Asymmetric Hubbert Curve in Indonesia Oil Production

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***Abstract*—Hubbert's Curve has been the most popular method to create an oil production projection since its success in determining the U.S. peak production in 1970. This method assumes the oil production profile of a large region to be symmetrical. This is not the case for many regions including Indonesia. Accordingly, two novel asymmetric Hubbert Curve variants used to create projection of Indonesia's Oil Production are proposed. The first model is based on Original Hubbert Curve with dynamic variance and the second model is based on Generalized Logistic Function. Results are compared to previously published models i.e. Hubbert Curve and its modifications and Arps Decline Curve Analysis. Using Indonesia oil production data, the first model outperforms the other models with the smallest Akaike Information Criterion (AIC) value of calculated oil production and the smallest error of historical cumulative oil production. Furthermore, it shows reasonable estimation of remaining reserve which is relatively close with the exponential decline analysis. The model shows not only great accuracy in calculating Indonesia historical oil production but also realistic projection of Indonesia's remaining reserve.**

Keywords—Hubbert curve, oil production, mathematical modelling, Information theory, statistics distribution

I. Introduction

Oil is an important resource. Its products underpin current modern life, increasing the standard of living. In fact, oil is currently the most important transportation fuels, with 90% of all fuels are generated from crude oil, enabling people to travel all around the world. In addition to that, oil is also an essential resource which powers industry and becomes raw materials for many products such as medicines, plastics, cleaning product and many others [1]. Despite of that, its nature as non-renewable resource is implying that one day the resources availability will come to an end. 2017 BP Statistical review of world energy reported that more than 63% of countries/regions suffer the oil production decline on 2016 [2].

Because of its importance and its limited availability, oil production projection needs to be generated. Even though it is impossible to predict the future of crude oil production, but a good oil production projection model will give people an idea of what to expect [3]. Having a good projection model helps to create a strategic planning of upstream and downstream sectors which are parts of an integrated system of journey of oil products from discovery through to final consumptions [4]. Moreover, a prediction of oil production is important for the macro economy condition of a country especially if the country relies heavily on its export or import of oil resources [5]

Efforts to create an oil projection model are general interests since nearly the beginning of commercial exploitation of oil. One of the most well-known models is Hubbert Curve. This curve states that oil production in a large region over time takes a shape of a bell shaped curve [6]. Following his theory, Hubbert used the first derivative of logistic function to model the oil production [7]. Deffeyes showed later that for symmetrical model, another bell-shaped curve, Gaussian Model fitted better than the logistic function [8]. Furthermore, the original theory believed that the oil production over time would be symmetrical to its peak time. This was not the case for most of the regions [9]. Thus, several modifications to the symmetrical models were made, introducing the asymmetrical models. The asymmetrical models built are the asymmetrical Gaussian model [9] and Gompertz function [10]. The other common models to predict the oil production are the Arps Decline curves, which model the oil depletion and ignore the increasing side of the production [11].

Up until today, the author has not found any asymmetric modification to the logistic curve, even though Hubbert originally introduced his model using the logistic function.

Moreover, there is no study about the fitting comparison of all of those models except for the symmetrical and asymmetrical Gaussian model by Brandt [9]. In this paper, an asymmetric logistic Hubbert Curve with dynamic variance and asymmetric generalized logistic function will be proposed. This paper aims to test the plausibility of the proposed models by comparing them with other models.

II. LITERATURE STUDY

A. Original Hubbert Model

In 1959, M. King Hubbert, an American geophysicist for Shell Oil that time, stated that cumulative oil production overtime would follow a logistic curve, implying that the yearly production would follow the first derivative of this curve [7]. Several assumptions made in Hubbert modelling includes: oil production over time follows a bell-shaped curve; it is symmetrical, meaning that the decline rate of production is exactly the mirror of increasing rate of oil production. [8]

Recalling that according to Hubbert, cumulative oil production overtime would follow a logistic curve. Let $Np(t)$ be the cumulative oil production as a function of time and Np_{max} be the ultimate recoverable resources of oil or the cumulative oil production $Np(t)$ at $t \to \infty$, Hubbert equation can be expressed as:

$$Np(t) = \frac{Np_{max}}{\left(1+e^{k\left(t-T_{peak}\right)}\right)} \tag{1}$$

with k is inverse decay time, and T_{peak} is peak production time [12].

