## Assignment2

Q1(a) There is the code to complete the request:

```
1. import pandas as pd
2. import numpy as np
3. import random
4. import matplotlib.pyplot as plt
5. from sklearn.datasets import make_classification
6. from sklearn.preprocessing import StandardScaler
7. from sklearn.tree import DecisionTreeClassifier
8.
9. #Ouestion 1a
10. #generate data
11. X, y = make_classification(n_samples=1000, n_features=20, n_informative=5,
n_redundant=15, shuffle=False, random_state=4)
12.
13. #Normalize the data
14. scaler = StandardScaler()
15. scaler.fit(X)
16. X = scaler.transform(X)
17.
18. #shuffle the data
19. shuffle_idxs = np.random.default_rng(seed=0).permutation(X.shape[1])
20. X shff = X[:,shuffle idxs]
21.
22. #Desicion Tree Classifier
23. clf = DecisionTreeClassifier(criterion='entropy', random_state=4)
24. clf.fit(X_shff, y)
25. importances = clf.feature_importances_
26. #descending order
27. indices = np.argsort(importances)[::-1][:5] #top 5 features
28. #map the top 5 features to the original data
29. top5 = shuffle_idxs[indices]
30. #count the number of times each feature is in the top 5
31. count=np.sum(top5<5)</pre>
32. print(f"Number of true informative features in top 5: {count}")
34. #plot the top 5 features
35. plt.bar(range(len(importances)), importances[np.argsort(importances)[::-1]])
36. plt.xlabel('Feature Rank')
37. plt.ylabel('Feature Inportance Score')
38. plt.title('Feature Importance Ranking')
39. plt.show()
```

```
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import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification
from sklearn.preprocessing import StandardScaler
from sklearn.tree import DecisionTreeClassifier

#Question 1a
#generate data
X, y = make_classification(n_samples=1000, n_features=20, n_informative=5, n_redundant=15, shuffle=False,random_state=4)

#Normalize the data
scaler = StandardScaler()
scaler.fit(X)
X = scaler.transform(X)

#shuffle the data
shuffle_idxs = np.random.default_rng(seed=0).permutation(X.shape[1])
X_shff = X[:,shuffle_idxs]

#Desicion Tree Classifier
clf = DecisionTreeClassifier(criterion='entropy', random_state=4)
clf.fit(X_shiff, y)
import numpy as np
#descending order
indices = np.argsort(importances)[::-1][:5] #top 5 features
#descending order
indices = np.argsort(importances)[::-1][:5] #top 5 features
#may the top 5 features to the original data
top5 = shuffle_idxs[indices]
```

```
#count the number of times each feature is in the top 5
true_importance = np.arange(5)

count = len(set(true_importance).intersection(set(indices)))

print(f"Number of true informative features in top 5: {count}")

#plot the top 5 features

plt.bar(range(len(importances)), importances[np.argsort(importances)[::-1]])

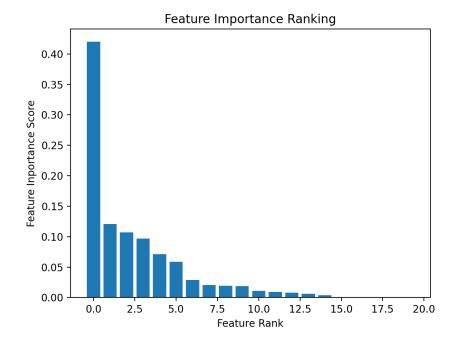
plt.xlabel('Feature Rank')

plt.ylabel('Feature Inportance Score')

plt.title('Feature Importance Ranking')

plt.show()
```

The result of above code is like followings:



And the number of true informative feature in top 5 is:

# Number of true informative features in top 5: 2

Q1(b) The first question: Gini Importance calculate the importance of feature by summing the impurity decrease it causes across all trees in the forest. If we assume that at the split points, each point impurity decrease  $\Delta I$ , and if we at the point t then the proportion of the number of samples corresponding to this node to the total number of samples is  $\frac{N_t}{N}$ . Thus, for all point the reduction of total impurities is  $\sum_{every\ point\ j\ in\ this\ feature} \Delta I \frac{N_j}{N}$ , where the N is the number of all sample, the  $N_t$  is the number of samples corresponding to the point. When we split the point we just want to have less impurity in the next layer, so if the feature is not used to split, which means it has no attribution to decreasing the impurity, and its  $\Delta I = 0$ , and its importance is also 0.

