Appendix

A. Indexes, sets, parameters and decision variables

Table A1. Indexes, sets, parameters and decision variables

Indexes:	
i, j	Index of activities
$k = 1, \dots, K$	Index of the resource types
t	Index of time periods
$e, e' = 1,2, \dots, E$	Index of choices
Sets:	
$N = \{0,1, \dots, n, n+1\}$	Set of activities
A	Set of precedence relationships
ACT_t	Set of activities being performed at time period t
M	Set of mandatory activities
Q	Set of optional activities
B	Set of dependent activities
Q_e	Set of optional activities corresponding to choice e
B_j	Set of activity j's dependent activities
Ω	Set of scenarios
Parameters:	
δ_{ij}	The minimum finish-start time lag between activities i and j
$\boldsymbol{d}_i$	Stochastic duration of activity i
$\bar{d}$	Project deadline
r_{ik}	Requirement for the kth resource type of activity i
$a(e)$	The activity that triggeres choice e
lp_{ij}	Longest path length between activities i and j
c_k	Unit penalty cost of resource type k
C_1	Penalty cost per time unit for the project delay
C_2	Unit penalty cost when two activities overlap
$\hat{d}$	Upper bound of the realized project completion time
T	A large enough positive number
ω	A scenerio
$p(\omega)$	Probability of the occurrence of scenario ω
$d_{i,\omega}$	Duration of activity i in scenario ω
$es_{\omega,i}$	Earliest start time of activity i in scenario ω
$ls_{\omega,i}$	Latest start time of activity i in scenario ω
$lp_{\omega,ij}$	Longest path length between activities i and j in scenario ω
Decision variables:	
$\boldsymbol{s}_i$	Start time of activity i
$\boldsymbol{u}_{kt}$	Total usage of resource type k during time period t
x_{it}	Indicate whether activity i starts at time period t
Δ_{ij}	The amount of time that activity i overlaps with j
y_i	Indicate whether activity i is implemented
$x_{\omega it}$	Indicate whether activity i is started at time period t in scenario ω
$u_{kt\omega}$	Total usage of resource type k during time period t in scenario ω
$\Delta_{\omega ij}$	The amount of time that activity i overlaps with j in scenario ω
$\varphi_{kt\omega}$	Auxiliary variable to linearize $\lvert \boldsymbol{u}_{k,t+1} - \boldsymbol{u}_{kt} \rvert$
$\mu_{\omega ij}$	Auxiliary variable to linearize constraint (20)

B. Proof of Proposition 2

Proof of Proposition 2: In a completely serial project network, since the activities can only be executed sequentially, it is obvious that all activities must be implemented; otherwise, the project will not be finished. Further, since each activity has only one successor in a completely serial project network and the activities are topologically numbered, there exisits only one execution order that can be obtained by ranking activity numbers in ascending order. Therefore, the implementation activity list in the optimal policy can only be in this order.

For the shift key list, we first consider $\delta_{ij} \leq 0$. Then Proposition 2 means that the shift key list is obtained by letting each activity start immediately after its predecessor. To prove this case, let us first consider a completely serial project consisting of two activities a and b. There are three cases (Figure A1) for the possible solutions.

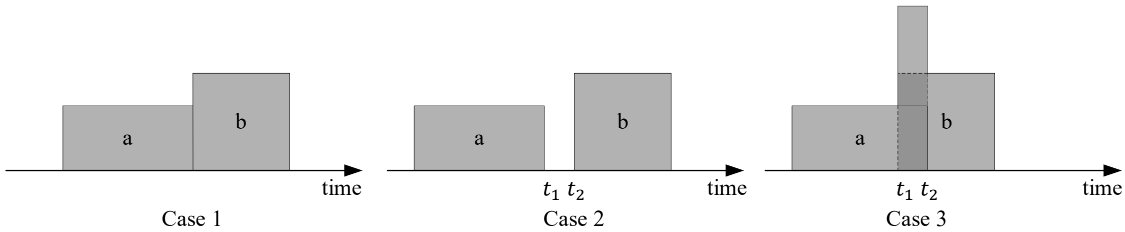

Figure A1. Three cases for the possible solutions corresponding to the shift key list ($\delta_{ij} \leq 0$)

(1) Case 1, in which activity b is started immediately after activity a is finished. We denote the value of objective function (1) in this case as $OBJ^{(1)} = O1 + O2 + O3$.