The production rate $P(t)$ can be expressed as the first derivative of $Np(t)$:

$$P(t) = \frac{Np_{max}\,k}{\left(e^{-\frac{k}{2}\left(t-T_{peak}\right)}+e^{\frac{k}{2}\left(t-T_{peak}\right)}\right)^2}. \tag{2}$$

A plot of equation (2) looks like a Gaussian, just like a derivative of logistic curve should be, but it is not. The shape of the curve $P(t)$ is clearly symmetrical at $t = T_{peak}$ and with a simple mathematical approach, one can simply find the P_{max} , maximum value of production rate.

$$P_{max} = \frac{Np_{max}\,k}{4} \tag{3}$$

Thus, by substituting equation (3) to (2), the $P(t)$ can be rewritten as:

$$P(t) = \frac{4\,P_{max}}{\left(e^{-\frac{k}{2}\left(t-T_{peak}\right)}+e^{\frac{k}{2}\left(t-T_{peak}\right)}\right)^2}. \tag{4}$$

Equation (4) will be used to create the Symmetrical Logistic Hubbert Model in this paper.

B. Gaussian Hubbert Model

Hubbert published his analysis using logistic curve and did not justify his choice of method. Because of this reason, Deffeyes conducted a comparison of curve fitting analysis for oil production overtime with wider freedom using 3 common symmetric bell-shaped curve: Gaussian, Logistic and Cauchy curve [8]. He did the comparison using U.S. historical oil production and found that the Gaussian matched with the production profile the best, the logistic fitted less well and the Cauchy missed badly.

This model follows general Gaussian function and the production rate $P(t)$ can be expressed as follows:

$$P(t) = P_{max}.\,e^{-\frac{\left(t-T_{peak}\right)^2}{2\sigma^2}} \tag{5}$$

where σ is the standard deviation of the production curve.

Equation (5) will be used to create the Symmetrical Gaussian Hubbert Model in this paper.

Furthermore, one of the curve that Brandt used for testing the symmetrical Gaussian Hubbert Model was an Asymmetrical Gaussian Model [9]. This model uses dynamic values of standard deviation to build an asymmetrical curve. The standard deviation is a logistic function of time. The equation can be written as follows:

$$P(t) = P_{max}.\,e^{-\frac{\left(t-T_{peak}\right)^2}{2\sigma(t)^2}} \tag{6}$$

where

$$\sigma(t) = \sigma_{dec} - \frac{\sigma_{dec}-\sigma_{inc}}{\left(1+e^{\left(t-T_{peak}\right)}\right)} \tag{7}$$

σ_{dec} = right side standard deviation (for $t \gg T_{peak}$, $\sigma = \sigma_{dec}$)

σ_{inc} = left side standard deviation (for $t \ll T_{peak}$, $\sigma = \sigma_{inc}$).

The curve of standard deviation over time can be found in Fig. 1.

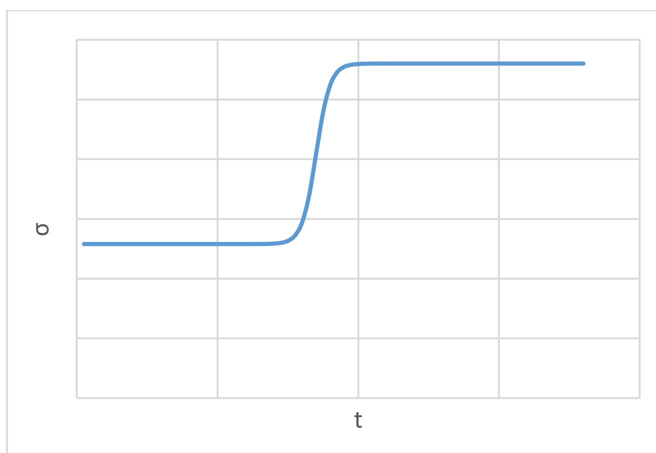

Fig. 1. Standard Deviation vs Time.

Equation (6) and (7) will be used to create the Asymmetrical Gaussian Hubbert Curve.

C. Gompertz Model

Carlson used Gompertz Function as one of the sigmoid function to model World's oil production from 2009 onward to accommodate for additional oil resources, resulting in asymmetrical production profile [10]. Gompertz function is an asymmetrical growth curve developed by Gompertz in 1825 [13] modifying the symmetric logistic function. This function is expressed as:

$$Np(t) = Np_{max}\, e^{-e^{-k(t-T_{peak})}}. \tag{8}$$

Thus,

$$P(t) = \frac{d\,Np(t)}{dt} = Np_{max}.k.e^{-e^{-k(t-T_{peak})}}.e^{-k(t-T_{peak})}. \tag{9}$$

Equation (9) will be used to create Gompertz Model.