The second question: The feature importance of 0.15 means this feature accounts for 15% of the total importance in all splits.

Q1(c) Set the random state =I, then the completely code is as follows:

```
1. counts=[]
2. for i in range(1000):
      X,y=make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=1
5, shuffle=False, random state=i)
4.
5.
       #Normalize the data
       scaler = StandardScaler()
7.
       scaler.fit(X)
       X = scaler.transform(X)
8.
9.
10.
       #shuffle the data with seed=i
11.
       shuffle_idxs = np.random.default_rng(seed=i).permutation(X.shape[1])
12.
       X_shff = X[:,shuffle_idxs]
13.
14.
       #Desicion Tree Classifier
15.
       clf = DecisionTreeClassifier(criterion='entropy', random state=4)
16.
       clf.fit(X_shff, y)
17.
       importances = clf.feature_importances_
18.
       #descending order
19.
       indices = np.argsort(importances)[::-1][:5] #top 5 features
20.
       #map the top 5 features to the original data
21.
22.
       top5 = shuffle idxs[indices]
```

```
23.
        #count the number of times each feature is in the top 5
24.
        count=np.sum(top5<5)</pre>
25.
        counts.append(count)
26.
27.
        #plot the top 5 features
28.
        plt.hist(counts,bins=np.arange(0,6)-0.5,edgecolor = 'black')
29.
        plt.xticks(range(0,6))
        plt.xlabel('Number of True Features Recovered')
30.
31.
        plt.ylabel('Frequency')
        plt.title('Histogram of True Features Recovered (Decision Tree)')
32.
33.
        plt.show()
34.
        average=np.mean(counts)
35.
        print(f"Average number of good feature recovered:{average}")
```

```
#Q1(c)
counts=[]
for i in range(1000):
    X,y=make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=15,shuffle=False, random_state=i)

#Normalize the data
scaler = StandardScaler()
scaler.fit(X)

X = scaler.transform(X)

#shuffle the data with seed=1
shuffle_idxs = np.random.default_rng(seed=i).permutation(X.shape[1])

X_shff = X[:,shuffle_idxs]

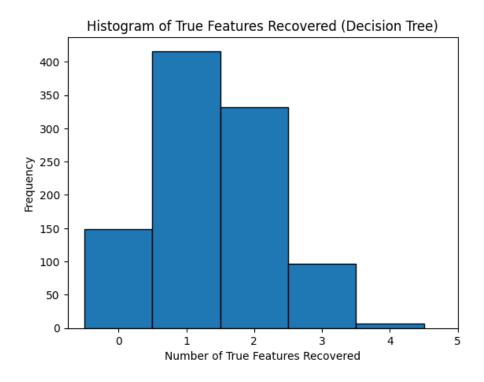
#Desicion Tree Classifier
clf = DecisionTreeClassifier(criterion='entropy', random_state=4)
clf.fit(X_shff, y)
importances = clf.feature_importances_
#descending order
indices = np.argsort(importances)[::-1][:5] #top 5 features

#map the top 5 features to the original data

top5 = shuffle_idxs[indices]
#count the number of times each feature is in the top 5
count=np.sum(top5<5)
counts.append(count)</pre>
```

```
#plot
plt.hist(counts,bins=np.arange(0,6)-0.5,edgecolor = 'black')
plt.xticks(range(0,6))
plt.xlabel('Number of True Features Recovered')
plt.ylabel('Frequency')
plt.title('Histogram of True Features Recovered (Decision Tree)')
plt.show()
average=np.mean(counts)
print(f"Average number of good feature recovered:{average}")
```

And the result is as follows:



The average is: Average number of good feature recovered:1.396

According to sklearn's documentation, when shuffle = False, the order of features is in the following order: n\_informative, n\_redundant, n\_repeated, random\_state. Therefore, regardless of the random\_state, as long as shuffle = False, the first 5 features are informative, followed by redundancy. Therefore, in this part, the first 5 features in the data generated by each trial i are always informative, followed by redundancy. Then when shuffling the feature order, we use a random seed of i. Therefore, in each trial i, the first 5 features of the generated data are informative, and the order of the scrambled features is random, determined by seed = i. Therefore, in statistics, the number of correct identifications depends on whether the model can identify those of the original top 5 among the scrambled features.