(2) Case 2, in which activity b is started after activity a has been finished for some time units. In this case, $OBJ^{(2)} = O1 + O1' + O2 + O3 + O3'$, where $O1'$ represents the additional resource fluctuation cost between time point t_1 and t_2, $O1' > 0$; $O3'$ is the cost caused by the possible delay of activity b, $O3' \geq 0$. Therefore, $OBJ^{(2)} > OBJ^{(1)}$.

(3) Case 3, in which activities a and b are overlapped. In this case, $OBJ^{(3)} = O1 + O1' + O2 + O2' + O3$, where $O1' > 0$; $O2'$ is the cost of overlapping activities a and b, $O2' > 0$. Therefore, $OBJ^{(3)} > OBJ^{(1)}$.

Obviously, case 1 corresponds to an optimal shift key list that is a zero vector. Note that the relative size of activities a and b's resource requirements does not affect the analysis results.

Then, we consider $\delta_{ij} > 0$. In this case, Proposition 2 means that the shift key list is obtained by starting each activity j immediately after its predecessor i has been finished for δ_{ij} time units. This is obvious since delaying the start of activity j may cause a delay cost, which will increase the objective function value.

The above results can be generalized to a project with more than two activities because a solution can be constructed by sequentially considering each pair of adjacent activities. ■

C. Generating an initial individual

Step 1: Generating IL randomly.

Step 1.1: All activities in the set M must be implemented, i.e., let $il_i = 1, i \in M$. Set the values for all other il_i to 0. Sort the choice numbers in ascending order.

Step 1.2: Take the first choice e from the current ordered choice numbers.

Step 1.3: If choice e is triggered (i.e., $il_{a(e)} = 1$), then we randomly choose an activity i to implement from the optional activity set Q_e (i.e., $il_i = 1$). The rest activities in Q_e are not implemented.

Step 1.4: If the above implemented activity i has a dependent activity set B_i, then all its dependent activities must be implemented, i.e., $il_j = 1, j \in B_i$.

Step 1.5: Remove e from the current ordered choice numbers. If all choice numbers have been removed, then stop; else go to step 1.2.

Step 2: Generating AL according to the regret-based random sampling.

Step 2.1: Let $AL = (0)$.

Step 2.2: Based on the current AL, determine the eligible set that consists of implemented activities (whose predecessors have been placed in AL) indicated by IL.

Step 2.3: Select an activity i using the roulette wheel based on the cumulative probability of the activity from the eligible set. The selection probability of an activity is calculated based on regret-based random sampling. The regret value ρ_i of activity i is $\max\{v(j) - v(i),\ i \in E_\ell\},\ v(i) = LST(i)$. The late start time ($LST$) is calculated based on the critical path method in the project network formed by the implemented activities indicated by IL. During the calculation, we use the mean of the stochastic duration as the duration of each activity. Then, the probability of activity i $p_i = \frac{(\rho_i+\varepsilon)^\alpha}{\sum_{j\in E_\ell}(\rho_j+\varepsilon)^\alpha}$, $\varepsilon = \alpha = 1$.

Step 2.4: Put i to the end of AL. If all implementation activities have been put to AL, then randomly insert the non-implemented activities into AL and stop; else go to step 2.2.

Step 3: Generating SL randomly.

Step 3.1: For each implemented activity i that is indicated by IL, the value of the shift key sl_i is randomly chosen from the interval [0,1].

Step 3.2: The shift keys of the rest activities are set to 0.

