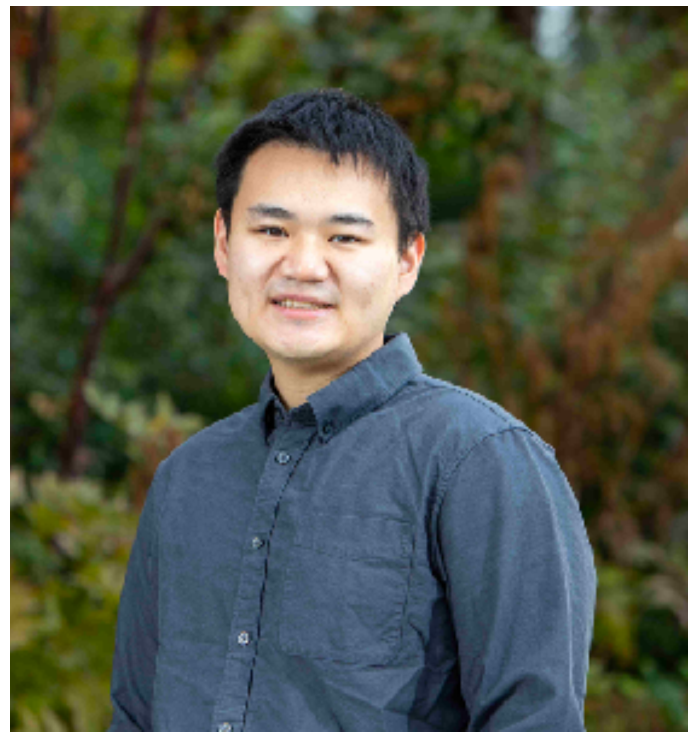

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Impressive Reinforcement Learning (RL) Results

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Board Game

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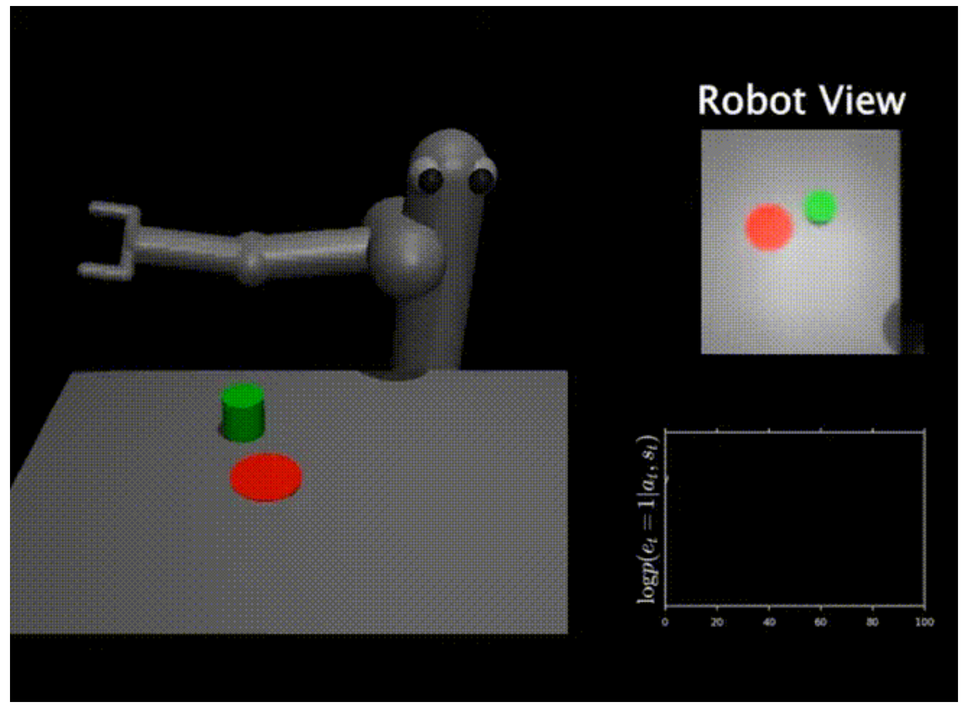
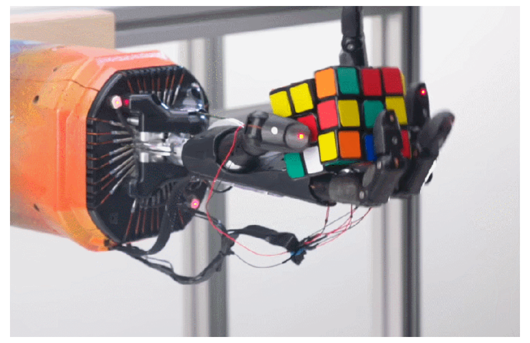
Robotic Manipulation





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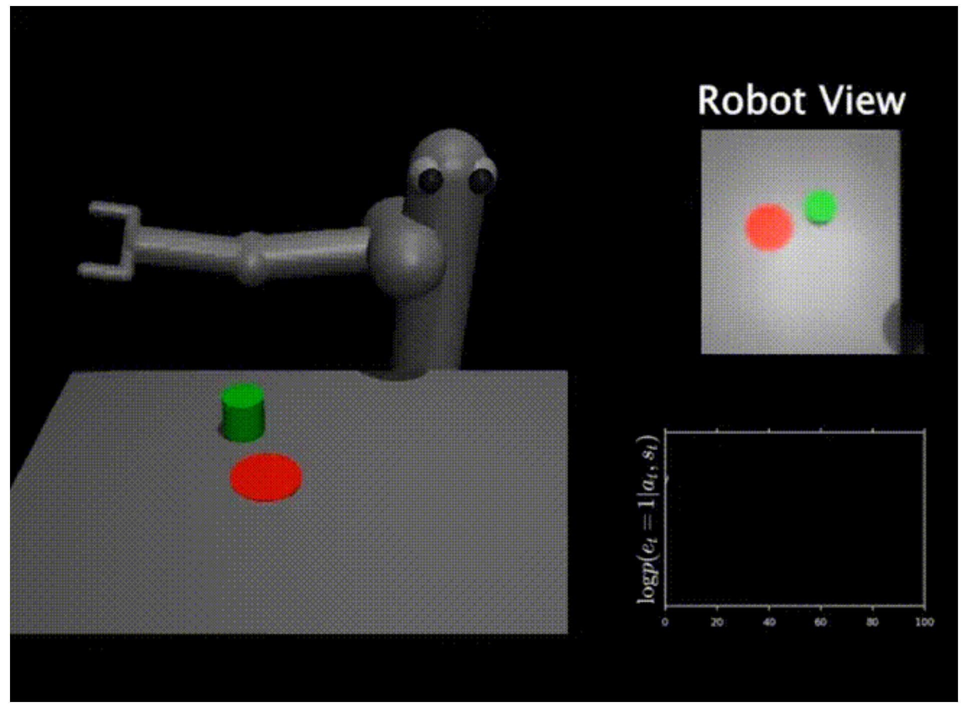
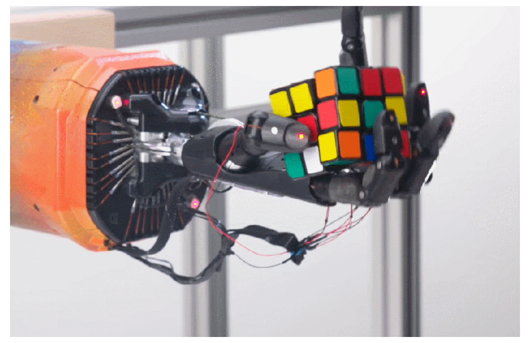
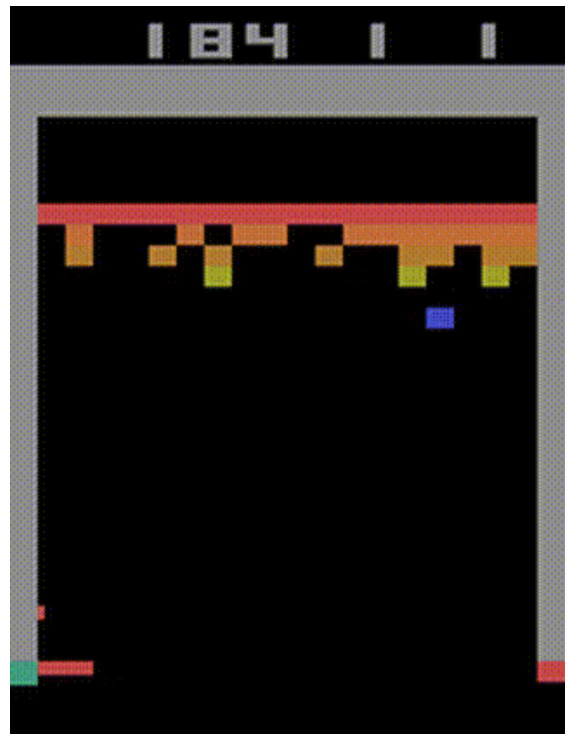
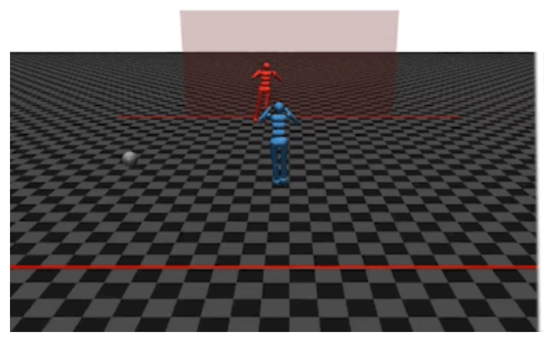
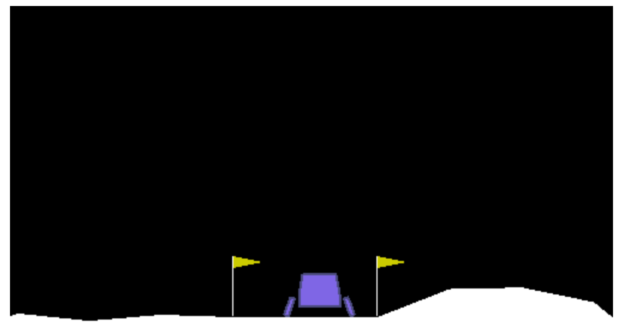