Q1(d) This part use logistic regression without penalty, the code is as following:

```
1. #Q1(d)
     #without scaling
     3. counts_nscale = []
     4. #repeat Q1c
     5. for i in range(1000):
     6. X,y =
\label{local_make_classification} \verb|(n_samples=1000, n_features=20, n_informative=5, n_redundant=15, shuffle=1000, n_features=20, n_informative=5, n_informative=5, n_informative=1000, n_informative=10000, n_informative=1000, n_informative=1000, n_informative=1000, n_informative=1000, n_informa
False, random_state=i)
     7.
     8.
                                                   #shuffle the data with seed=i
     9.
                                                   shuffle_idxs = np.random.default_rng(seed=i).permutation(X.shape[1])
10.
                                                   X_shff = X[:,shuffle_idxs]
11.
```

```
12.
       #Logistic Regression
13.
       lr = LogisticRegression(penalty=None,max iter=1000,random state=4)
14.
       lr.fit(X_shff,y)
15.
       coef = np.abs(lr.coef_[0])
16.
       #descending order
17.
       indices = np.argsort(coef)[::-1][:5] #top 5 features
18.
       #map the top 5 features to the original data
19.
       top5 = shuffle_idxs[indices]
20.
       #count the number of times each feature is in the top 5
21.
       count=np.sum(top5<5)</pre>
22.
       counts_nscale.append(count)
23.
24. #with scale
25. counts_scale = []
26. for i in range(1000):
27.
       X,y =
make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=15,shuffle=
False, random_state=i)
28.
29.
       #Normalisation
30.
       scaler = StandardScaler()
31.
       X = scaler.fit_transform(X)
32.
33.
       #shuffle the data with seed=i
34.
       shuffle_idxs = np.random.default_rng(seed=i).permutation(X.shape[1])
35. X_shff = X[:,shuffle_idxs]
36.
37.
       #Logistic Regression
       lr = LogisticRegression(penalty=None,max_iter=1000,random_state=4)
39.
       lr.fit(X shff,y)
       coef = np.abs(lr.coef_[0])
40.
41.
       #descending order
42.
       indices = np.argsort(coef)[::-1][:5] #top 5 features
43.
       #map the top 5 features to the original data
       top5 = shuffle_idxs[indices]
44.
45.
       #count the number of times each feature is in the top 5
46.
       count=np.sum(top5<5)</pre>
47.
       counts_scale.append(count)
48.
49. #plot with scale
50. plt.hist(counts_nscale,bins=np.arange(0,6)-0.5, alpha=0.5, label='Unscaled')
51. plt.hist(counts_scale,bins=np.arange(0,6)-0.5, alpha=0.5, label='Scaled')
52. plt.legend()
53. plt.xlabel('Number of True Features Recovered')
```

```
54. plt.ylabel('Frequency')
55. plt.title('Histogram of True Features Recovered (Logistic Regression)')
56. plt.show()
57. average_nscale=np.mean(counts_nscale)
58. average_scale=np.mean(counts_scale)
59. print(f"Average number of good feature recovered(LR without scale):{average_nscale}")
60. print(f"Average number of good feature recovered(LR with scale):{average_scale}")
```

```
#with out scaling
counts_nscale = []
#repeat Qtc
for i in range(1000):

#shuffle the data with seed=i
shuffle_idxs = np.random.default_rng(seed=i).permutation(X.shape[1])

#counts_nscale = []

#Logistic Regression
Ir = LogisticRegression(penalty=None,max_iter=1000,random_state=4)
Ir.fit(X_shff,y)
coef = np.abs(Ir.coef_[0])
#descending order
indices = np.argsort(coef)[::-1][:5] #top 5 features
#map the top 5 features to the original data
top5 = shuffle_idxs[indices]
#count the number of times each feature is in the top 5
counts_nscale = []
for i in range(1000):

X,y = make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=15,shuffle=False,random_state=i)

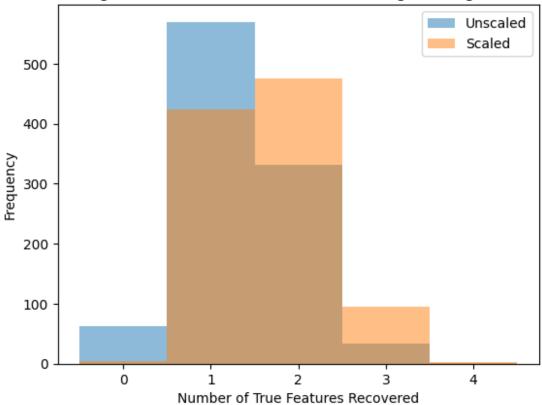
X,y = make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=15,shuffle=False,random_state=i)

X,y = make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=15,shuffle=False,random_state=i)
```

```
scaler = StandardScaler()#Normalisation
    X = scaler.fit transform(X)
    shuffle_idxs = np.random.default_rng(seed=i).permutation(X.shape[1])
    X_shff = X[:,shuffle_idxs]
    lr = LogisticRegression(penalty=None,max_iter=1000,random_state=4)
    lr.fit(X_shff,y)
    coef = np.abs(lr.coef_[0])
    #descending order
    indices = np.argsort(coef)[::-1][:5] #top 5 features
    top5 = shuffle_idxs[indices]
    count=np.sum(top5<5)</pre>
    counts_scale.append(count)
plt.hist(counts_nscale,bins=np.arange(0,6)-0.5, alpha=0.5, label='Unscaled')
plt.hist(counts_scale,bins=np.arange(0,6)-0.5, alpha=0.5, label='Scaled')
plt.legend()
plt.xlabel('Number of True Features Recovered')
plt.ylabel('Frequency')
plt.title('Histogram of True Features Recovered (Logistic Regression)')
plt.show()
average_nscale=np.mean(counts_nscale)
average_scale=np.mean(counts_scale)
print(f"Average number of good feature recovered(LR without scale):{average_nscale}")
print(f"Average number of good feature recovered(LR with scale):{average_scale}")
```

