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T1 ____________	**7**	F1 ____________
T2 ____________		F2 ____________
T3 ____________	Problem Chosen	F3 ____________
T4 ____________	**A**	F4 ____________

2019
Mathematical Modeling Course ZJU
Summary Sheet

Summary

In this paper, we will give some advice on where to build charging stations and the quantity of charging piles in each station. It' s necessary for us to do so due to the increasing selling of new energy cars.

On one hand, charging stations should be built at the suitable places that most people could use them conveniently. On the other hand, we should reduce the number of charging stations in order to cut down the budgets. To meet these requirements, we first collected data from Google map, Baidu map and Hangzhou Bureau of Planning and Natural Resources. Then we created a probability map which would tell us the demand of charging in each area. By using K-means clustering algorithm we get the centroids of these points and we determined where to build charging stations. Then we use Queuing Theory to determine the number of charging piles in each station. We also created a graph concerning the distance of mileage and the number of charging stations. With this graph, we tried to reduce the quantity of charging stations to a reasonable number.

Finally, we got some results. Several pictures that shows the location of charging stations and the quantity of piles were created and we also got a chart which shows the relationship between the distance of mileage and number of charging stations. under the circumstance we set, it is possible to reduce the number of charging stations to 120.

Keywords: Clustering;Prediction;Queuing Theory;K-Means;

Contents

1 Introduction

1.1 Background

With the increasingly severe environmental and energy situation, the interaction and constraints between energy supply and demand, greenhouse gas emissions, and economic growth have received widespread attention from countries around the world. Energy and the environment have become the most concerned issues in the world. As a country with rapid economic development, China's energy demand is increasing, and coal and oil consumption accounts for the vast majority of energy consumption. This energy consumption structure has already posed a serious threat to China's energy security. At the same time, this situation has brought many negative effects to the environment, bringing many environmental problems to the society, and environmental problems have also restricted the economic development. Therefore, promoting the application of clean and efficient new energy sources and reducing the dependence of economic development on petroleum resources is of great practical significance for adjusting China's energy consumption structure, improving environmental pollution, and achieving sustainable development of energy and environment.

The use of new energy vehicles and the gradual replacement of traditional fuel vehicles is a means of solving energy and environmental problems. At present, new energy vehicles in the Chinese market mainly include BEV, PHEV and other types of vehicles, of which BEV refers to cars that use only batteries as an energy source, and PHEV refers to cars that use gasoline engines and batteries as energy sources. At present, a major problem that restricts the development of new energy vehicles is that the battery unit density capacity is small, the one-time cruising range is short and the charging time is long. Therefore, in order to promote the sales and use of new energy vehicles in large-scale cities, it is necessary to increase the number and density of supply stations and reduce the worries of potential customers.

The construction of new energy vehicle supply stations involves many aspects. Supply stations often require a large space. Therefore, the chosen location must be economical and have little impact on the living and working of the surrounding residents. The supply station often needs to use a charging device with a large current, and a special power supply network needs to be built. Therefore, the construction of the supply station has a high cost, and the number of supply stations should be reduced due to the limited budget. It should be reduced on the basis of little impact on users' satisfaction. Supply stations must provide timely service to users, especially when their car is running out. The distance between the supply stations should not be too large, so that the user will be able to find the charging pile to charge in time when the car is exhausted.

We will use mathematical modeling to find the best way to build new energy vehicle supply stations in Hangzhou.

1.2 Restatement of problem

The distribution of supply stations will affect the user experience of using new energy vehicles, while the unreasonable distribution of supply stations will limit the use of new energy cars. By analyzing different limiting factors, a plan of setting new energy vehicle supply stations in Hangzhou City is made on the premise of maximizing user experience satisfaction and reducing users' travel restrictions.
Assuming that the cruising range of new energy vehicles is improved because of factors such as scientific and technological progress, we can reduce the number of supply stations to reduce construction and maintenance costs. Of course, this is based on the basis that users still have the same satisfaction. We use mathematical modeling analysis to minimize the number of supply stations under this premise.