D. Arps Decline Curve Analysis

Currently, Arps' decline curve analysis is the most common method to analyze declining production and forecasting future performance of hydrocarbon wells. This method is a graphical method which fits a line into production history. The basic assumption for this method is that whatever affects the production trend in the past will continue to affects the future trend in uniform manner

J.J. Arps wrote in his paper about the previous ideas of decline curve analysis and defined exponential, hyperbolic and harmonic declines [11]. Hyperbolic decline is the general equation for the decline rate, while exponential and harmonic declines are the special case of the hyperbolic case. Exponential decline is the most conservative method among the decline curve analysis, while harmonic decline is the optimistic one [14]. Ling showed that exponential decline occurred when the oil was produced from a reservoir with close-boundary and partial water support [15]. On the other hand, hyperbolic and harmonic cases occur when there is strong pressure support such as partial water support with constant rate and waterflood [15] [16].

The general equation (Hyperbolic function) can be written as follows:

$$P(t) = \frac{P_i}{(1+b.D.(t-t_i))^{\frac{1}{b}}} \tag{10}$$

With P_i is production rate right before the decline, t_i is defined as time right before the production decline, D is decline rate, and b is the hyperbolic decline constant. For hyperbolic case, $0 < b < 1$ applies.

The special case for exponential decline occurs when $b = 0$:

$$P(t) = P_i.\, e^{-D(t-t_i)} \tag{11}$$

and harmonic case occurs when $b = 1$:

$$P(t) = \frac{P_i}{1+D.(t-t_i)} \tag{12}$$

The term b has no unit. This term causes and maintains the shape of the curve especially in long term, thus different value of b will lead to different estimates of Ultimate Recoverable Reserves. However different value of b will not have significant distinctions in the short term, which makes it difficult to determine its value. As the production matures, the reliability of b estimation will also increase, thus making it better for the Ultimate Recoverable Reserve estimates.

Even though the hyperbolic decline is the general form of the other two curves, and making it the most commonly encountered method, exponential and harmonic decline are more frequently used because of the simplicity [17]. In particular exponential curve is the most common method [11].

Unfortunately, decline curve analysis was made only to predict the decline in pseudo steady state condition [11] and cannot be used to model increasing production. Increasing production can be found in the early period of oil production of a certain region whether it is a field or a country. Thus this method cannot be used to describe a complete period of a region's oil production.

III. Proposed Model

A. Generalized Logistic Function

The generalized logistic function is an extension of logistic functions which allows for more flexible S-shaped curves. Numerous applications of this model can be found easily, for example growth phenomena by Richard [18] including the growth of tumors.

The first derivative of this function, has been modified to be used to forecast production for a single well producing from tight reservoir [19]. They found that the modified model suited for forecasting a single well producing from low permeability reservoir falls into a specific subcategory called hyper-logistics, mirroring the behavior of oil flow in low permeability reservoir which declines hyperbolically.

Analogous with Hubbert's Logistic Function, generalized logistic function is used to model cumulative oil production over time. The function can be expressed as:

$$Np(t) = \frac{Np_{max}}{(1+e^{k(t-T^*{}_{peak})})^{\frac{1}{v}}} \tag{13}$$

With v is a factor which affects the curve near which the asymptote occurs and $T^*{}_{peak}$ refers to a time reference [18].

Hubbert model and Gompertz models are two special cases of the generalized logistics function. If $v = 1$, the equation above becomes the Hubbert Model, and if $v \to 0$, the equation above becomes Gompertz Function [20].

The first derivative of this function is oil production rate:

$$P(t) = \frac{Np_{max} .k.e^{k\left(t-T^{*}{}_{peak}\right)}}{v(1+e^{k\left(t-T^{*}{}_{peak}\right)})^{\frac{1}{v}+1}} \tag{14}$$

Equation (14) will be used to create Asymmetrical Generalized Logistic Curve.

Note that in this function $T^{*}{}_{peak} \neq T_{peak}$ because of its asymmetrical nature. To find the real T_{peak}, one can calculate for t in which the second derivative of $Np(t)$ equals to 0.

B. Asymmetrical Logistic Hubbert Model

This model is based on the original Hubbert model, the logistic curve. The form of this model follows the first derivation of logistic curve in predicting the oil production, equation (4) but with dynamic value of k. Similar to the asymmetrical Gaussian Model, the value of k is a sigmoid function of time. The oil production rate is expressed as:

$$P(t) = \frac{4\,P_{max}}{(e^{-\frac{\sigma(t)}{2}\left(t-T_{peak}\right)}+e^{\frac{\sigma(t)}{2}\left(t-T_{peak}\right)})^{2}} . \tag{15}$$

The function for $\sigma(t)$ can be taken from equation (7), which is a sigmoid function that changes value at $t \to T_{peak}$. The typical $\sigma(t)$ curve can be found at Fig. 1.

