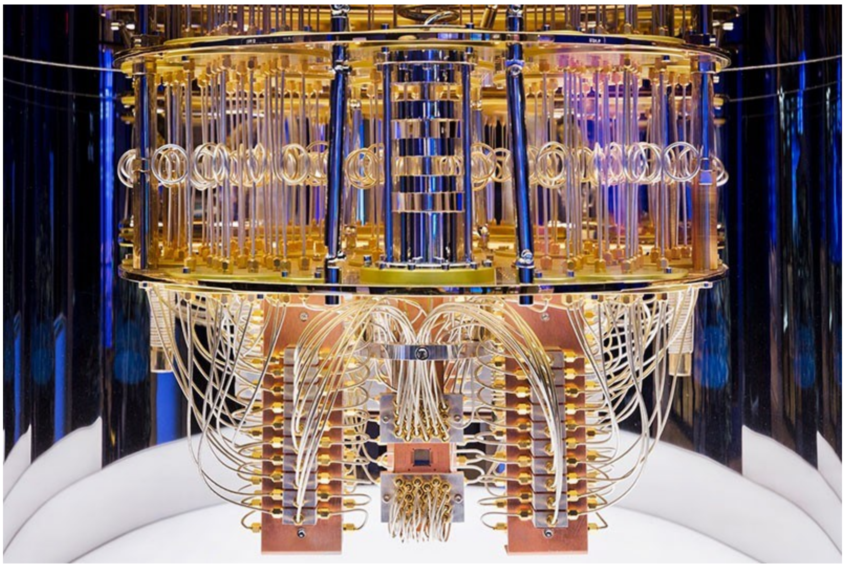
Technologies of Computing Systems

2nd Project

(guide for the groups with even number assigned)

# Quantum Computing
# Designing and Simulating Quantum Circuits



**1st Semester, Second Quarter 22/23**

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# 1 Introduction

In the lectures, students have analysed quantum computation as a new type of reversible and invertible computation [1][2]. Representing the most basic piece of quatum data bit (qubit) as a two elements vector, the two states of a classical bit can be represented by:

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}\ ; \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}.$$

A quantum state may then be found in any quantum superposition $|\psi\rangle$ of the two classical states $|0\rangle$ and $|1\rangle$:

$$|\psi\rangle = \rho|0\rangle + \sigma|1\rangle = \begin{pmatrix}\rho\\\sigma\end{pmatrix}\ ; \ ||\rho||^2 + ||\sigma||^2 = 1$$

When a measurement is performed in a quantum system, the state $|\psi\rangle$ can be observed as a probability vector. If the quantum state is measured to check whether the state is $|0\rangle$ or $|1\rangle$, $||\rho||^2$ is the probability of observing the zero state and $||\sigma||^2$the probability of observing the one state ($||\rho||^2$and $||\sigma||^2$ are designated the quantum amplitudes).

The state of $|\psi\rangle$ changes by applying quantum gates. The application of quantum gates correspond to the linear algebra matric multiplication. An example of a simple gate is the quantum NOT:

$$NOT \equiv \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\ ; \ NOT|0\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} * \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}\ ; \ NOT|1\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} * \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

Multi-qubits can be derived by applying tensor algebra to single qubits, except when quantum entanglement is registered (quantum entanglement is a physical phenomenon that occurs when each particle of a group of particles cannot be described independently of the state of the others).

$$|00\rangle = |0\rangle \otimes |0\rangle \begin{pmatrix}1\begin{pmatrix}1\\0\end{pmatrix}\\0\begin{pmatrix}1\\0\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix};\ |01\rangle = |0\rangle \otimes |1\rangle \begin{pmatrix}1\begin{pmatrix}0\\1\end{pmatrix}\\0\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix};\ldots$$

An important example of a quantum gate with two qubits is the controlled NOT gate, which can then be represented by the following matrix:

$$CNOT \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The main goal of this project is to simulate simple quantum circuits, identifying the functions they compute, taking advantage of superposition and solving dilemmas. Although this project is supported on the Qirk quantum circuit simulator [3], you may use any of the multiple frameworks publicly available, such as the simulators and physical computers from IBM [4] and TU Delft [5]. This is an optional component of the work, which is not required to achieve the maximum grade.

## 2 Qirk Simulator

Quirk is a quantum circuit simulator that aims people to learning about quantum computing. The code of Quirk can be found on github [1]. However, if you do not change the code of Quirk, it does not require installing or configuring or scripting. You go directly to the following URL

[algassert.com/quirk](algassert.com/quirk)

Quirk runs in a web browser, as depicted in Fig.1. It provides drag-and-drop circuit editing, interacts, simulates and animates in real time, and supports bookmarkable/linkable circuits. The main limitation of the simulator is on the maximum number of qubits supported, 16 qubits.