1.3 Overview of Our Work

- **We analyzed the problem.** The topic asks us to set up new energy vehicle supply stations in Hangzhou. By looking up the Internet literature, we found the factors associated with the supply station setup, determined several factors of the model, and how important they were to the supply station setup.
- **We have collected relevant information.** On the website of Hangzhou Planning Bureau, we downloaded the master plan of Hangzhou city. And we downloaded the population density map and road network map on Baidu Map. We found relevant literature and determine the proportion of each graph in the decision.
- **We processed the collected data.** The population density map is grayed out according to the original color. The grayer pixels indicate the higher population density, and the road network distribution map is combined with it to generate the traffic probability density map. The urban functional zoning map weights the probability density of each region. The probability of building supply station in a residential area will be lower, because the residents themselves may have higher possibilities of setting up charging stations, and the need to build a public supply station is reduced.
- **We use the K-Means clustering algorithm** to process the graph obtained in the previous step, and get several aggregation points, which are the recharge stations we expect to set.
- **By changing the expecting number of target replenishment stations, we get different average distance to the station of the users.** Under the premise of the improved mileage of new energy vehicles, the number of replenishment stations will be reduced, which will not change the satisfaction of the customers.

- **We analyzed the advantages and disadvantages of the established model.** We made some simple comparisons between the locations in simulation and in reality. And we analyzed the degree of reflection of our model.

2 Assumptions

- Assumption: The scope of Hangzhou City is mainly within the expressway around the city,from Liangzhu to Xiaoshan.
  Justification: The population is mainly distributed here and the restriction policy of cars mainly located here.

- Assumption: The number of cars on the road is close to that in the busy business district.
  Justification: More vehicles are driving on the road while in residential areas, they parked more. Cars on the road are possibly need to be recharged at any time.

- Assumption: There are about 168,000 PHEVs or EVs in Hangzhou City and about 58% of the them will drive on the road every day.
  Justification: According to the statistic in Q1,2018.

- Assumption: Users can wait for a charging within 15 minutes and they choose a fast charging of 30 minutes to increase the batter power from 20% to 85%.
  Justification: 15 minutes is generally short enough for ordinary users. The power of new super charging piles is very high, but batteries shouldn't be overcharged or overdischarged as usual.

Here are the notations and their meanings in our paper.

Notation	Meaning
n	Number of charging piles
λ	Number of cars driving to the charging station every minute
T_c	Time of charging per car
N_{wait}	Average number of queuing cars
T_{wait}	Average queuing time
P_{loss}	Loss probability
$power$	Average mileage of each vehicle
dis	Average distance to the nearest charging station
T	Average queuing time
w	Coefficient of T
Eva	Evaluation of satisfaction

3 Model

3.1 Layer Overlay and Image Processing

For the third problem, we consider the restrictions on user travel and the satisfaction of user experience. The comprehensive analysis shows that the three factors are mainly affected by traffic network density, population density and different functional zones of the city. We make the following analysis based on the travel chain theory and the principle of population density.

- **Traffic network map**
  In the city, we analyze the demand distribution of charging stations in their geographic location. Based on some literatures, we believe that similar to gas stations, road traffic flow has a greater impact on the distribution of charging stations. The larger the traffic flow, the greater the demand for charging stations. Where the traffic network map is denser, the larger the traffic flow is, the larger the traffic flow is. Therefore, we can judge by the denser degree of traffic network map. Demand Distribution of Break Charging Pile in Geographical Position. In our image, the wider the road, the more pixels, reflecting the greater traffic flow.
  We obtained the map of Hangzhou city, removed some irrelevant factors by image processing software, extracted the road skeleton, and processed the image into gray-scale images, and obtained an image of 2160 $\times$ 1440 pixels.