Note that $\sigma(t)$ for this model is different from the $\sigma(t)$ acquired from Asymmetrical Gaussian Hubbert Model, even though both of them affect the growth rate of the cumulative curves.

Equation (15) will be used to create the Asymmetrical Logistic Hubbert Model.

Unlike the Generalized Logistic function, it is difficult to calculate for the cumulative oil production by integrating the $P(t)$ of this model. Instead, the sum of cumulative yearly production will be used to estimate the cumulative oil production of certain year t.

It can be observed at Fig. 2 that both the proposed models fit well with the actual cumulative oil production and form sigmoid growth function. The only observable aberrations can be found at 1980-1995 which is the period around peak production and after 2020. Judging from the difference between the two curves after 2020, the ultimate recoverable reserves calculated by the Asymmetric Logistic Model is greater than the Generalized Logistic Model.

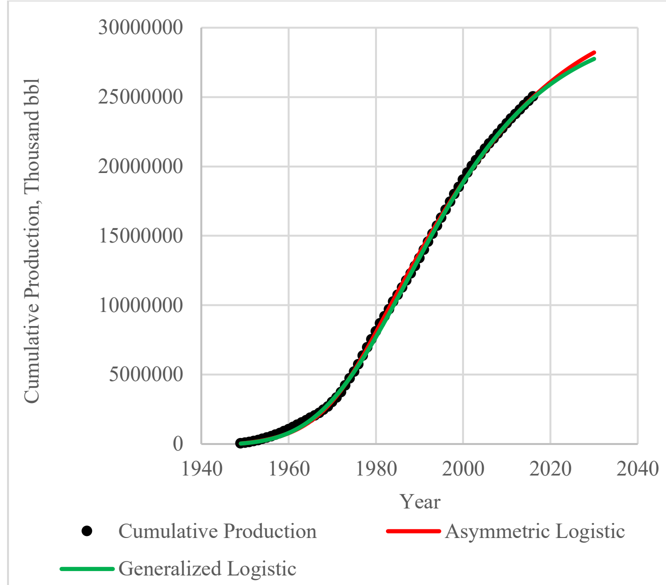

Fig. 2. Cumulative Oil Production Overtime - Proposed Model

IV. Data and Methodology

In this paper, three things will be analyzed. First of all, the proposed models will be compared with the other modified Hubbert oil projection models mentioned in the section earlier to find the best-fit model. Secondly, the depletion section of the best-fit model, will be compared with the Arps Decline curves, which are the commonly used method of determining the oil projection in the future. Finally, all of the models calculated historical cumulative oil production will be compared with the actual data and the remaining reserves forecasted by the models will be analyzed.

A. Datasets used

The historical oil production profile used is Indonesia's historical oil production profile from 1949 until 2016 as the most recent one. All of the models will be fit into Indonesia's oil production history which implies that the best-fitting model will be chosen to forecast Indonesia's oil production and this do not conclude that the best-fitting model fits into nor can be used to forecast the other countries' or even world's oil production profile. Another study will need to be done to achieve those objectives.

Indonesia's historical oil production data is acquired from BP statistical review of world energy 2017 report [2] which starts from 1965 up until 2016. Complementing the data, another source will be used to acquire production data from 1949 up until 1965 [21].

The summary of datasets used can be find in Table I.

TABLE I. DATASETS USED IN ANALYSIS

Regional Level	Source	Years
Indonesia National	[21]	1949-1964
	[2]	1965-2016

B. Methodology to Determine Best-Fitting Model

Using a curve fitting software, the parameters were iterated so that the model simulate the best-fitting results to the actual production data. The summary of models and their parameters to be fit can be found in Table II.

TABLE II. SUMMARY OF HUBBERT & MODIFIED HUBBERT MODELS

Model	Number of Parameters	Parameters to be fit
Symmetrical Logistic Hubbert	3	P_{max}; T_{peak}; k
Symmetrical Gaussian Hubbert	3	P_{max}; T_{peak}; σ
Asymmetrical Gaussian Hubbert	4	P_{max}; T_{peak}; σ_{dec}; σ_{inc}
Gompertz Model	3	P_{max}; T_{peak}; k
Asymmetrical Generalized Logistic	4	P_{max}; T_{peak}; k; υ
Asymmetrical Logistic Hubbert	4	P_{max}; T_{peak}; σ_{dec}; σ_{inc}

After the best value of those parameters were calculated, the quality of fit across models needs to be compared. Sum of squared error (SSE), which is a common way to do this, is not applicable to the models because of different number of parameters used. The higher number of parameters tend to give smaller value of SSE because the flexibility of the model with higher number of parameters [9].

Several approaches are available to deal with this problem. In this paper, Akaike's Information Criterion (AIC) will be used because it allows models comparison of different complexity accounting for the advantage the more complex model has in fitting [22].