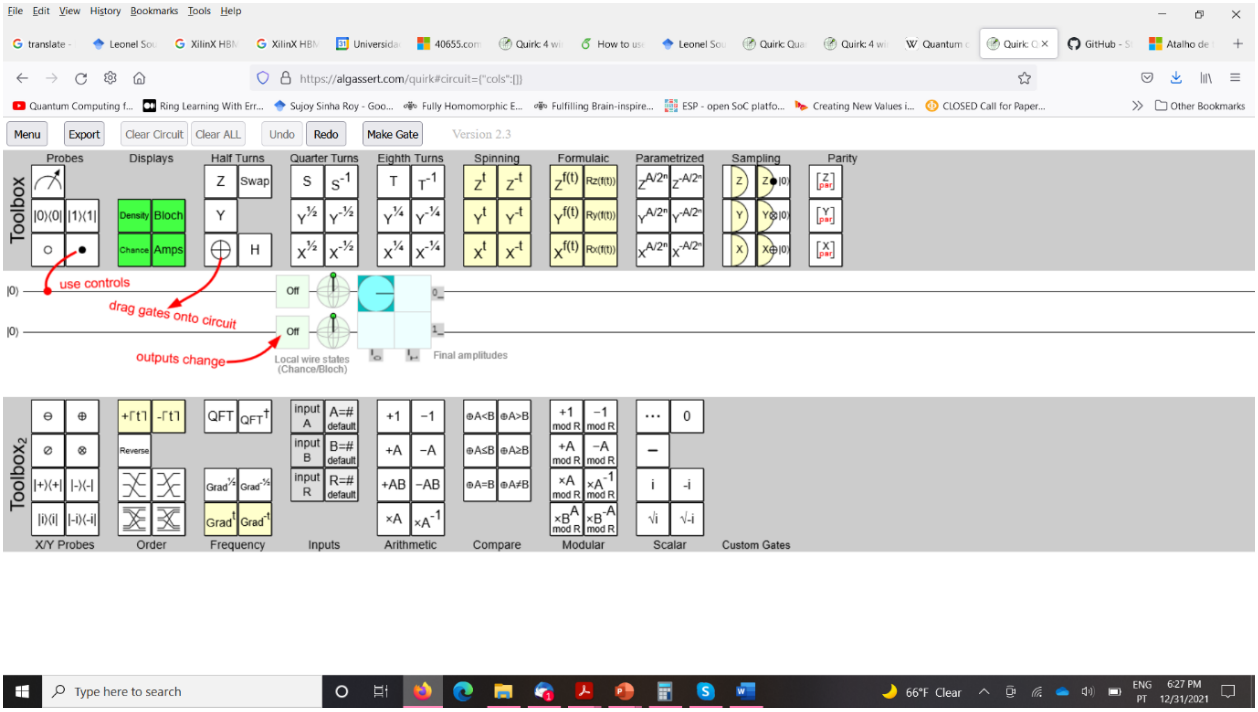


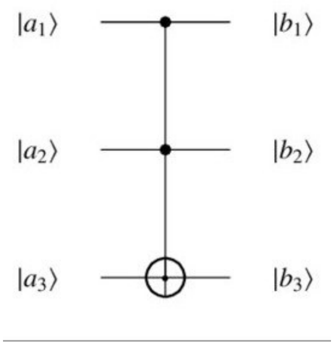

Fig. 1- Screenshot of Qirk.

# 3. Project: first week

To become familiar with the Quirk operation and usage, design simple quantum circuits with studied gates. You should, at least, verify the operation, and register the results of the following quantum gates:

1. Not gate
2. CNOT gate
3. Hadamard gate.

By using only simple quantum gates, design a Tofolli gate represented by the symbol and matrix represented in Fig.2.




$$Toffoly \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Fig. 2- Toffoli gate.

It ca be observed that with the Toffoli gate, the target output |b3> takes the complemented logic value of |a3> when both control bits |a1> and |a2> are equal to 1. Otherwise, |b3> takes the value of |a3>.

# 4. Project: second week

Analyse and simulate the quantum circuit in Fig. 3.