Figure 1: Map of roads

- **Population thermogram**
  The use of new energy vehicles is inseparable from people, and charging new

energy vehicles requires people and vehicles to complete together. Therefore, we can judge the geographic distribution of charging stations by the dense population distribution in the population thermodynamic map. The more densely populated the area, the greater the demand for charging stations.
In image processing, we mainly adopt the method of layer superposition. We obtained the population thermodynamic map at 12 noon and 12 midnight respectively. After processing into gray-scale image, we took the average gray-scale value of each pixel one by one, and obtained the population thermodynamic map layer. Next, in order to overlay the traffic network layer with the population thermogram layer, we use the maximum overlay method to compare each pixel point of the two pictures one by one, and select the larger gray value as the gray value of the composite picture.

Figure 2: Map added with population thermogram

- **Functional zoning map**
Based on the travel chain model, we believe that different functional zones in a city correspond to different demand for charging stations. By consulting relevant information and literature, we believe that if the demand for charging stations is 80% in residential areas, the demand for charging stations is 100% in commercial areas, 110% in industrial areas and 0% in prohibited areas (such as green space, lakes, etc.). In the implementation of the algorithm, we obtain the coordinates of each pixel in the composite graph obtained in the first two steps, find its position in the functional partition graph, i.e. the functional partition to which it belongs, and use the probabilistic pixels to select the more intensive points where the gray scale is larger. Therefore, we make the probability of each pixel retaining as a black point its original gray value/255*W, and W as the previous paragraph ‘s percentages. Finally, we

get a composite image of three previous layers and resize it to a 300 × 200 pixel image.

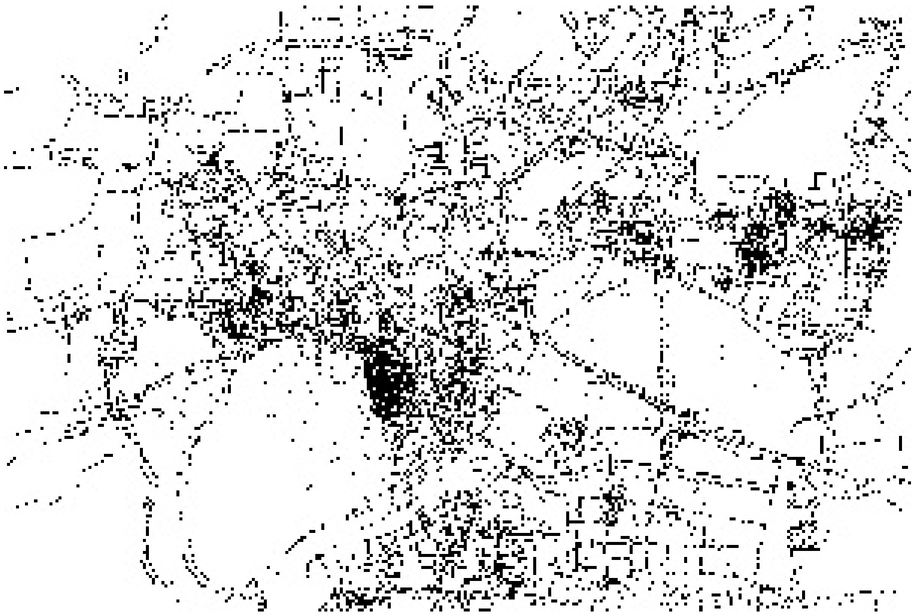

Figure 3: Map added with population thermogram

3.2 K-Means Clustering Model

For how to set up charging stations, we mainly use K-means clustering algorithm based on the previous layer overlay and image processing model. The idea of the algorithm is as follows: First, K objects are randomly selected as the initial clustering center. Then the distance between each object and each seed cluster center is calculated, and each object is assigned to the nearest cluster center. Cluster centers and objects assigned to them represent a cluster. Once all the objects are allocated, the cluster center of each cluster is recalculated according to the existing objects in the cluster. This process will be repeated until a termination condition is met.

In our program, we cluster the black spots in the previous images, and take all the clustering centers as the locations of the final charging stations. We divide all the black spots into K clusters according to the clustering algorithm, and calculate the sum of the distances from all the black spots to their clustering centers. By changing the number of K, we get the curve relationship between the sum of charging pile points and distances(*inertia*).