AIC score is expressed as:

$$AIC_c = N.\ln\left(\frac{SSE}{N}\right) + 2K + \frac{2K(K+1)}{N-K-1} \tag{16}$$

where:

AIC_c= corrected AIC score

N = number of data points in data series

K = number of model parameters.

The model with smallest value of AIC_c is the most likely to be the best-fitting model. AIC is based on information theory instead of statistics, thus the model cannot be rejected or accepted statistically [22]. This method is applicable if two or more models are being compared, and the probability of one model being correct compared to the others can be calculated using:

$$\text{Probability} = \frac{e^{-0.5.\Delta AIC}}{1+e^{-0.5.\Delta AIC}} \tag{17}$$

with

$\Delta AIC = AIC\ (best\ fitting) - AIC\ (second\ best\ fitting).$

C. Methodology to Analyze Comparison of Best-Fitting Model with Arps Decline Curve

The comparison between best-fitting model and the Arps Decline Curve will be analyzed visually by plotting decreasing part of the oil production. The best-fitting model cannot be compared quantitatively with the Arps Decline Curve because of the difference in parameters and number of points since the decline curves only account for the oil rate depletion [11]. The comparison made will be done by extrapolating the curves to year 2050. This is because parameter b will only show clear distinction in long term.

V. RESULTS AND ANALYSIS

A. Best-Fitting Model Result

Model fitting results can be seen at Fig. 3. It can be observed that all of the asymmetrical models act similarly at the starting period of production in 1950s. The clear distinction starts to be observable near the peak production in 1980-1990. The Asymmetric Logistic reaches the peak production the fastest followed by Asymmetric Gaussian, while both symmetric curves reach the peak production the slowest. In the longer term, symmetric curves show faster drop while asymmetric logistic model shows the slowest drop in the oil production, bearing the most optimistic result.

The best fit model, which has the lowest AIC value, is the asymmetric logistic model as can be seen at Fig. 4. The second best-fitting model is the asymmetric Gaussian model followed by asymmetric generalized logistic and Gompertz model. Asymmetric logistic model has a ~98% probability of being correct model compared to the second best, Asymmetric Gaussian model.

Having said that, Asymmetric Generalized Logistic model and Gompertz model, have an advantage compared to both asymmetric logistic and Gaussian models. Asymmetric logistic and Gaussian models need the right side of the production profile to be able to fit the σ_{dec}, which is not always the case especially when a region just recently started the production. Meanwhile, Asymmetric Generalized Logistic and Gompertz model can be used to model the peak and depletion part of the production profile asymmetrically even when the production data is still in the inclining part. Generalized Logistic model has slightly lower AIC than the Gompertz, and the Generalized Logistic model's probability of being right when being compared to the Gompertz model is ~51%, implying that both of the model is nearly equally correct [22].