Game Playing

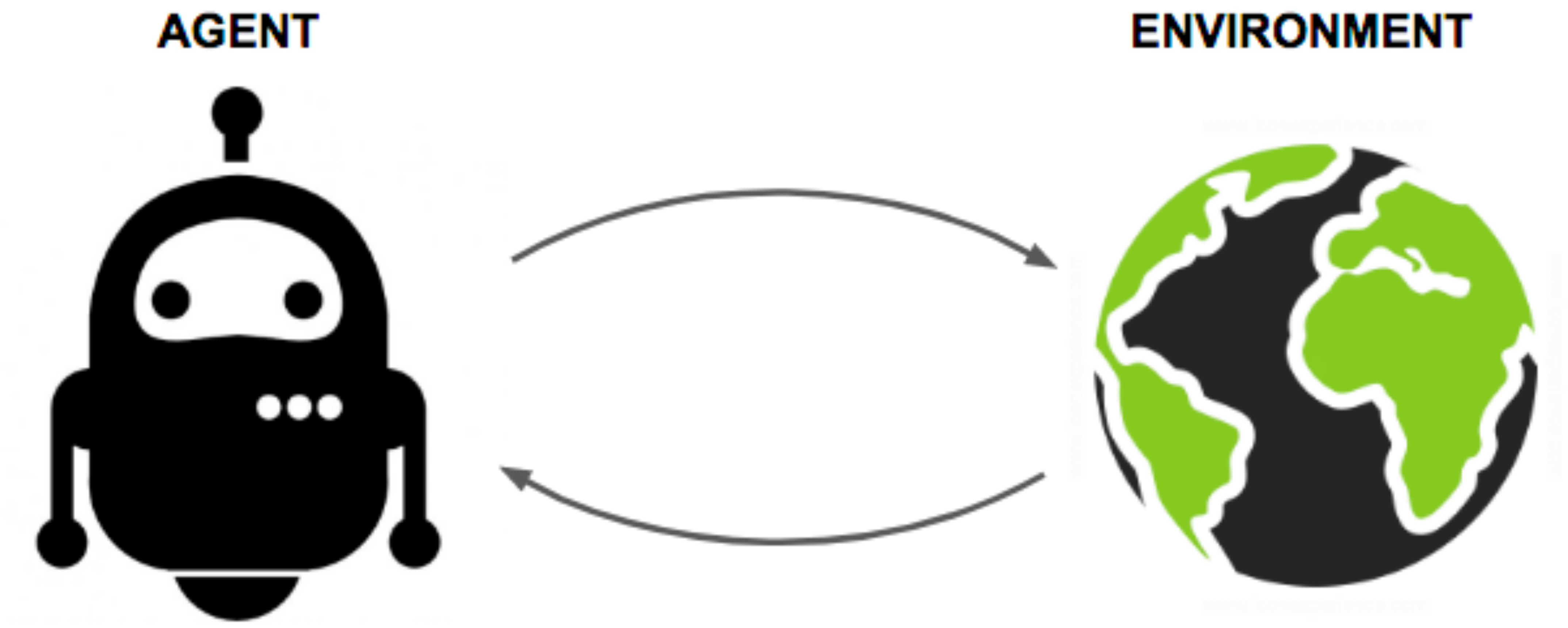






Reinforcement Learning

Sequential decision making problems



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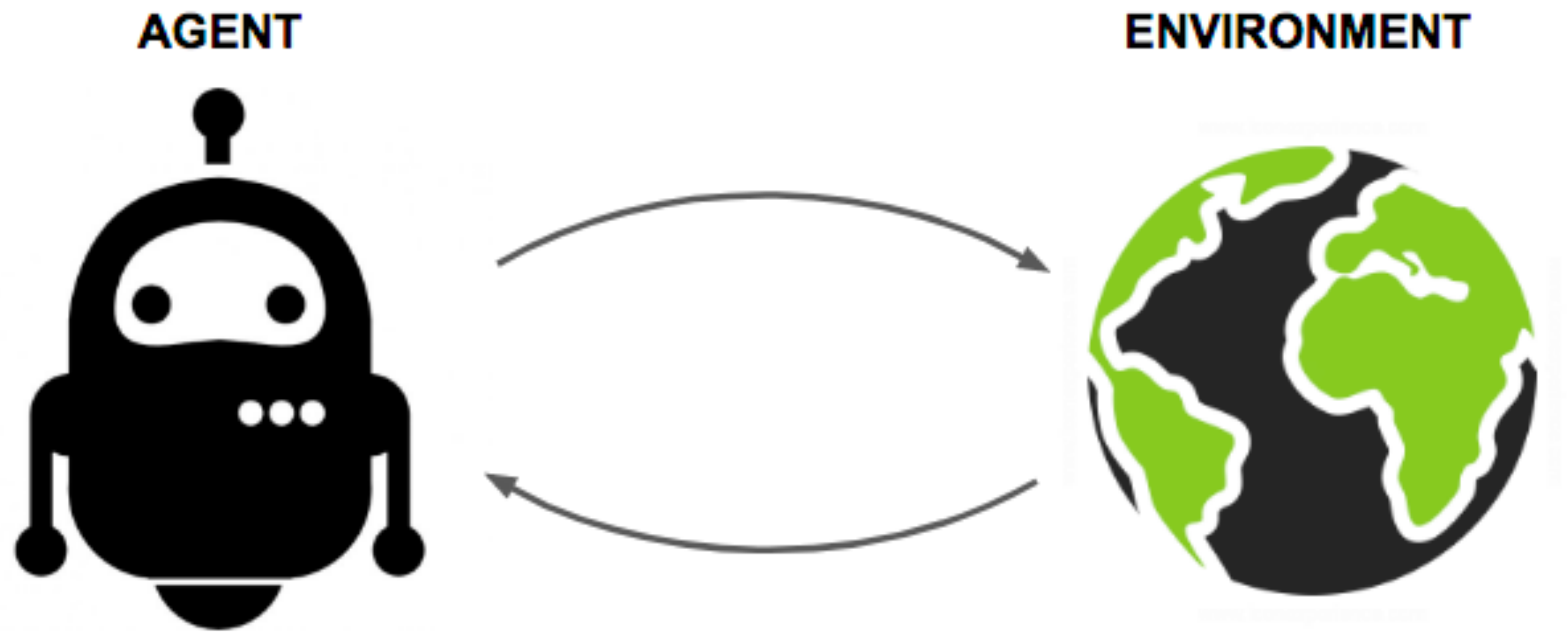

Markov decision Process (MDP)

Reinforcement Learning

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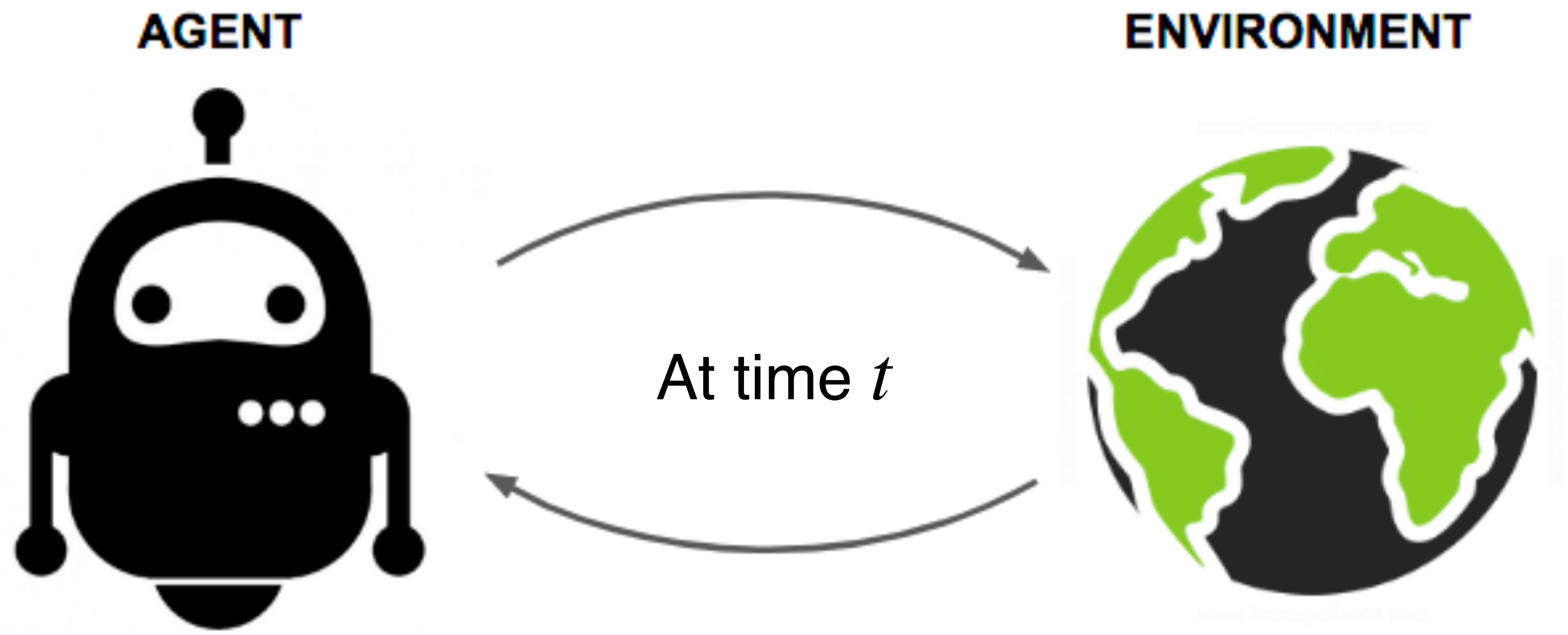