3.3 User Satisfaction Model

As for customer satisfaction, we believe that it is mainly affected by the average queuing time T and the minimum charge required. The average queuing time refers to the average queuing waiting time of the car according to the queuing model, while the average minimum power required to charge refers to the average distance from any point on the map to the nearest charging pile, and power refers to the average mileage of each vehicle.

$$Eva{=}W{\times}T{+}dis/power$$

For the fourth question, we need to ensure that the number of charging stations is reduced without decreasing user satisfaction. So to keep Eva at a certain level, we keep T and dis/power at a certain level. That is to say, we ensure that the average queuing time is constant by configuring the number of charging stations in each charging pile selection point. With the increase of the endurance power of new energy vehicles, we can get it. The larger dis makes the dis/power constant. Therefore, for a certain endurance power, we calculate the corresponding dis. From the relationship curve between the number of charging stations and the sum of distance we obtained before, we read out the number of charging stations, i.e. the least number of charging stations, and then use K-means clustering algorithm to obtain the corresponding endurance (promotion), without reducing user satisfaction, the least number of charging stations ‘geographical location distribution.

3.4 Queuing Theory

Queuing model consists of service organization and service object. The arrival time and service time of the service object are random. The model consists of three parts: input process, queuing rules and service organization. The input process examines the law of customer arrival in the service system. It can be described by the number of arrivals of customers in a certain time or the interval between successive arrivals of two customers. Queuing rules are divided into waiting system, loss system and mixed system. A service organization may be one or more service desks. Multiple service desks can be arranged in parallel or in series. Service time is generally divided into two types: deterministic and random.

In the fourth question, to ensure that user satisfaction does not decrease, we use queuing model for the queuing time part of our user satisfaction model. We need to get the number of vehicles to be recharged at the location of each charging pile. The specific methods are as follows: counting the number of black spots belonging to each clustering center in the clustering results, and converting the number of black spots into the number of vehicles to be recharged. The conversion method is that the number of vehicles to be recharged is equal to the number of vehicles to be recharged (168000)× the active rate (58÷100)× the number of black spots × charging rate (33.3%)/ total Pixel Points (7876). After obtaining the number of vehicles to be charged, we use our queuing model algorithm to calculate the minimum number of charging stations required for each selected point under the condition that the average queuing time is less than 15 minutes. We assume that the average charging time of each vehicle is 30 minutes.

According to the Multi-channel Waiting System Queuing Model, a relationship model between the number of charging piles, charging time and average waiting time is established. We set the probability of loss $P_{loss} = 0$, because every car that needs to be charged has a shorter mileage and it is riskier to go to another charging station. Let

$$\rho = \lambda \times T_c$$

$$N_{wait} = \frac{\rho^{n+1}}{n \cdot n!}(1 - P_{loss})\frac{1}{(1 - \frac{\rho}{n})^2}$$

$$T_{wait} = \frac{N_{wait}}{\lambda}$$

Now that the charging time per car and required charging time are set, we can get different waiting time when we input different n. If we use a loop to test each waiting time by inputting different ns, we can get the minimal required charging piles by setting the maximum allowed waiting time. The final results are shown in the results section.

4 The Model Results

4.1 Distribution

For the third question, we analyzed the curve relationship between the number of charging stations and the sum of distance. Combining with the number of gas stations and the number of charging stations in Hangzhou, we think it is reasonable to set up 200 charging stations. We set the cluster in the clustering algorithm to 200, and take all the clustering centers as the location of our final charging pile. Finally, we print the coordinates of the cluster center on the map and get the following picture results.

Figure 4: Distribution of 200 charge stations

Then we use the Queuing Theory to determine the number of charging piles in each charging station.The final result is shown as follows.