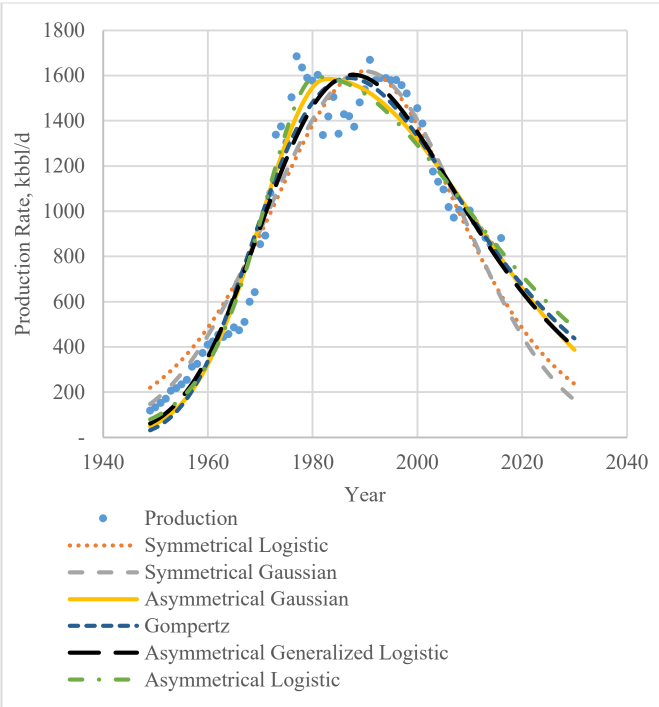

Fig. 3. Model Fitting Results

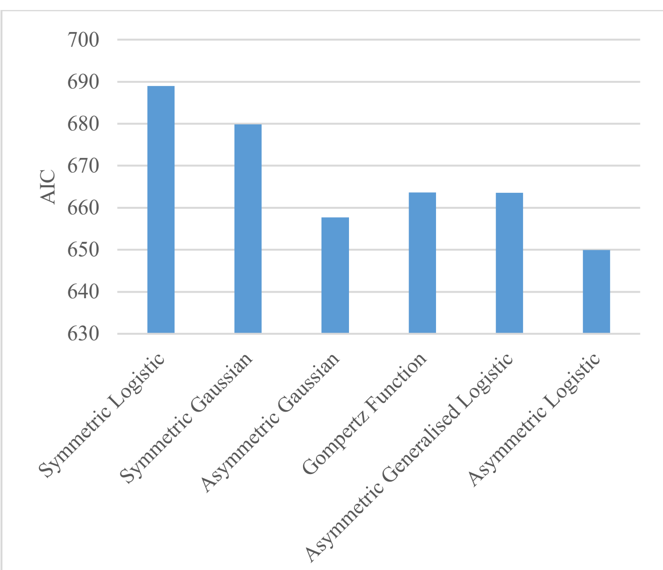

Fig. 4. AIC Comparison

B. Comparison with Arps Decline Curve

After the fitting process, hyperbolic decline shows that parameter $b > 1$, therefore hyperbolic decline curve will not be used in this analysis since Arps mentioned that $0 < b < 1$ [11].

Based on Fig. 5, from around 2010 up until 2035, the oil rate projection from asymmetric logistic curve is greater than exponential decline curve. While harmonic decline is generally higher than the other two curves. In the short term, the gradient of oil rate over time is steeper than the asymmetric logistic model implying that the oil production drops faster in both exponential and harmonic decline. In longer term, the oil production rate projected by asymmetric logistic model is lower than both the harmonic and exponential curve.

Based on Fig. 5, from around 2010 up until 2035, the oil rate projection from asymmetric logistic curve is greater than exponential decline curve. While harmonic decline is generally higher than the other two curves. In the short term, the gradient of oil rate over time is steeper than the asymmetric logistic model implying that the oil production drops faster in both exponential and harmonic decline. In longer term, the oil production rate projected by asymmetric logistic model is lower than both the harmonic and exponential curve.

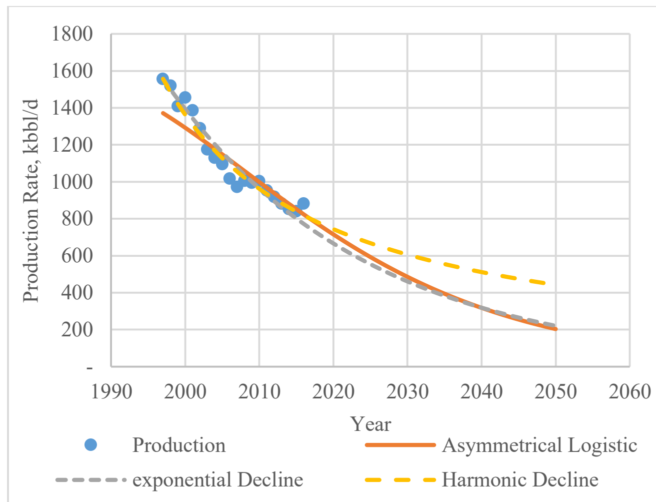

Fig. 5. Comparison of Asymmetric Logistic Curve with Arps Decline Curves

Based on the observation done visually to Fig. 5, both exponential and harmonic decline pose better results to the curve fitting than the asymmetric logistic model, thus making it better to use the Arps Decline Curve analysis to forecast the oil depletion profile if it has been declining long enough so that there is enough data available for the curve fitting.

C. Comparison of Historical Cummulative Oil Production and Remaining Reserves Forecasted by Each Models

In the 1949-2016 period, Indonesia had produced 25.04 billion barrels of oil [2]. Generally, all of the models calculate the cumulative oil production relatively close to this actual value with less than 0.7% error. The comparison of calculation done by each model to the actual cumulative oil production can be found in Table III.

In line with the result from AIC, Asymmetrical Logistic Model shows the least error when compared to the actual cumulative oil production data with only 0.21% of error. On the other hand, the generalized logistic model shows the second least error with 0.5%. This result deviates from the AIC result which shows this second proposed model as the third best-fitting model.