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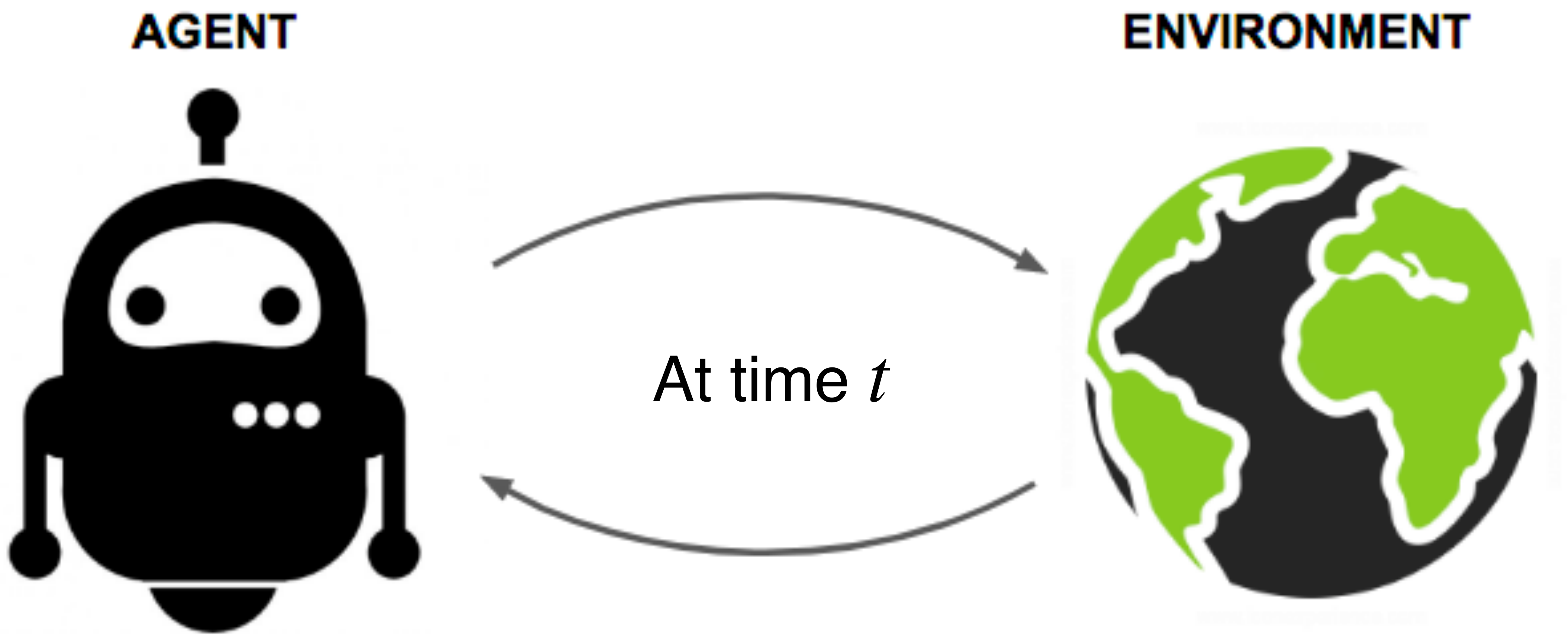

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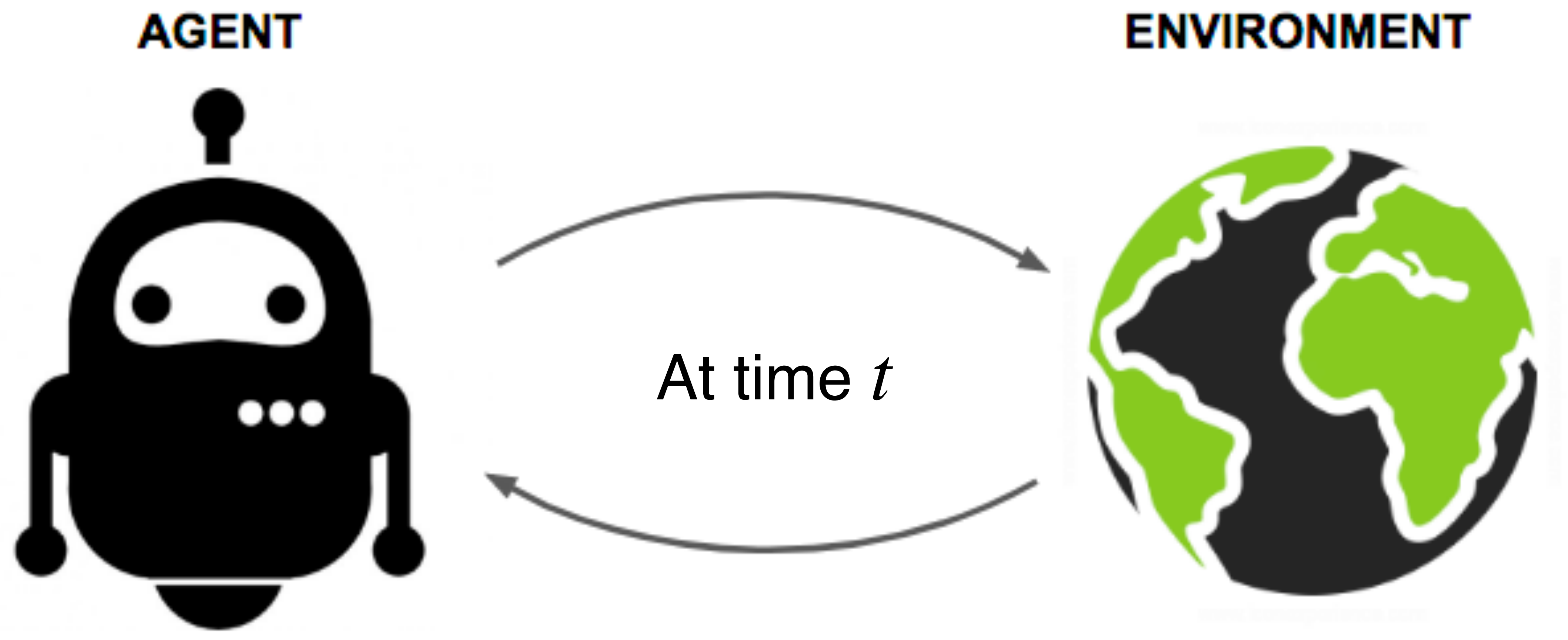

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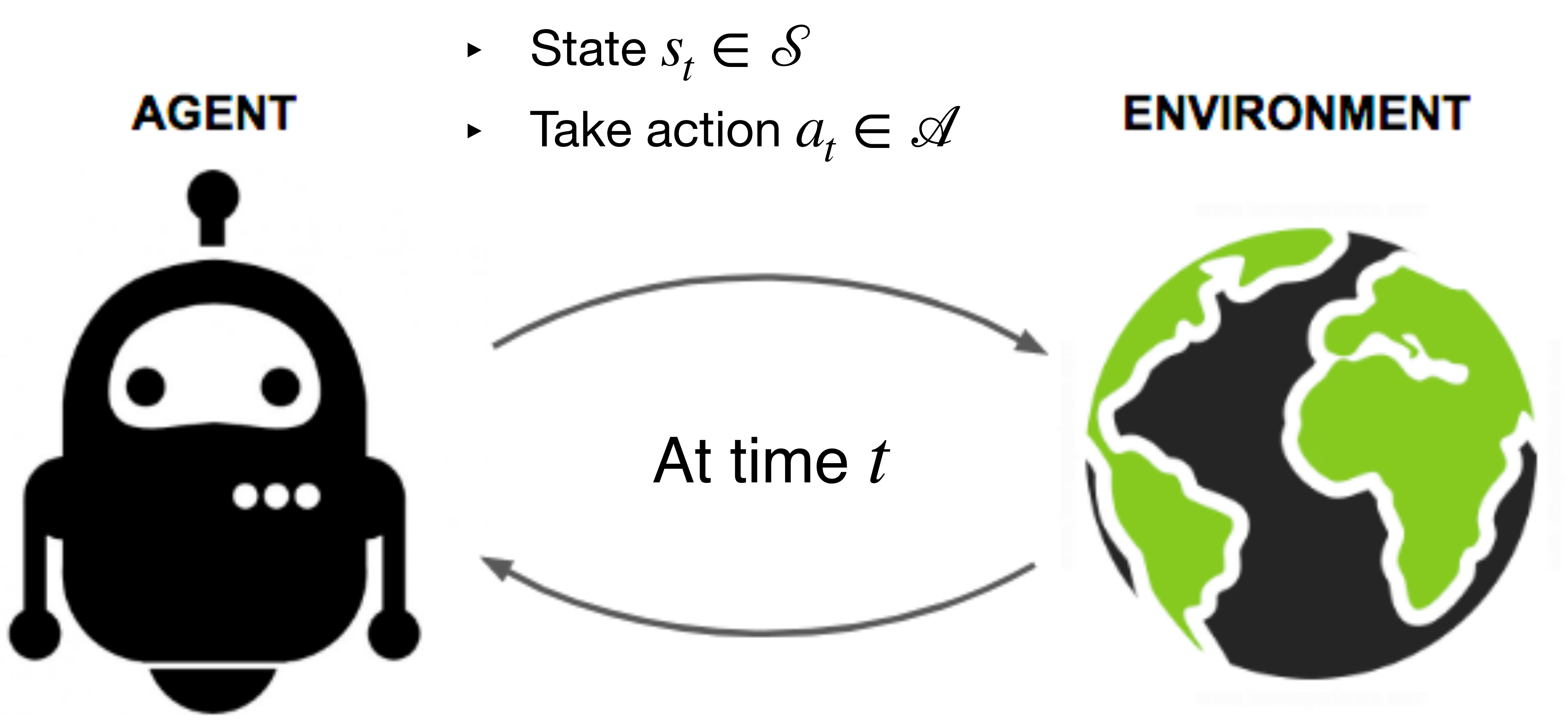