The histogram is like followings:





The averages are as following respectively:

Average number of good feature recovered(LR without scale):1.34

Average number of good feature recovered(LR with scale):1.667

In logistic regression without scaling, the size of the coefficients may be affected by the feature scale, so the feature importance may not be accurate. However, after scaling, the features are in the same dimension, and the coefficient size can more accurately reflect the importance. Therefore, logistic regression should perform better after scaling.

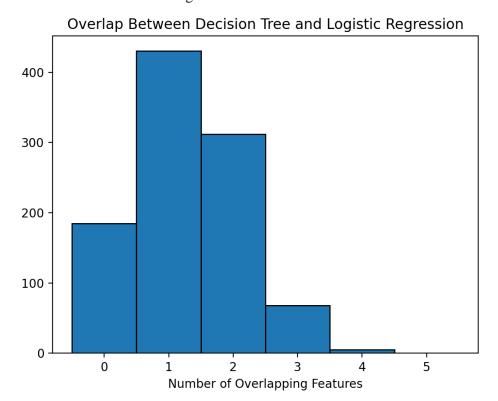
Q1(e) No, scaling does not affect decision trees since they split based on feature value order, not magnitude.

Q1(f) Using the followings code to complete the requirement:

```
1. #Q1(f)
2. overlaps=[]
3. for i in range(1000):
      X,y=make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=1
5, shuffle=False, random_state=i)
       #Normalize the data
6.
       scaler = StandardScaler()
       X = scaler.fit_transform(X)
7.
       #shuffle the data with seed=i
8.
9.
       shuffle_idxs = np.random.default_rng(seed=i).permutation(X.shape[1])
       X_shff = X[:,shuffle_idxs]
10.
11.
       #Desicion Tree Classifier
```

```
12.
       dtc = DecisionTreeClassifier(criterion='entropy', random_state=4)
13.
       dtc.fit(X_shff, y)
       importances = clf.feature_importances_
14.
15.
       #descending order
16.
       indices = np.argsort(importances)[::-1][:5] #top 5 features
17.
       #map the top 5 features to the original data
18.
       top5_tree = shuffle_idxs[indices]
19.
       #Logistics Regression
       lr = LogisticRegression(penalty=None, random_state=4)
20.
21.
       lr.fit(X_shff,y)
22.
       coef = np.abs(lr.coef_[0])
23.
       #descending order
       indices = np.argsort(coef)[::-1][:5] #top 5 features
       top5_lr = shuffle_idxs[indices]
25.
       overlaps.append(len(set(top5_tree)&set(top5_lr)))
26.
27. #plot
28. plt.hist(overlaps,bins=np.arange(7)-0.5,edgecolor='black')
29. plt.xlabel('Number of Overlapping Features')
30. plt.title('Overlap Between Decision Tree and Logistic Regression')
31. plt.show()
```

The result shows as following:



Q1(g) Disadvantages of Model-Based Feature Importance:

1. Model bias: Different models emphasize different features, like decision tree and logistics regression.

- 2. Redundant feature: For the different redundancies will have different results. If the redundancy is too high, which will dilute importance scores and make true features harder to identify.
- 3. Just like the example, if the model prefer some feature types, the model-based will miss true associations.