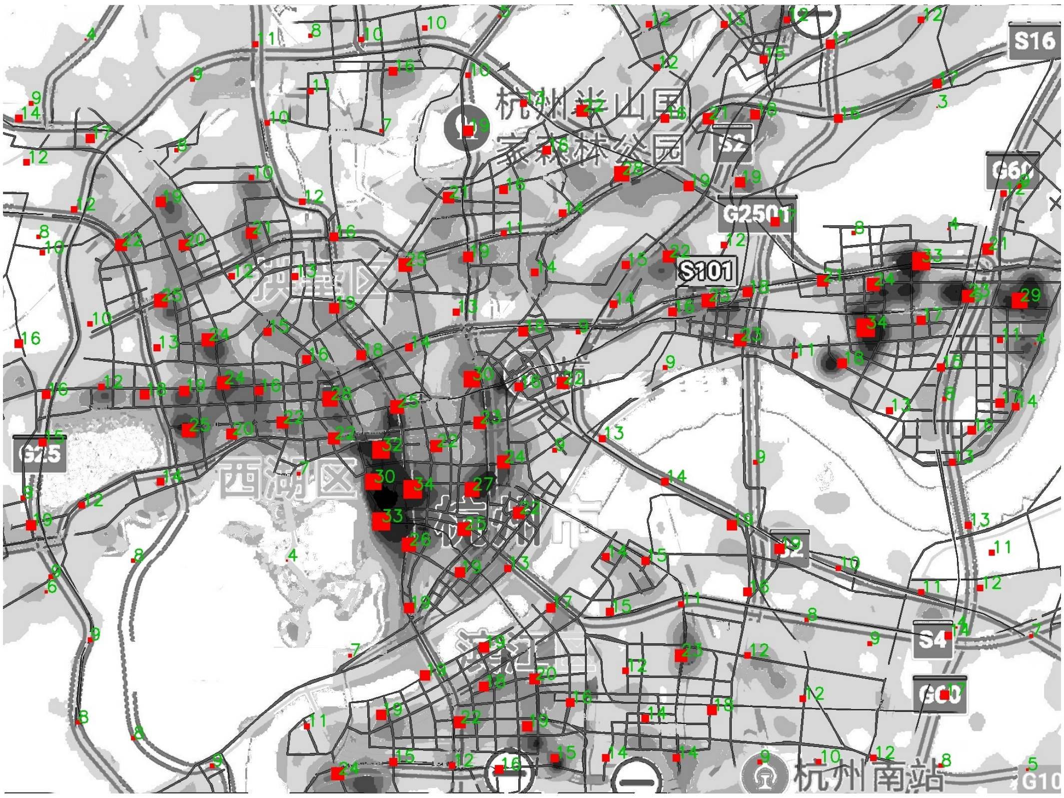

Figure 5: The number of charging piles in each station

4.2 How to decrease

For the fourth question, we need to ensure that the number of charging stations is reduced without decreasing user satisfaction. So to keep Eva at a certain level, we need to keep T and dis/power at a certain level. That is, we can ensure that the average queuing time is constant by configuring the number of charging stations in each charging pile selection point, and with the increase of new energy vehicle's endurance power, we can get bigger dis and make dis/power constant. Therefore, for a certain endurance power, we calculate the corresponding dis. From the relationship curve between the number of charging stations and the sum of distance we obtained before, we read out the number of charging stations, that is, the least number of charging stations, and then K-means clustering algorithm is used to obtain the minimum distribution of charging stations without reducing user satisfaction. The following two graphs show the relationship between the number of charging stations and the sum of their distances.

Next, we will give an example to illustrate. Assuming that the endurance of new energy vehicles is doubled, the corresponding dis is doubled. It can be seen from