TABLE III. COMPARISON OF CALCULATED AND ACTUAL HISTORICAL CUMULATIVE OIL PRODUCTION 1949-2016 (BBBL)

Model	Cumulative Production (Bbbl)			Remaining Reserve (Bbbl)	
	Calculated	Actual	Error (%)	Projected	Proven
Symmetrical Logistic	25.20	25.04	0.64	3.24	3.3
Symmetrical Gaussian	25.20		0.66	2.35	
Asymmetrical Gaussian	24.88		0.62	4.55	
Gompertz	24.87		0.69	6.10	
Asymmetrical Generalized Logistic*	24.91		0.50	5.40	
Asymmetrical Logistic*	24.99		0.21	7.03	
Exponential Decline	-		-	7.48	
Harmonic Decline**	-		-	20.45	
Hyperbolic Decline***	-		-	22.06	

* Proposed models

** Harmonic Decline comes with optimistic result [14], usually occurs when there is strong reservoir pressure support [15] [16].

*** Hyperbolic Decline is invalid because of $b > 1$.

In addition to the historical cumulative oil production, the models are also used to forecast the remaining reserve. Usually reserves are classified on the degree of certainty. Proven Reserve is associated to a high probability level of confidence (90% or P90 in the probabilistic approach of the SPE/WPC/AAPG rules). This means that there is a 90% possibility that the actual remaining reserve is greater than the proven reserve [23]. 2017 BP Statistical Review reported that Indonesia's Proven Reserve at the end of 2016 is 3.3 Bbbl [2].

In line with the definition of proven reserve, the remaining reserves forecasted by most of the models are higher than Indonesia's Proven Reserve (Table III). The remaining reserve projected by Symmetrical Logistic Model poses the closest result to the proven reserve at 3.24 Bbbl, while the most optimistic result is showed by the harmonic decline with 20.45 Bbbl (Note that the hyperbolic decline shows $b > 1$). The proposed model Asymmetric Logistic predicts similar remaining reserve to the exponential decline as the most commonly used method to determine remaining reserve [15], with 7.03 and 7.48 Bbbl respectively, implying that the projection made by Asymmetric Logistic Model is reasonable. The second proposed model Asymmetric Generalized Logistic shows more pessimistic result with 5.4 Bbbl, close with the Gompertz model with 6.1 Bbbl.

These optimistic projection values can be achieved by continuing to do everything that has been done in the past. An assumption used in Arps Decline Analysis, which is whatever affects the production trend in the past will continue to affects the future trend in uniform manner [11], also applies to all of the empirical models mentioned in this paper. Thus, the projected remaining reserves can be recovered by continuing the efforts to arrest declines and finding new resources, i.e. Enhanced Oil Recovery (EOR) efforts, exploration, workover etc.

VI. CONCLUSION

This work develops two new models to forecast production of large region that shows asymmetrical behavior. The plausibility of the two models is tested by comparing them with the other commonly used models and their modifications using Indonesia oil production data.

Among all the Hubbert's curve and its modifications, the best-fitting model to Indonesia's oil production data is one of the proposed models, Asymmetric Logistic Model with dynamic variance. Using Akaike Information Criterion (AIC), the probability of this model being a correct model if compared to the second best model, i.e. Asymmetric Gaussian Model is 98%.

Referring to the same method, Generalized Logistic Model and Gompertz Model show third and fourth best AIC value respectively. Even though their AIC value is not as good as both the Asymmetrical Logistic and Gaussian Model, they have their own merit. These two models can be used to create oil production projection in the case of no declining part of the production available.

Besides the Hubbert's curve and its modifications, the other commonly used projection method is the Arps' Decline Curves Analysis. This method consists of three methods, which are Exponential, Harmonic and Hyperbolic. In Indonesia's Oil production case, the hyperbolic curve cannot be used since the fitting results in $b > 1$. Based on visual observation, both the exponential and harmonic curves show better fit than the Asymmetric Logistic Model. However, the Arps decline curve can only be generated if a long period of decline exists. Thus, Arps Decline Curves should be used to forecast the oil production if such condition applies.

Both the proposed models show better accuracy in calculating the historical cumulative oil production than all of the other Hubbert's curve and its modifications. Compared to Indonesia actual oil production data, the Asymmetrical Logistic Model shows only 0.2% of error while the Generalized Logistic Model shows 0.5% of error.

Generally, most of the models mentioned in this paper show more optimistic remaining reserves projections compared to Indonesia's Proven Reserves. The Harmonic Curve is the most optimistic method of forecasting with 20.5 Bbbl of Remaining Reserves. Following this, Exponential Decline as the most commonly used method in forecasting the remaining reserve, shows the closest result of projection to the proposed model, Asymmetric Logistic Model, with 7.48 Bbbl and 7.03 Bbbl respectively. This means that the remaining reserve projection by the Asymmetric Logistic Model is reasonable.

Using Indonesia oil production data, the Asymmetric Logistic Model has showed not only great accuracy in determining Indonesia past production data and cumulative oil production, but also realistic future projection of remaining reserve.