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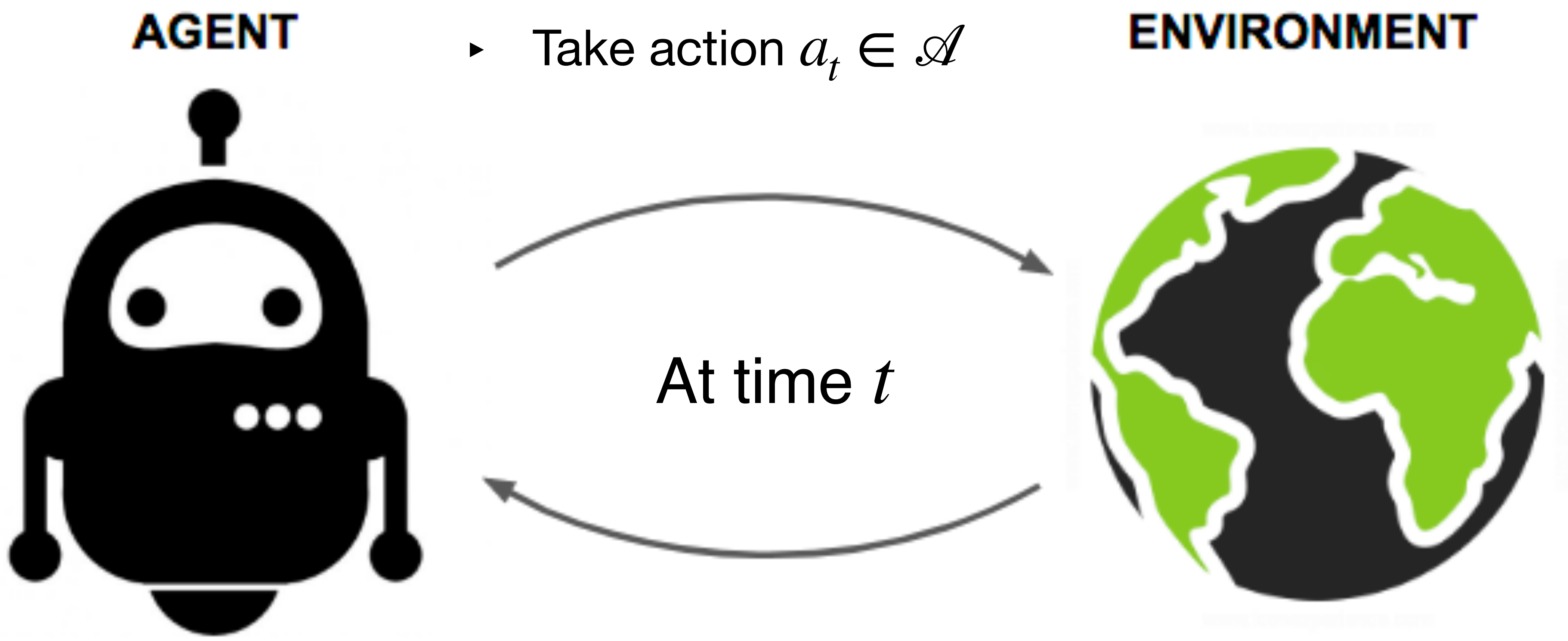

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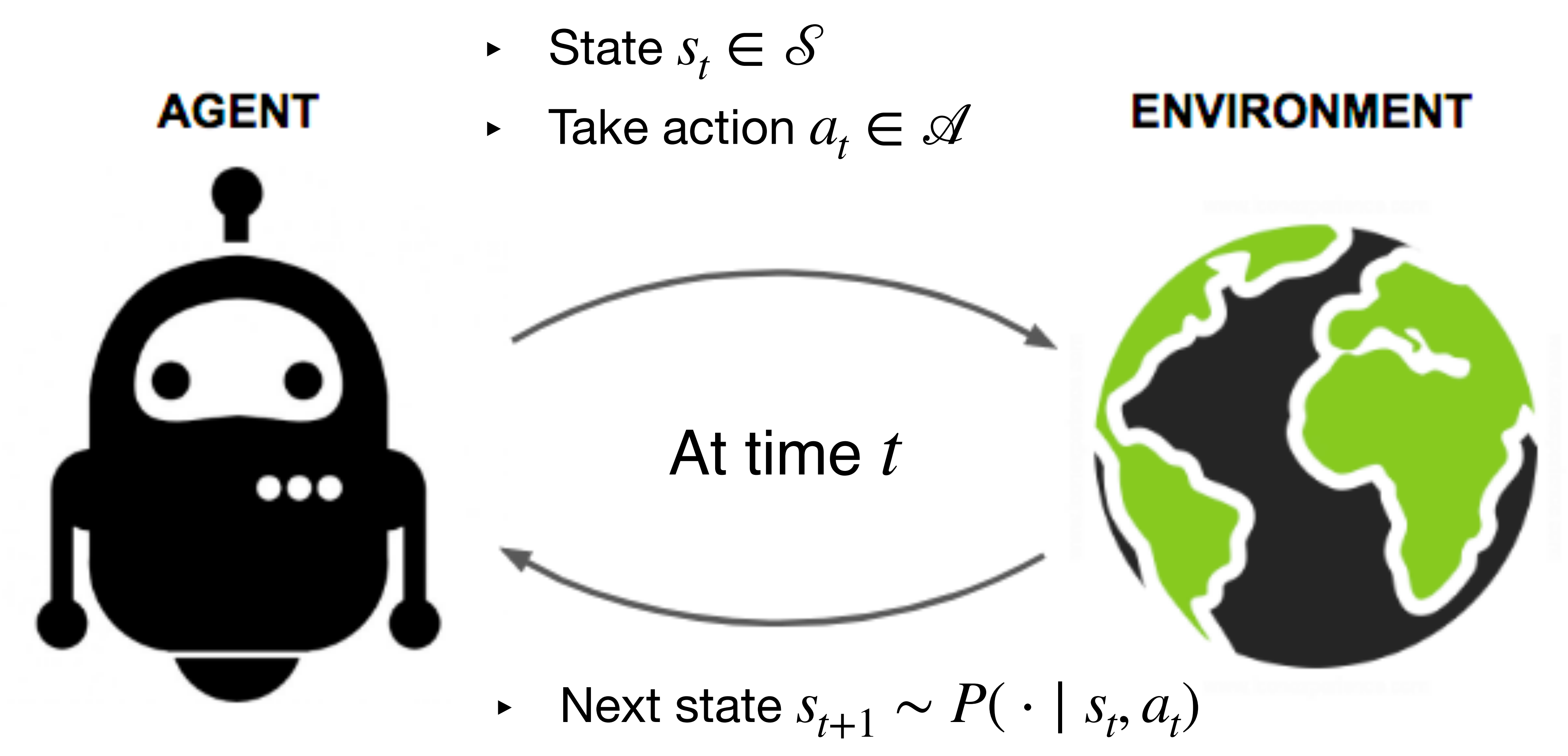

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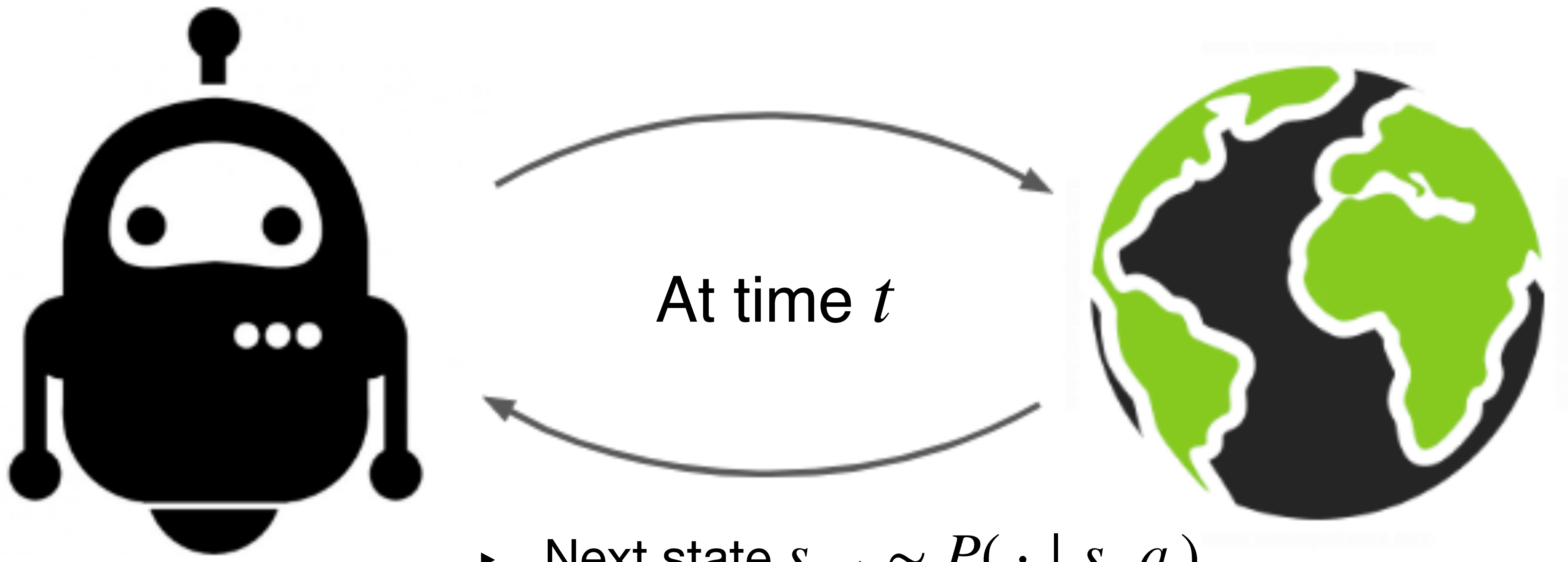
AGENT **ENVIRONMENT**



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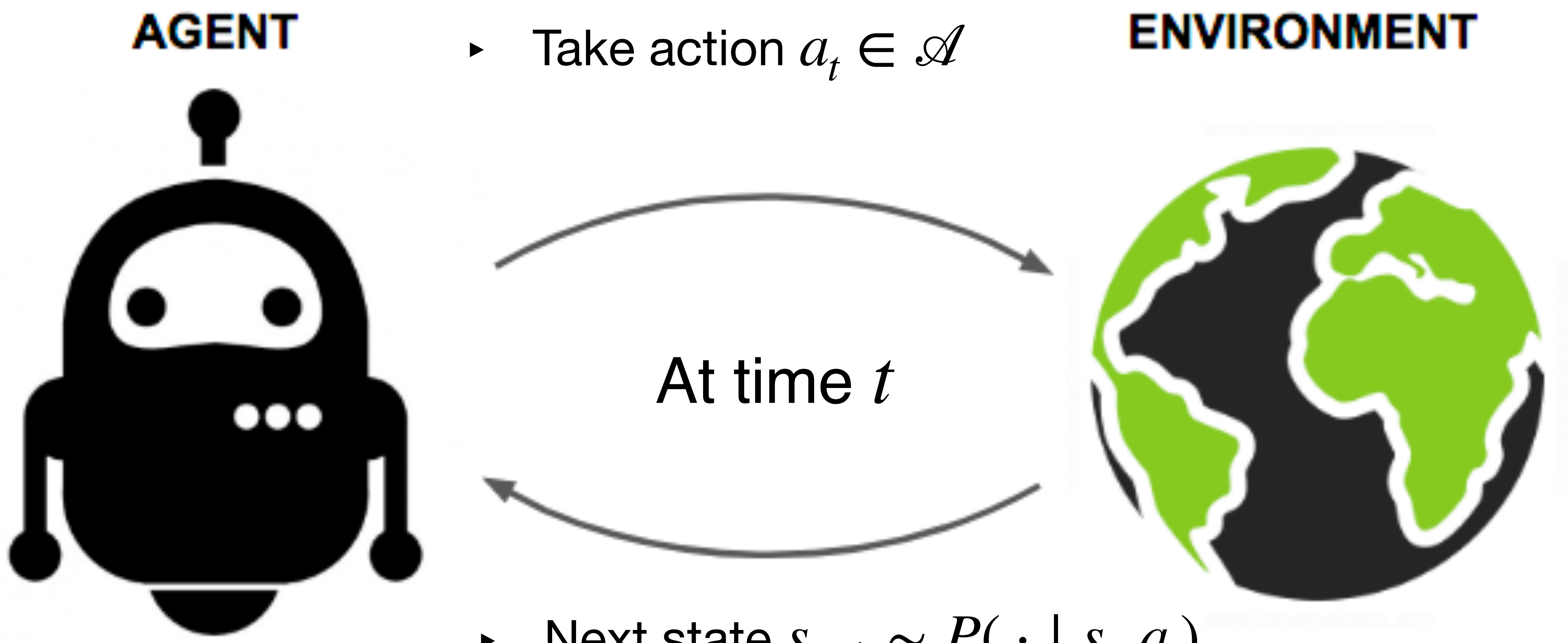