D. Dataset generation

First, we generate instances with fixed project structures. These instances belong to four sub-datasets (i.e., SET5, SET10, SET30 and SET60) that will be finally adapted to form SET. Each instance in SET5 (SET10, SET30, SET60) has 5 (10, 30, 60) non-dummy activities. The instances in SET5 and SET10 are generated using the project scheduling instance generator ProGen (Kolisch et al., 1995) with the parameter settings shown in Table A2. The network complexity (NC) defines the average number of arrows associated with each node. The resource factor (RF) reflects the average number of resource types required by each activity. The resource strength (RS) expresses the scarceness of the resources. For SET5, we produce five instances under each parameter combination, and this results in a total of $1 \times 1 \times 2 \times 2 \times 1 \times 5 = 20$ instances. Similarly, 20 instances are also generated for SET10. We select 20 instances (instance number being $6,16,26,\cdots,196$) from the J30 and J60 dataset of PSPLIB (Kolisch & Sprecher, 1996) to form SET30 and SET60, respectively.

Table A2. Parameter settings of SET5 and SET10

Dataset	$\lvert N \rvert$	NC	RF	RS	$\lvert K \rvert$
SET5	7	1.5	0.3, 0.7	0.2, 0.8	4
SET10	12	1.5	0.3, 0.7	0.2, 0.8	4

Then, based on the parameter settings shown in Table A3, we extend the above instances such that the resulting instances have flexible structures. In Table A3, N^E represents the number of choices in an instance. N^Q indicates the number of optional activities in a choice. N^C denotes the number of optional activities that can trigger dependent activities. N^B is the number of dependent activities of an optional activity. After the extension, the number of instances in SET5/SET10 is still 20. The number of instances in SET30/SET60 becomes $2 \times 2 \times 2 \times 2 \times 20 = 320$. Therefore, there are a total of $20 + 20 + 320 + 320 = 680$ instances in the benchmark dataset SET.

Table A3. Parameter settings for the flexible project structure

Data set	N^E	N^Q	N^C	N^B
SET 5	1	2	1	1
SET 10	1	2	1	2
SET 30	2; 4	2; 3	1; 2	1; 3

For each non-dummy activity i and for each scenario ω, we sample a random integer from the uniform distribution U~$[d_i - \sqrt{d_i}, d_i + \sqrt{d_i}]$ as its duration $d_{i\omega}$, where d_i is the deterministic duration given by ProGen and PSPLIB. It can be seen that the mean of the stochastic activity duration is d_i. For the minimum time lag δ_{ij} between activities i and j, we sample a random integer from the uniform distribution U~$[-0.5d_i, 0]$.

E. MILP model M3 for the DRLP-PS

(M3) Minimize $\sum_{k=1}^{K} \sum_{t=1}^{\hat{d}-1} c_k \lvert u_{k,t+1} - u_{kt} \rvert + C_1 \sum_{t=\bar{d}}^{\hat{d}} (t - \bar{d}) \cdot x_{n+1,t} + C_2 \sum_{(i,j)\in A} \Delta_{ij}$ (A1)

Subject to:

(2)-(5),(9)-(10)

$\sum_{t=1}^{\hat{d}} (t + \bar{d}_i + \delta_{ij}) \cdot x_{it} \le \sum_{t=1}^{\hat{d}} t \cdot x_{jt} + T(1 - \sum_{t=1}^{\hat{d}} x_{jt}) \quad (i,j) \in A$ (A2)

$$u_{kt} = \sum_{i \in N} r_{ik} \sum_{\tau = max\{es_i, t - \bar{d}_i + 1\}}^{min\{t, \hat{d}\}} x_{i\tau} \qquad k = \{1, \dots, K\}; t = \{1, \dots, \hat{d}\} \tag{A3}$$

$$\Delta_{ij} = \max\{0, \sum_{t=1}^{\hat{d}} (t + \bar{d}_i) \cdot x_{it} - \sum_{t=1}^{\hat{d}} t \cdot x_{jt} - T(1 - \sum_{t=1}^{\hat{d}} x_{jt})\} \qquad (i, j) \in A \tag{A4}$$

where objective function (A1) is obtained by replacing the uncertain parameters in (1) with deterministic ones. Compared with M0, the stochastic duration variable $\boldsymbol{d_i}$ is replaced with the mean duration $\bar{d}_i$. Constraints (A2)-(A4) have the same meaning as constraints (6)-(8) of model M0.