Q2(a) Because at each round, we remove the j-th feature from the model based on the drop in the value of a certain metric. We eliminate the feature corresponding to the smallest drop in the metric, which is only the local optimal choice and not the global optimal choice. The pitfalls of greedy algorithms are that we will miss the global optimal choice , remove one feature might degrade the utility of others and computational cost a lot.

Q2(b) The code e implementing the backward elimination algorithm as followings:

```
1. #Q2(b)
2. #backward elimination algorithm
3. from sklearn.metrics import accuracy score
4. def backward_elimination(n_feature_keep=5,seed=0):
       \label{eq:classification} \textbf{X}, \textbf{y} = \texttt{make\_classification} (\textbf{n\_samples=1000}, \textbf{n\_features=20}, \textbf{n\_informative=5}, \textbf{n\_redundant=1} \\
5, shuffle=False, random state=seed)
6.
        #Normalize the data
7.
        X = scaler.fit_transform(X)
8.
        #shuffle the data with seed
9.
        shuffle_idxs = np.random.default_rng(seed=seed).permutation(X.shape[1])
        X_shff = X[:,shuffle_idxs]
10.
11.
        features = list(range(X_shff.shape[1]))
12.
        while len(features) > n feature keep:
13.
            scores=[]
14.
            for f in features:
15.
                 X_subset = X[:,[x for x in features if x!=f]]
16.
                 lr = LogisticRegression(penalty=None, random_state=4)
17.
                 lr.fit(X_subset,y)
18.
                 scores.append(accuracy_score(y,lr.predict(X_subset)))
19.
            smallest_feature = features[np.argmin(scores)]#find the feature corresponding
to th smallest drop in the metric
            features.remove(smallest feature)
20.
        origin_left = shuffle_idxs[features]
21.
22.
        correct = np.sum(origin_left<5)</pre>
23.
        print(f"Left features:{origin_left}")
        return correct
25. correct = backward_elimination(n_feature_keep=5, seed=0)
26. print(f" Number of correct features: {correct}")
```

```
from sklearn.metrics import accuracy_score
def backward_elimination(n_feature_keep=5,seed=0):
   X,y=make classification(n samples=1000,n features=20,n informative=5,n redundant=15,shuffle=False, random state=seed)
    X = scaler.fit transform(X)
    shuffle_idxs = np.random.default_rng(seed=seed).permutation(X.shape[1])
    X shff = X[:.shuffle idxs]
    features = list(range(X_shff.shape[1]))
           X_subset = X[:,[x for x in features if x!=f]]
1r = LogisticRegression(penalty=None, random_state=4)
             scores.append(accuracy_score(y,lr.predict(X_subset)))
        smallest_feature = features[np.argmin(scores)]#find the feature corresponding to th smallest drop in the metric
        features.remove(smallest feature)
    origin left = shuffle idxs[features]
    correct = np.sum(origin_left<5)</pre>
    print(f"Left features:{origin_left}")
correct = backward_elimination(n_feature_keep=5, seed=0)
print(f" Number of correct features: {correct}")
```