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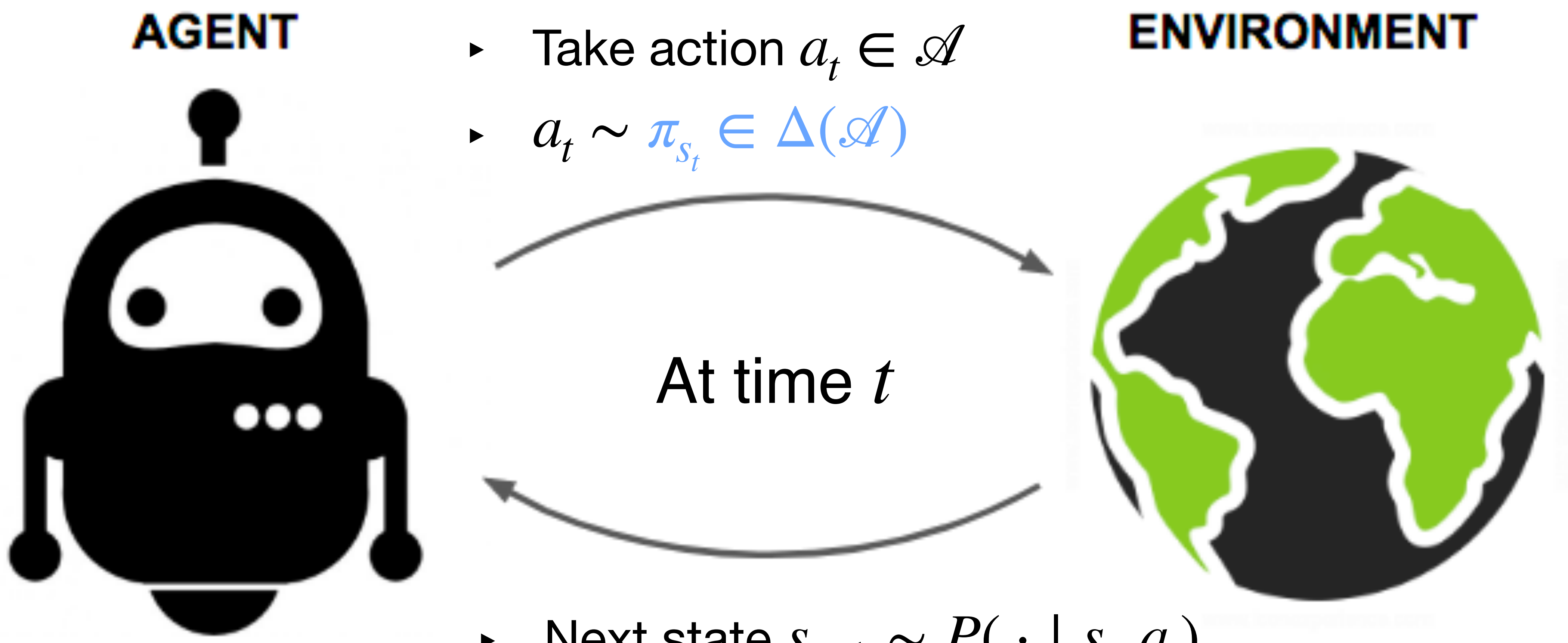

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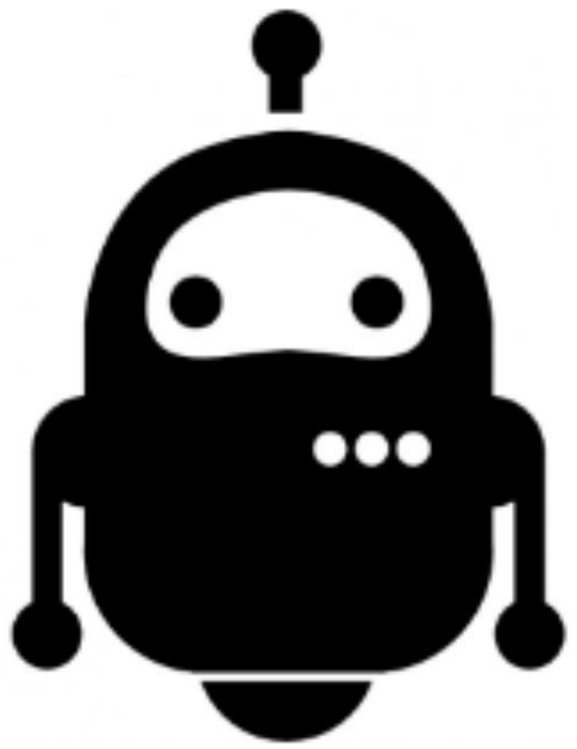

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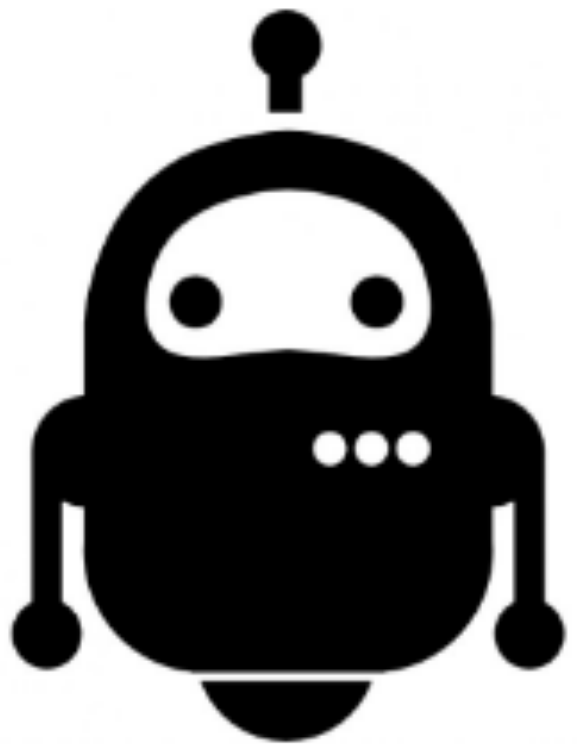

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Solve an MDP to minimize total expected cost (a.k.a. policy optimization)

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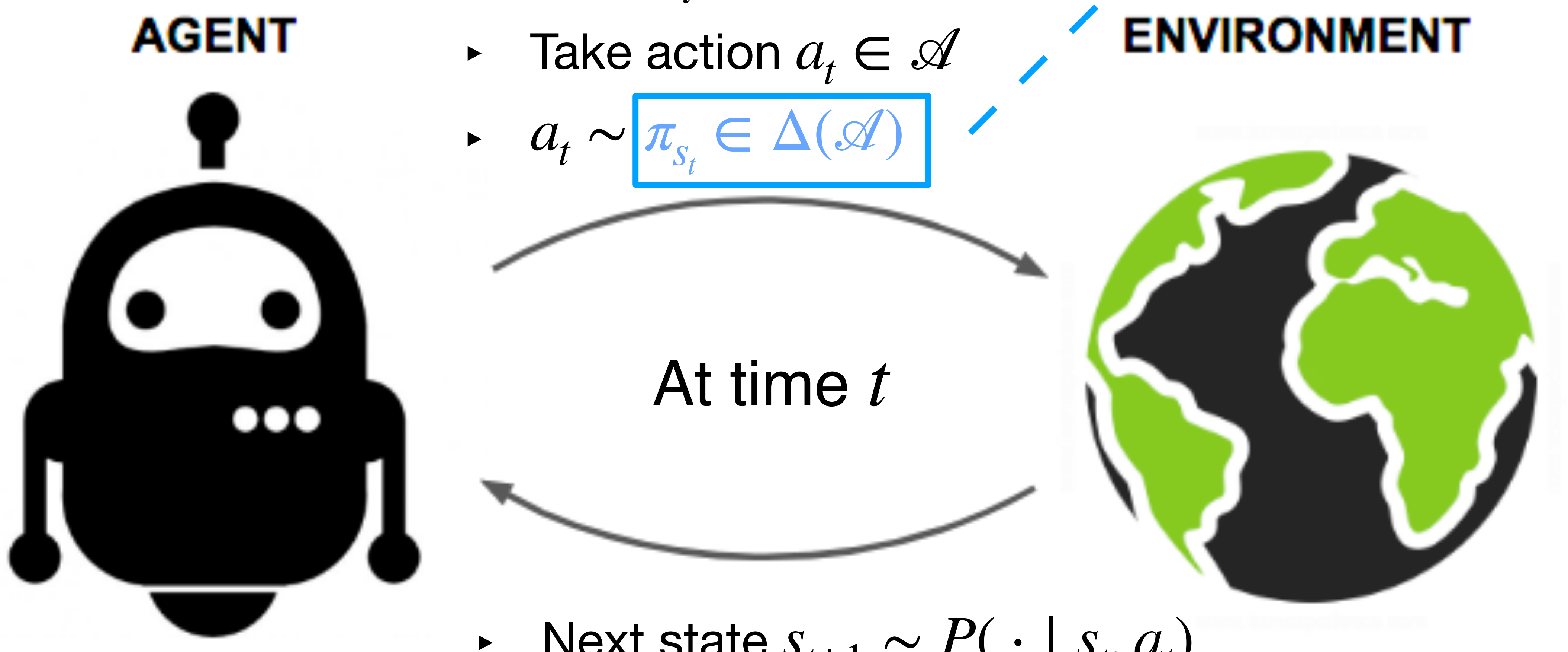