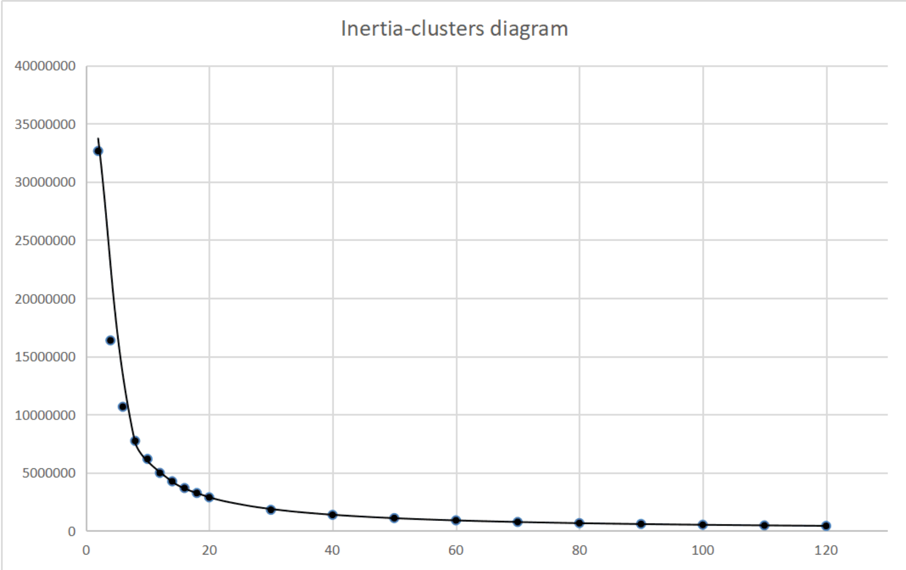

Figure 6: Inertia-clusters diagram 1

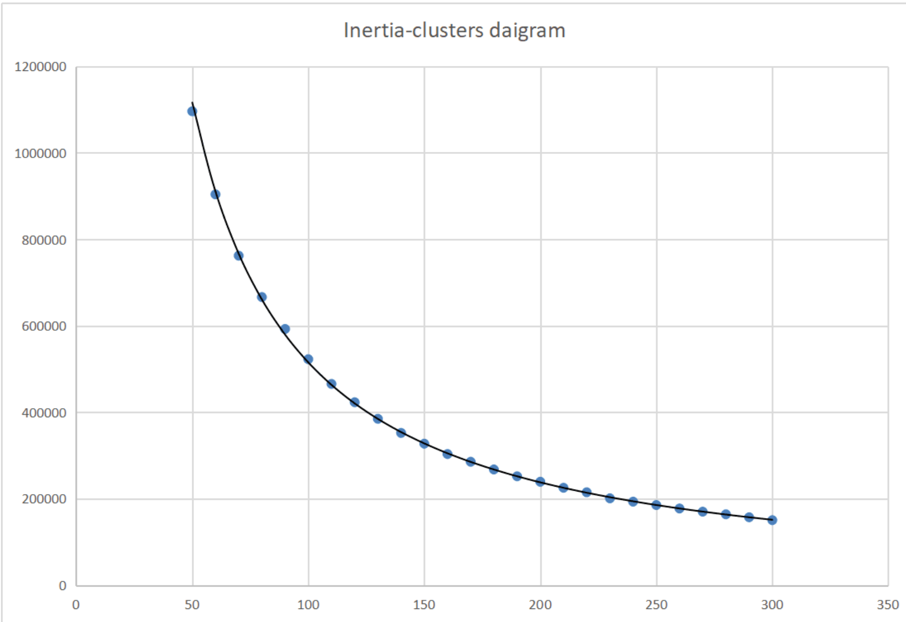

Figure 7: Inertia-clusters diagram 2

the graph that when the original 200 charging stations are used, the sum of the average distance (*inertia*) is 210000, then after the change, the sum of the average distance (*inertia*) needs to be changed to 420000. It can be seen from the curve that the number of charging stations needed is about 120. Therefore, we set $K = 120$. The locations of charging stations and the number of charging piles in each station are shown in the following picture.