References

[1] OPEC, "I need to know - An Introduction to the Oil Industry & OPEC," 2013.

[2] BP, "BP Statistical Review of World Energy," British Petroleum, 2017.

[3] U. Bardi and A. Lavacchi, "A Simple Interpretation of Hubbert's Model of Resource Exploitation," *Energies,* pp. 646-661, 2009.

[4] C. Robinson and A. Scott, Strategic Planning for the Oil and Gas Industry, Edinburgh, 2016.

[5] J. D. Hamilton, "Oil and the Macroeconomy," *Palgrave Dictionary of Economics,* 2005.

[6] K. M. Hubbert, "Nuclear energy and the fossil fuels," in *Meeting of the Southern District, Division of Production, American Petroleum Institute*, San Antonio, Texas, 1956.

[7] K. M. Hubbert, "Techniques of prediction with application to the Petroleum Industry," in *44th Annual meeting of the American Association of Petroleum Geologists*, Dallas, Texas, 1959.

[8] K. S. Deffeyes, Hubbert's Peak - The Impending World Oil Shortage, Reissue ed., Princeton, New Jersey: Princeton University Press, 2008.

[9] A. R. Brandt, "Testing Hubbert," in *Energy in World of Changing Costs and Technologies*, 2006.

[10] W. B. Carlson, "The Modeling of World Oil Production Using Sigmoidal Functions," *Energy Sources, Part B: Economics, Planning, and Policy,* vol. 6, no. 2, pp. 178-186, 2011.

[11] J. J. Arps, "Analysis of Decline Curves," in *Houston Meeting*, Houston, Texas, 1944.

[12] J. Claerbout and F. Muir, "Stanford Exploration Project," [Online]. Available: http://sepwww.stanford.edu/sep/jon/hubbert.pdf.

[13] B. Gompertz, "On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies," *Philosophical Transactions of the Royal Society,* vol. 182, pp. 513-585, 1825.

[14] W. A. Beninger and R. H. Caldwell, "Estimating Decline Curves with Confidence," *Oil and Gas Journal,* vol. 89, no. 40, 1991.

[15] K. Ling, "Theoretical Bases of Arps Empirical Decline Curves," in *Abu Dhabi International Petroleum Exhibition and Conference*, Abu Dhabi, UAE, 2012.

[16] R. O. Baker, T. Anderson and K. Sandhu, "Using Decline Curves to Forecast Waterflooded Reservoirs: Fundamentals and Field Cases," in *Canadian International Petroleum Conference*, Calgary, Alberta, Canada, 2003.

[17] E. Shirman, "Universal Approach to the Decline Curve Analysis," in *49th Annual Technical Meeting of The Petroleum Society*, Calgary, Alberta, Canada, 1998.

[18] F. J. Richard, "A Flexible Growth Function for Empirical Use," *Journal of Experimental Botany,* pp. 290-300, 1959.

[19] A. J. Clark, L. W. Lake and T. W. Patzek, "Production Forecasting with Logistic Growth Models," in *SPE Annual Technical Conference and Exhibition*, Denver, Colorado, 2011.

[20] X. Yin, J. Goudriaan, E. A. Lantinga, J. Vos and H. J. Spiertz, "A Flexible Sigmoid Function of Determinate Growth," *Annals of Botany,* vol. 91, no. 3, pp. 361-371, 2003.

[21] V. S. Purba, "Penentuan Total Cadangan Minyak Nasional Indonesia dengan Metoda Perhitungan Kurva Puncak Hubbert dan Pendekatan Numerikal terhadap Grafik Produksi Minyak Nasional Indonesia," Institut Teknologi Bandung, Bandung, 2007.

[22] H. Motulsky and A. Christopoulos, Fitting Models to Biological Data using Linear and Nonlinear Regression - A Practical Guide to Curve Fitting, 1 ed., New York: Oxford University Press, 2004.

[23] J. Laherrere, "Forecasting Future Production from Past Discovery," *Int. J. Global Energy,* vol. 18, no. Nos.2/3/4, pp. 218-238, 2002.

[24] P. F. Verhulst, Recherches mathématiques sur la loi d'accroissement de la population, Brussels: Nouv. mém. de l'Academie Royale des Sci. et Belles-Lettres de Bruxelles, 1845.

[25] M. Hook, R. Hirsch and K. Aleklett, "Giant Oil Field Decline Rates and Their Influence on World Oil Production," *Energy Policy,* vol. 37, no. 6, pp. 2262-2272, 2009.

[26] M. I. Kabir, "Normalised Plot - A Novel Technique for Reservoir Characterisation and Reserves Estimation," in *SPE Asia Pacific Oil and Gas Conference*, Adelaide, Australia, 1996.