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Cost function

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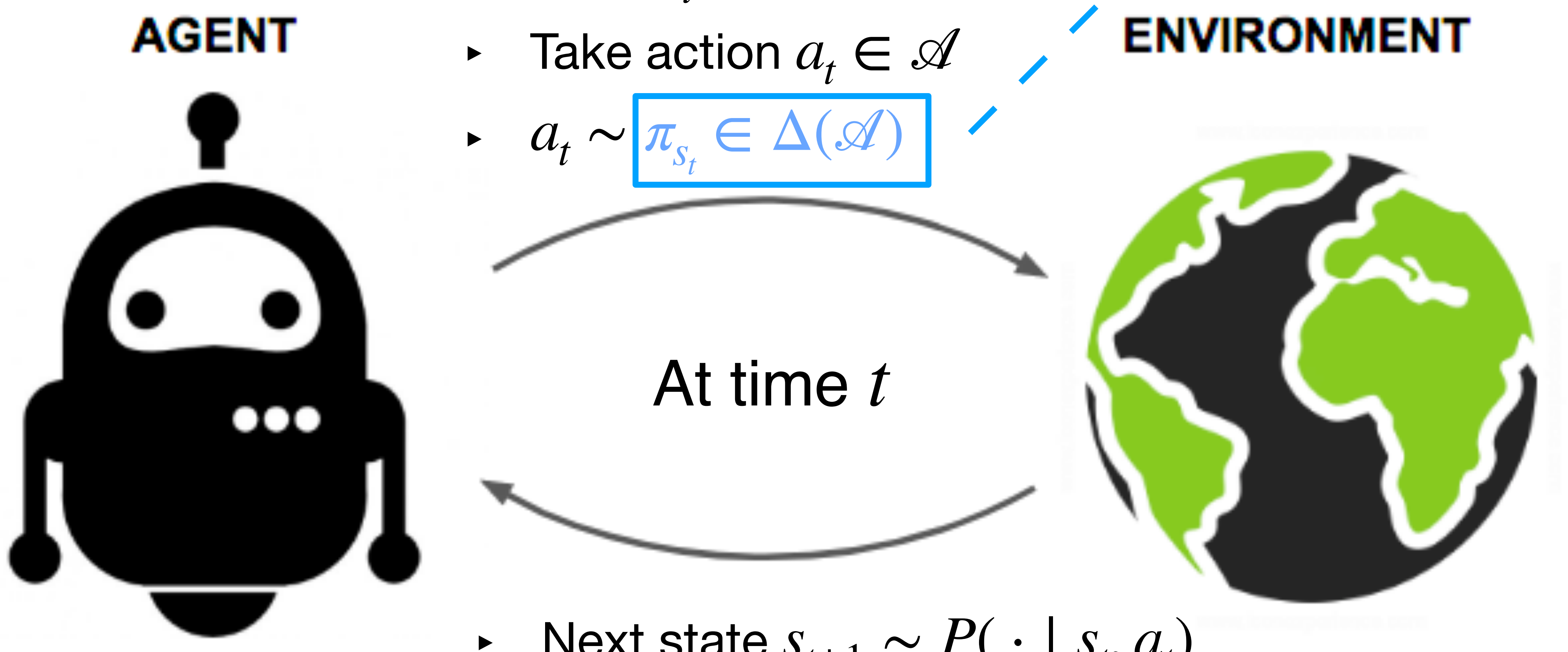

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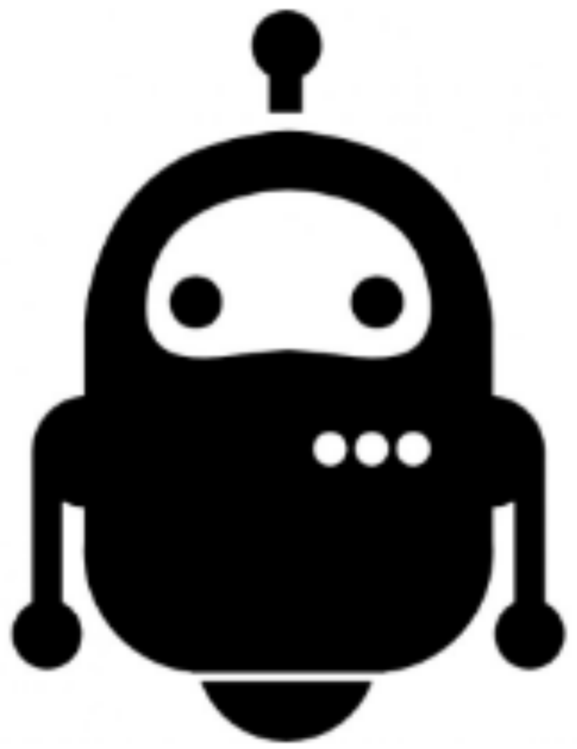

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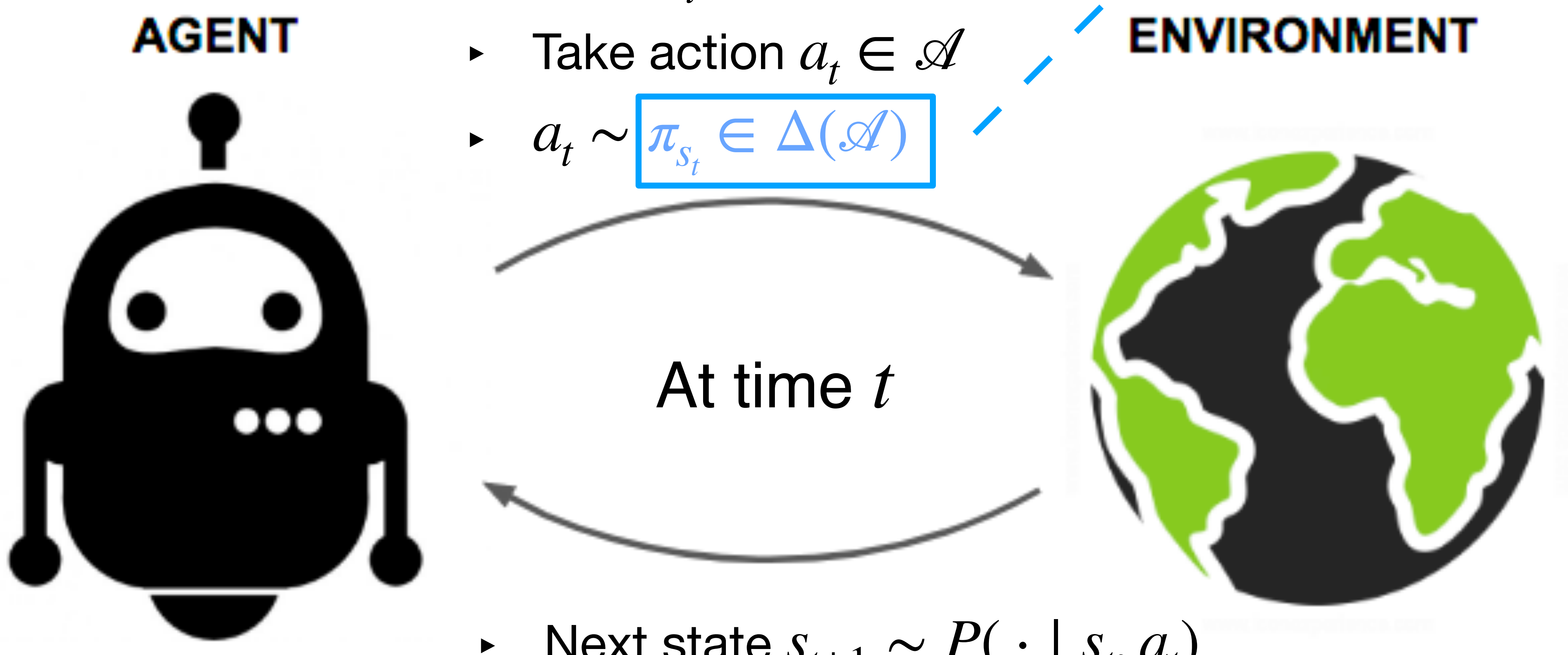

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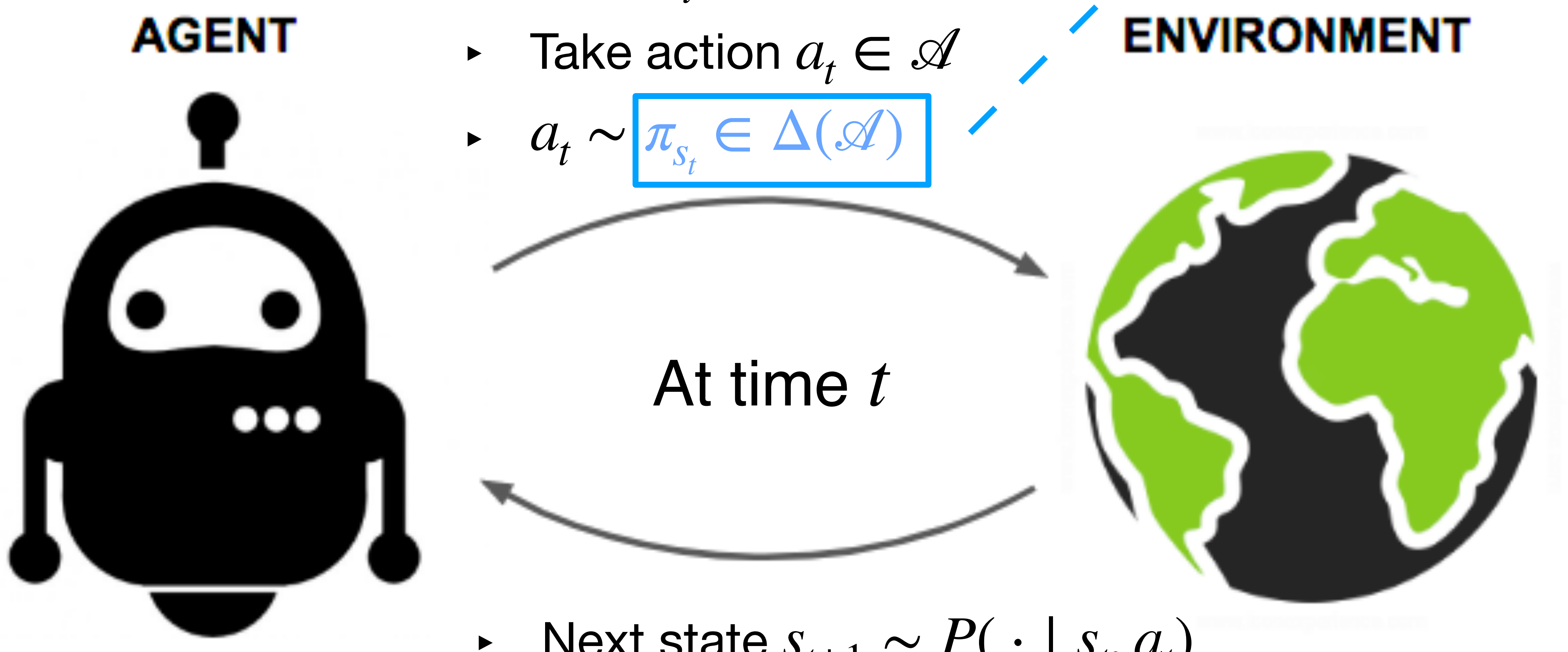