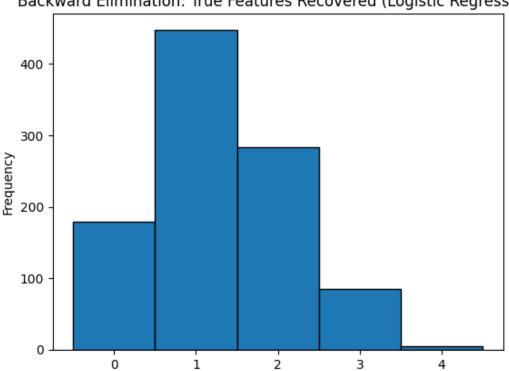
# Left features: [ 1 16 12 3 11] Number of correct features: 2

Q2(c) Repeat part a for 1000:

The result is:

```
1. #Q2(c)
2. recovered_f=[]
3. for i in range(1000):
4.    correct = backward_elimination(n_feature_keep=5,seed=i)
5.    recovered_f.append(correct)
6. plt.hist(recovered_f, bins=np.arange(0,6)-0.5, edgecolor='black')
7. plt.xlabel('Number of True Features Recovered(Backward Elimination)')
8. plt.ylabel('Frequency')
9. plt.title('Backward Elimination: True Features Recovered (Logistic Regression)')
10. plt.show()
11. average_re=np.mean(recovered_f)
12. print(f"Average number of recovered feature (Backward Elimination):{average_re}")
```

The histogram chart is like followings:



### Backward Elimination: True Features Recovered (Logistic Regression)

The average number of recovered feature is:

#### Average number of recovered feature (Backward Elimination):1.289

Number of True Features Recovered(Backward Elimination)

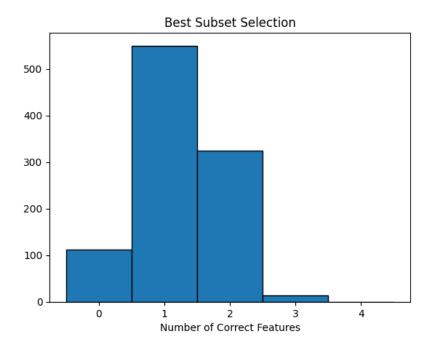
Q2(d) Best subset selection could avoid to select the local optimal choice, so it will perform well on selecting global optimal choice. But this will cost a lot time more than Backward elimination, since we need find all possible subsets and train a model on each subset. Therefore, best subset selection will not always outperform backward elimination. The disadvantages of best subset selection are that for large data it can't work and prone to overfitting with limited data.

#### Q2(e) Implement Best subset selection:

```
1. #Q2(e)
2. from itertools import combinations
3.
4. def best_subset(seed=0):
5.    X,y=make_classification(n_samples=1000,n_features=7,n_informative=3,n_redundant=4, shuffle=False, random_state=seed)
6.    #Normalize the data
7.    X = scaler.fit_transform(X)
8.    #shuffle the data with seed
```

```
9.
       shuffle_idxs = np.random.default_rng(seed=seed).permutation(X.shape[1])
10.
       X shff = X[:,shuffle idxs]
       best_score = -np.inf
11.
       best_subset = None
12.
13.
14.
       for subset in combinations(range(X_shff.shape[1]),3):
15.
           X_subset = X_shff[:,subset]
           lr = LogisticRegression(penalty=None,random_state=4)
16.
17.
           lr.fit(X subset,y)
18.
           acc = accuracy_score(y,lr.predict(X_subset))
19.
           if acc > best_score:
20.
               best_score=acc
21.
               best_subset=subset
       origin_subset = [shuffle_idxs[idx] for idx in best_subset]
22.
23.
       correct_best = sum(1 for idx in origin_subset if idx < 3)</pre>
24.
       return correct best
25. #repeat 1000
26. recoveries=[]
27. for i in range(1000):
       correct_best = best_subset(seed=i)
29.
       recoveries.append(correct_best)
30. plt.hist(recoveries,bins=np.arange(0,6)-0.5, edgecolor='black')
31. plt.xlabel('Number of Correct Features')
32. plt.title('Best Subset Selection')
33. plt.show()
34. print(f"Average correct(Best Selection):{np.mean(recoveries)}")
```