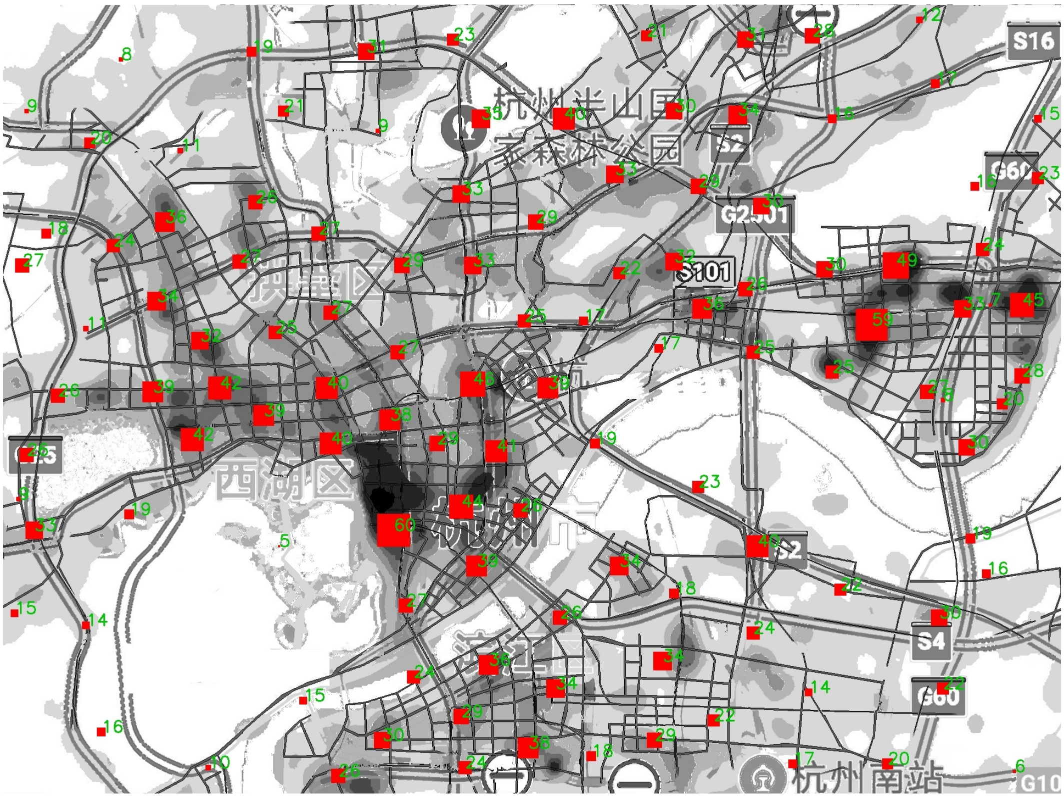

Figure 8: Distribution of 120 charge stations

5 Analysis

5.1 Strengths and Weakness

- **Strengths:**

  1. In image processing, our model adds the maximum overlay between different layers to extract features better.
  2. When we add layers of functional partitions, our algorithm actually superimposes different partitions on the previous layers according to different weights and probabilities, which is more reasonable.
  3. We use K-means clustering algorithm to select the geographic location of charging stations more accurately and minimize the average distance from charging stations.

- **Weakness:**

  1. The factors we may consider are not comprehensive enough. We can consider more factors that have an impact on the distribution of charging stations.
  2. We refer to some literature about the quantitative relationship between the impact of different functional zones on the demand of charging sta-

tions, but not specially for Hangzhou, there may be regional differences and other issues.

5.2 Model promotion

We can analyze more factors, collect more map data, and overlay more layers to achieve a more comprehensive analysis. We can do some special research on the impact of different functional zones on the demand of charging stations in Hangzhou area, so as to obtain more probabilistic basis for Hangzhou, make our model more pertinent and regional, and make the solution more reasonable.

Appendices

Appendix A First appendix: Kmeans.py

Here is the simulation program we used to cluster and draw the final map of charging stations