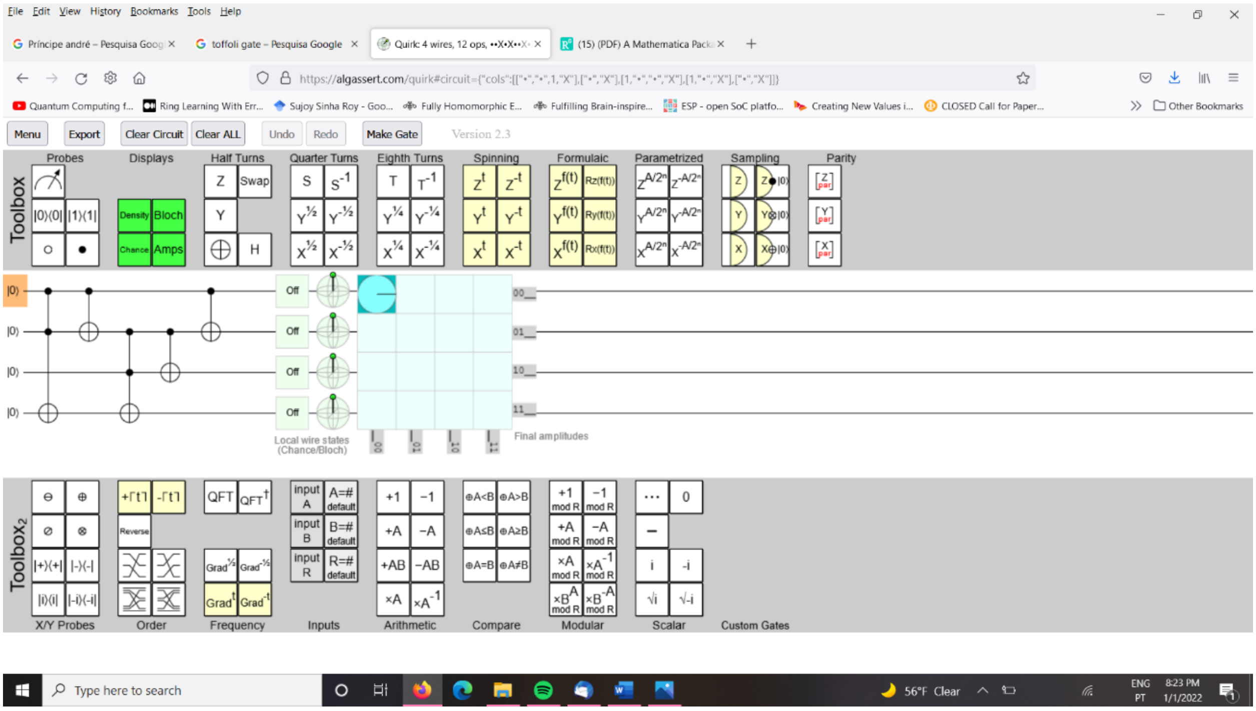


Fig. 3- Circuit to simulate in the lab.

Perform the following tasks.

a) Derive the function implemented by the circuit.
b) Simulate the circuit operation, by changing the state of the input quantum bits and registering the outputs.
c) Modify the circuit for taking advantage of the superposition, allowing the physical system to be in one of different configurations, corresponding to more than one state.

Optionally, you can setup an account on Quantum Inspire

https://www.quantum-inspire.com/

simulate and try to run your program in the quantum computer. As stated before, this step of the work is not mandatory, be aware that it is usually difficult to get a time slot in the physical quantum computer.

## 5. Reporting

Students are required to deliver a report (maximum of 4 pages), formatted according with the IEEE Manuscript Templates for Conference Proceedings, available at:

https://www.ieee.org/conferences/publishing/templates.html.

The report should be structured as follows:

- Abstract: 100-150 words summarizing the problem.
- 1. Introduction: Explain the basis of the quantum computing and the aim o the work.
- 2. Methodology: Analyse and discuss the simulated circuits.
- 4. Results: Describe the attained results and draw the main conclusions.
- 5 Conclusions: Summarize the report, including approach and results.

## 6. Deadlines

Week January 9 (last lab of this week): Project demo

January 20 (fenix): Submission of the report in PDF format.

## 7. References


[1] https://www.quantum-inspire.com/kbase/introduction-to-quantum-computing/.

[2] Yanofsky, N., & Mannucci, M., "Quantum Computing for Computer Scientists ", Cambridge University Press.

[3] Qirk Simulator, URL: https://github.com/Strilanc/Quirk.

[4] IBM Quantum Computing, URL: https://quantum-computing.ibm.com/.

[5] Quantum Inspire, URL: https://www.quantum-inspire.com/.