The histogram chart is below:



The average is: Average correct:1.24

Q2(f) Permutation Feature Importance: it will randomly shuffle the feature's value and measure the drop in model performance. A bigger drop means higher importance. And it does not rely on any model internals(such as coefficients). It's more fair to compare logistic regression and decision trees.

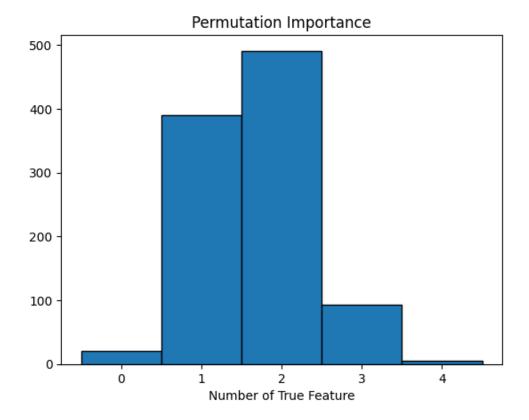
The implement code:

```
1. #Q2(f)
2. #Permutation Feature Importance score
3. from sklearn.inspection import permutation_importance
4. def permutation_importance(seed=0):
      X,y=make classification(n samples=1000,n features=20,n informative=5,n redundant=1
5, shuffle=False, random_state=seed)
6.
       #Normalize the data
7.
       X = scaler.fit_transform(X)
8.
       #shuffle the data with seed
       shuffle_idxs = np.random.default_rng(seed=seed).permutation(X.shape[1])
9.
10.
       X_shff = X[:,shuffle_idxs]
11.
       lr = LogisticRegression(penalty=None, random_state=4)
12.
       lr.fit(X_shff,y)
13.
       #calculate permutaion importance
```

```
14.
       importance = permutation_importance(lr, X_shff,y,n_repeats=10,random_state=4)
15.
       mean importance = importance.importances mean
       #top5
16.
17.
       indices = np.argsort(mean_importance)[::-1][:5]
18.
       top5 = shuffle_idxs[indices]
19.
       correct_pe = np.sum(top5<5)</pre>
20.
       return correct_pe
21. count_pe=[]
22. for i in range(1000):
23.
       correct_pe = permutation_importance(seed=i)
24.
       count pe.append(correct pe)
25. #plot
26. plt.hist(count_pe,bins=np.arange(0,6)-0.5,edgecolor='black')
27. plt.xlabel('Number of True Feature')
28. plt.title('Permutation Importance')
29. plt.show()
30. print(f"Average correct:{np.mean(count_pe)}")
```

```
from sklearn.inspection import permutation_importance
  X,y=make_classification(n_samples=1000,n_features=20,n_informative=5,n_redundant=15,shuffle=False, random_state=seed) #Normalize the data
def permutation_importance(seed=0):
    shuffle_idxs = np.random.default_rng(seed=seed).permutation(X.shape[1])
    lr = LogisticRegression(penalty=None, random_state=4)
    #calculate permutation importance
importance = permutation_importance(lr, X_shff,y,n_repeats=10,random_state=4)
    mean_importance = importance.importances_mean
    indices = np.argsort(mean_importance)[::-1][:5]
    correct_pe = np.sum(top5<5)</pre>
count_pe=[]
for i in range(1000):
    count_pe.append(correct_pe)
plt.hist(count_pe,bins=np.arange(0,6)-0.5,edgecolor='black')
plt.xlabel('Number of True Feature
plt.title('Permutation Importance')
print(f"Average correct:{np.mean(count_pe)}")
```

The histogram chart of permutation importance as below:



The average is:

Average correct:1.676