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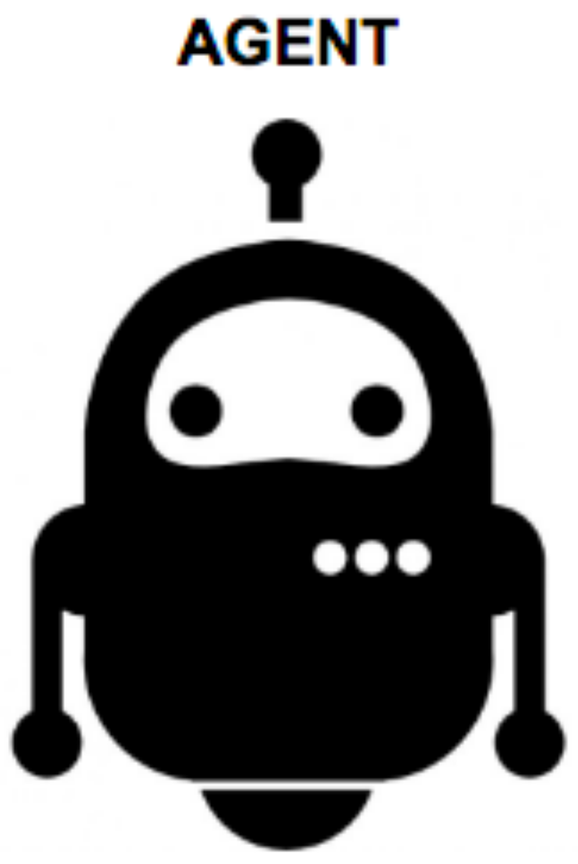

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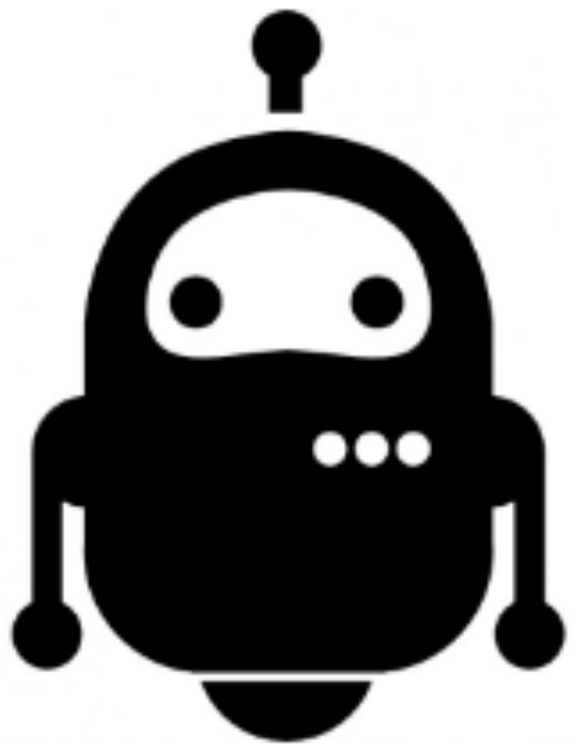

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Step size

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Gradient of $V_\rho(\theta)$

Policy gradient (PG) methods

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Gradient of $V_\rho(\theta)$

Step size

- Simplicity

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  - Natural PG[Kakade, 2001], policy mirror descent [Lan, 2022; Xiao, 2022], variance reduction techniques [Papini et al., 2018; Shen et al., 2019; Xu et al., 2020; Huang et al., 2020]

Policy gradient (PG) methods

Objective: $\arg\min_{\theta \in \mathbb{R}^d} V_\rho(\theta)$

$$\boxed{\theta^{(k+1)} = \theta^{(k)} - \underbrace{\boxed{\eta_k}}_{\text{Step size}} \underbrace{\boxed{\nabla_\theta V_\rho(\theta^{(k)})}}_{\text{Gradient of } V_\rho(\theta)}}$$

- Simplicity
  - Easy to implement and use in practice
  - Can solve a wide range of problems (e.g. partially-observable environments)
- Versatility
  - REINFORCE [Williams, 1992], PGT, GPOMDP [Baxter and Bartlett, 2001], actor-critic [Konda and Tsitsiklis, 2000]
  - Natural PG[Kakade, 2001], policy mirror descent [Lan, 2022; Xiao, 2022], variance reduction techniques [Papini et al., 2018; Shen et al., 2019; Xu et al., 2020; Huang et al., 2020]
  - Trust-region (e.g. TRPO [Schulman et al., 2015;]), proximal (e.g. PPO [Schulman et al., 2017])

Context

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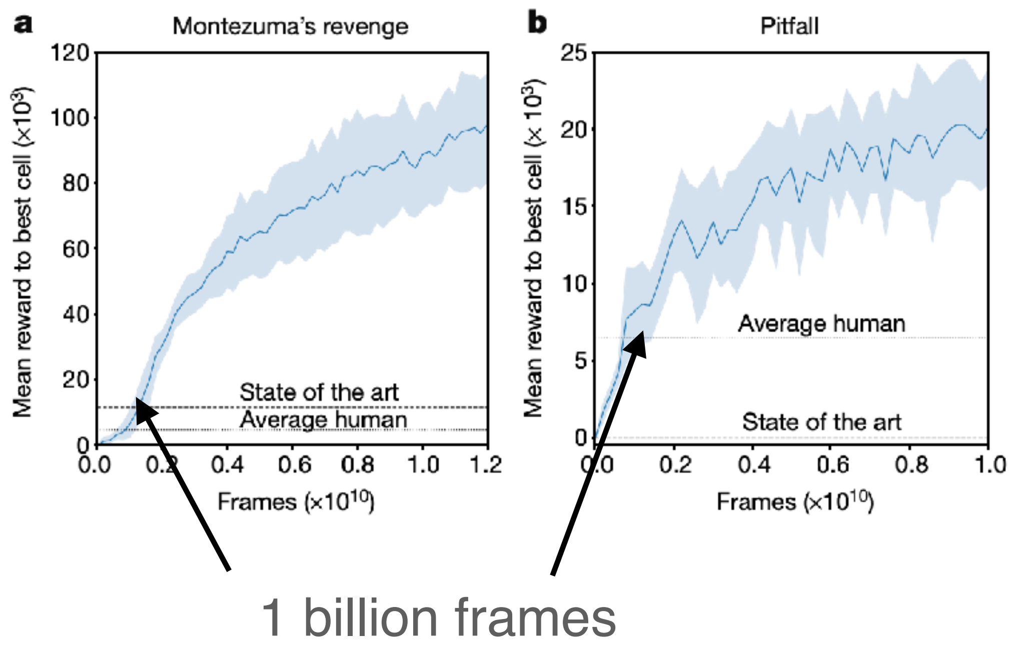
- Vanilla PG is not sample efficient



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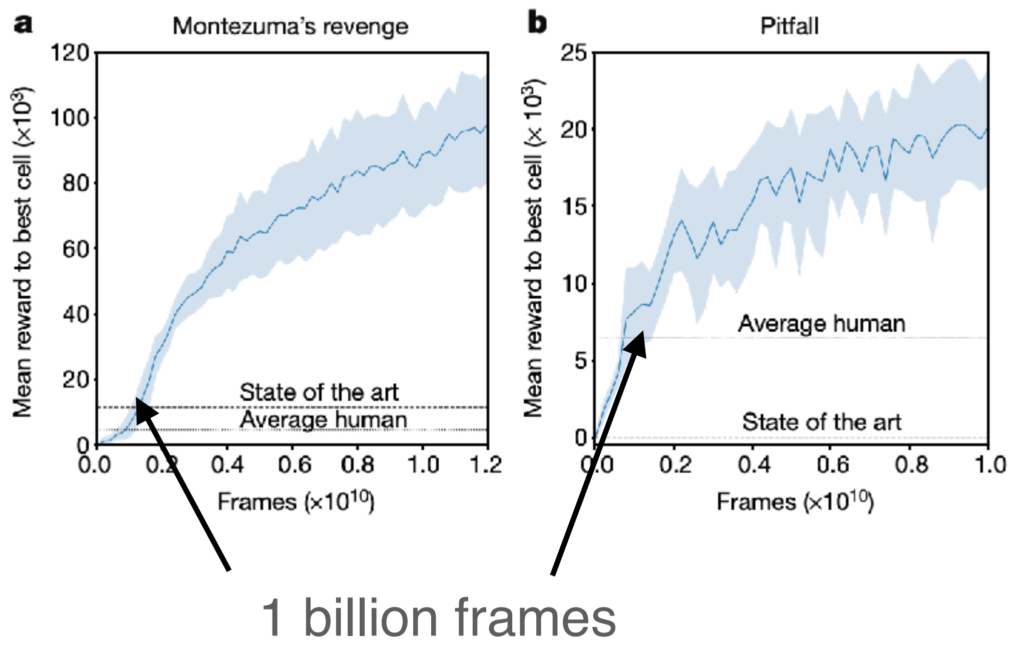