Input Python source:

```
from skimage import io
from sklearn.cluster import KMeans
import numpy as np
import matplotlib.pyplot as plt
import cv2
import os
import time

A = np.zeros((7876,2), dtype=float)
a1=np.zeros(1000)
#A = zeros((23,2), dtype=float)

f = open('test1.txt')
lines = f.readlines()
A_row = 0
for line in lines:
    list = line.strip('\n').split('\t')
    A[A_row:] = list[0:2]
    A_row += 1
print(A)
f.close()
#image = image.reshape(image.shape[0]*image.shape[1],1)
clus=110;
while(clus<120):
    clus+=10
    kmeans = KMeans(n_clusters = clus, n_init=50, max_iter=300) #K-Means by sklearn.I
    kmeans.fit(A)
    clusters = np.asarray(kmeans.cluster_centers_,dtype=np.uint8)
    labels = np.asarray(kmeans.labels_,dtype=np.uint8 )
    inertia=np.asarray(kmeans.inertia_)
    print(kmeans.inertia_)
#labels = labels.reshape(rows,cols)
#print(labels)
print (clusters)
fo = open("label.txt", "w")
for i in range(clus):
    a1[i]=0
for i in range(7876):
    a1[labels[i]]=a1[labels[i]]+1
for i in range(clus):
    fo.write(str(clusters[i][0])+' '+str(clusters[i][1])+' '+str(int(a1[i]))+'\n')
print("inertia:")
print(inertia)
```

```
fo.close()

os.system('label.exe') #calculate the number of piles for each station.
#It is generate by label.cpp, appendix B.
f = open('label1.txt')
lines = f.readlines()
A_row = 0
img1=cv2.imread("10.jpg")
for line in lines:                #draw the picture
  list = line.strip('\n').split(' ')
  Px=int(int(list[0])*7.2)
  Py=int(int(list[1])*7.2)
  si=int(int(list[3])/2)     #more the piles,bigger the point size.
  for ii in range(si*2):
    for jj in range(si*2):
        if ((Px+ii-si>0) and (Px+ii-si<1440) and (Px+jj-si>0) and (Px+jj-si<2160)):
            img1[Px+ii-si][Py+jj-si][2]=255
            img1[Px+ii-si][Py+jj-si][1]=0
            img1[Px+ii-si][Py+jj-si][0]=0
  cv2.putText(img1, str(int(list[3])), (Py,Px),cv2.FONT_HERSHEY_SIMPLEX, 1, (0,200,0)

cv2.imwrite("120P.jpg",img1)
```

Appendix B Second appendix: label.cpp

Here is the simulation program we used to calculate the number of charging piles for each station and label.exe will be generated.

Input C++ source:

```
#include<iostream>
#include<cstdio>
#include<cmath>
#include<cstdlib>
#include<string>
#include<algorithm>
using namespace std;
struct lll{
    int x,y,z;
}C[300];
bool cmp(lll a,lll b)
{
    if (a.z<b.z) return 1;else return 0;
}

const int maxn = 1007;
double jc(int k)
{
    double ans = 1;
    for (int i = 2; i <= k; i++)
        ans *= i;
    return ans;
```

```
}
int calc(int C_num)
{
    double lamda = 1.0*C_num/(12.0*60.0), u = 1.0/30;
    //lamda=(lamda-0.2)/25+0.2;
    for (int n=2;n<=100;++n)
    {
        double miu = lamda / u;
        double p0 = 1;
        double p_sun = 0;
        double Q = 1 - p_sun;
        double A = lamda * Q;
        double L_dui = pow(miu, n + 1) *p0 / (n * jc(n)) ;
        //cout<<L_dui<<endl;
        L_dui /= (1 - miu / n) * (1 - miu / n);
        double W_dui = L_dui / lamda;
        double K = miu * (1 - p_sun);
        double L_xi = L_dui + miu;
        double W_xi = L_xi/lamda;
        if(W_dui<=15.0) return n;
    }
}
int main()
{
    freopen("label.txt","r",stdin);
    freopen("label1.txt","w",stdout);
    int M=120;
    for(int i=1;i<=M;++i)
    {
        cin>>C[i].x>>C[i].y>>C[i].z;
    }
    sort(C+1,C+M+1,cmp);
    for(int i=1;i<=M;++i)
    {
        C[i].z=168000*88*C[i].z/(7876*152*3);
        printf("%d %d %d %d\n",C[i].x,C[i].y,C[i].z,calc(C[i].z));
    }
}
```