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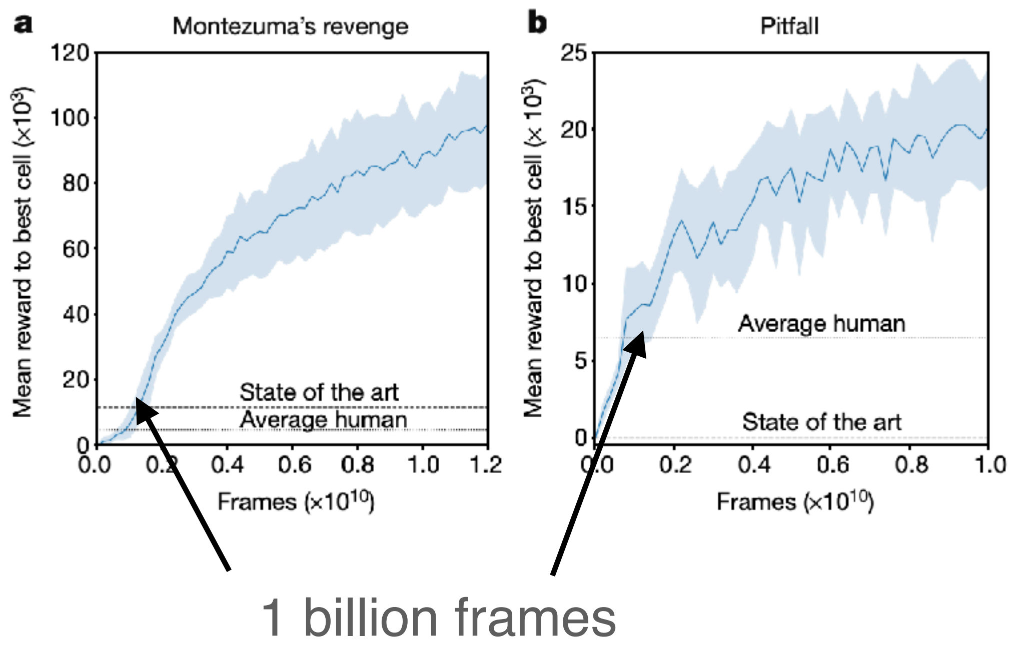

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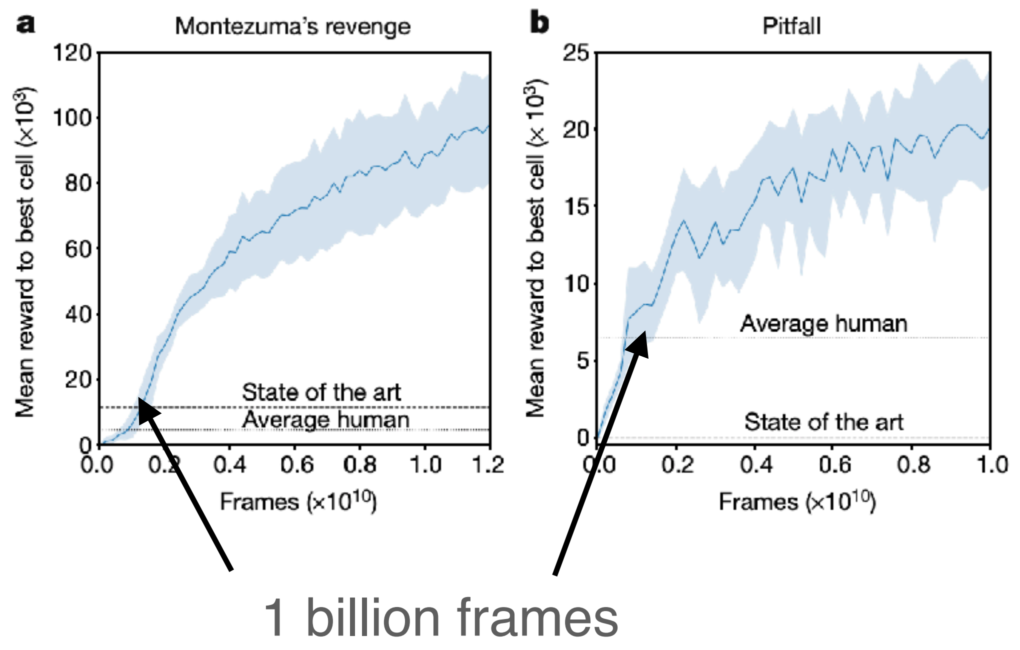

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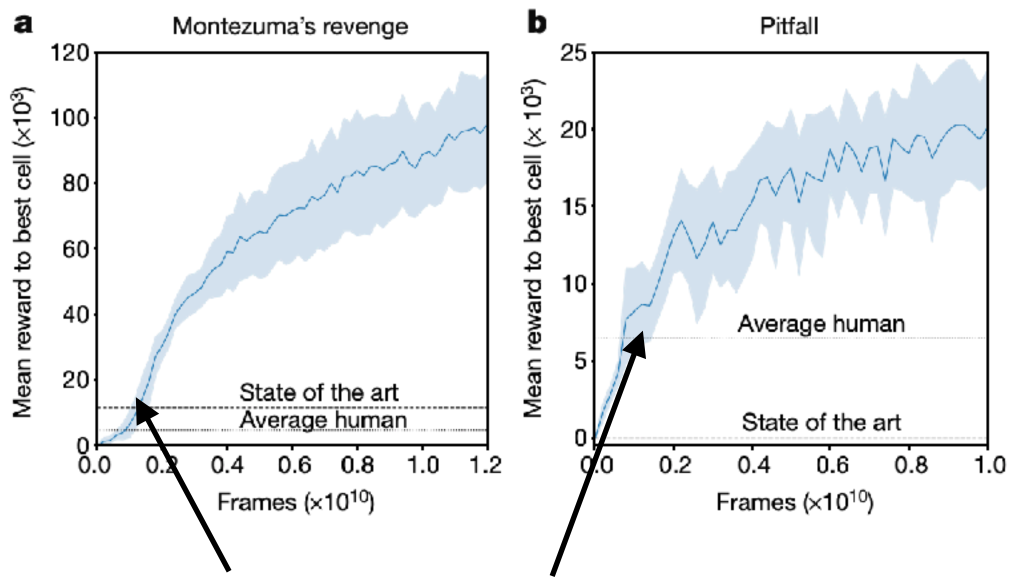

1 billion frames

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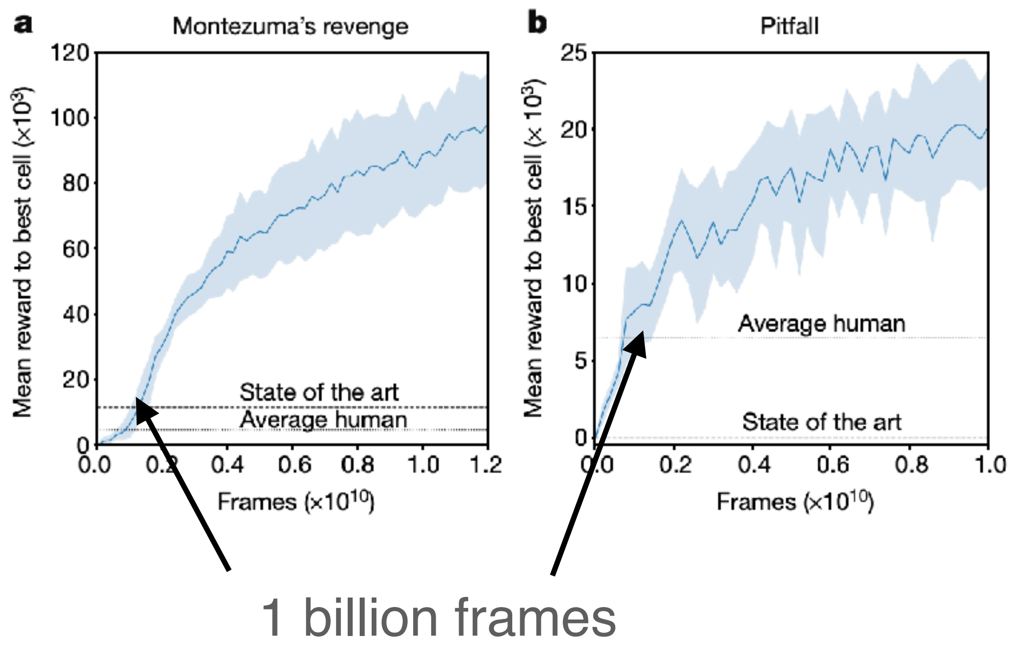

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Motivations

- Extend linear convergence of NPG from tabular to function approximation regime.

PG method: $\theta^{(k+1)} = \theta^{(k)} - \eta_k \nabla_\theta V_\rho(\theta^{(k)})$

Policy gradient theorem (PGT) [Sutton et al., 2000]

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$$Q_{s,a}(\theta) := \mathbb{E}_{a_t \sim \pi_{s_t}(\theta),\ s_{t+1} \sim P(\cdot|s_t,a_t)} \left[\sum_{t=0}^{\infty} \gamma^t c(s_t, a_t) \mid s_0 = s, a_0 = a \right]$$

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$$\begin{aligned} V_\rho(\theta) &= \mathbb{E}_{s\sim\rho}[V_s(\theta)] \\ &= \mathbb{E}_{s\sim\rho, a\sim\pi_s(\theta)}[Q_{s,a}(\theta)] \end{aligned}$$

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$$\nabla_\theta V_\rho(\theta) = \frac{1}{1-\gamma} \mathbb{E}_{s \sim d^{\pi(\theta)}(\rho), a \sim \pi_s(\theta)} \left[Q_{s,a}(\theta) \nabla_\theta \log \pi_{s,a}(\theta) \right]$$

Derivation of Policy Gradient Theorem

$$\nabla_\theta V_\rho(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d^{\pi(\theta)}(\rho),a\sim\pi_s(\theta)}\left[Q_{s,a}(\theta)\,\nabla_\theta \log \pi_{s,a}(\theta)\right]$$

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- *Proof:*

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- *Proof:* $\nabla_\theta V_\rho(\theta) = \nabla_\theta \sum_{s_0\in\mathcal{S},\ a_0\in\mathcal{A}} \rho(s_0)\pi_{s_0,a_0}(\theta)Q_{s_0,a_0}(\theta)$

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&\quad + \sum_{s_0,a_0} \rho(s_0)\pi_{s_0,a_0}(\theta)\,\nabla_\theta\left(c(s_0,a_0) + \gamma \sum_{s_1} P(s_1 \mid s_0,a_0)V_{s_1}(\theta)\right)
\end{aligned}$$

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Bellman Equation

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\end{aligned}$$

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Natural policy gradient

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- Natural policy gradient

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Behave more and more like policy iteration

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Discussion & Conclusion

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- We derive sample efficient policy gradient-based RL convergence theory

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- The linear convergence analysis of NPG with log-linear policy can be extended to general parametrization [Alfano et al., 2023]

Thank you !

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