Universidade do Minho
Escola de Engenharia

Processamento de Sinal
ENGENHARIA DE TELECOMUNICAÇÕES E INFORMÁTICA
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(Docente: José Manuel Tavares Vieira Cabral)

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Trabalho para casa

Séries de Fourier

Rui Filipe Ribeiro Freitas - a84121@alunos.uminho.pt

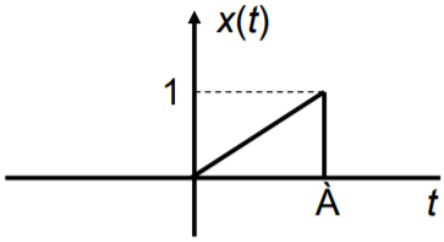
Tiago João Pereira Ferreira - a85392@alunos.uminho.pt

ENUNCIADO

Para este trabalho foi-nos proposta a realização de aproximações de funções através da Série Trigonométrica de Fourier (STF) e da Série de *Fourier-Legendre*, sendo os 2 primeiros exercícios utilizando a primeira e os 2 últimos a última.

Para a realização destes exercícios começamos por calcular os valores para 1 termo, 2 termos e 20 termos numa folha. Após estes cálculos achamos por bem realizar um programa em *MatLab* que nos ajudasse nas contas bem como na realização dos gráficos. Para isso escrevemos um script que nos facilitasse o trabalho onde o utilizador só tem de colocar o número de termos que quer e os valores da função para que o programa retorne um gráfico da aproximação à função inicial. Em seguida apresentaremos os gráficos bem como a demonstração de alguns cálculos relevantes.

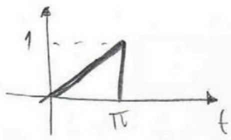
Slide 6 – Gráfico 1



$(0,0)$ $(-\pi,-1)$ $\frac{u-0}{-\pi} = -1$

$(\pi,1)$

$-u = -\pi y$ $\boxed{y = \frac{u}{\pi}}$ t



Trabalho de casa

n	an	bn	Cn
0	1	0	1
1	0	$-\frac{1}{\pi}$	$\frac{1}{\pi}$
2	0	$-\frac{1}{2\pi}$	$\frac{1}{2\pi}$
⋮	⋮	⋮	⋮
20	0	$-\frac{1}{20\pi}$	$\frac{1}{20\pi}$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t).\cos(n w_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t).\,\text{sen}(n w_0 t)\,dt$$

$$C_n = \frac{2n+1}{2}\int_{t_0}^{t_0+T}$$

$$w_0 = \frac{2\pi}{T} \qquad w_0 = \frac{2\pi}{\pi} = 2$$

$\boxed{a_0}$

$$\frac{2}{\pi}\int_0^{\pi} f(t).\cos(n w_0 t)\,dt = \frac{2}{\pi}\int_0^{\pi}\frac{t}{\pi}.\cos(n.2t)\,dt = \frac{2}{\pi^2}\int_0^{\pi} t.\cos(2nt)\,dt$$

$\boxed{C.a}$ $$\int_0^{\pi} t.\underbrace{\cos(2nt)}_{\cos(0)}\,dt = \int_0^{\pi} t.1\,dt = \left[\frac{t^2}{2}\right]_0^{\pi} = \frac{\pi^2}{2} - 0 = \frac{\pi^2}{2}$$

$$a_0 \rightarrow = \frac{2}{\pi^2}.\frac{\pi^2}{2} = 1$$

$\boxed{a_1}$

$$\frac{2}{\pi}\int_0^{\pi} f(t).\cos(w_0 t)\,dt = \frac{2}{\pi}\int_0^{\pi}\frac{t}{\pi}.\cos(2t)\,dt = \frac{2}{\pi^2}\int_0^{\pi} t.\cos(2t)\,dt$$

$\boxed{C.a}$ $$\int_0^{\pi} t.\cos(2t)\,dt = t.\frac{1}{2}\text{sen}(2t) - \int\frac{1}{2}\text{sen}(2t)\,dt = \left[\frac{t}{2}\text{sen}(2t) + \frac{1}{4}\cos(2t)\right]_0^{\pi} = 0$$

$$a_1 \rightarrow = \frac{2}{\pi^2}\times 0 = 0$$

$\boxed{a_2}$

$$\frac{2}{\pi}\int_0^{\pi} f(t).\cos(2\times 2t)\,dt = \frac{2}{\pi}\int_0^{\pi}\frac{t}{\pi}.\cos(4t)\,dt = \frac{2}{\pi^2}\int_0^{\pi} t.\cos(4t)\,dt$$

$\boxed{C.a}$ $$\int_0^{\pi} t.\cos(4t)\,dt = t.\frac{1}{4}\text{sen}(4t) - \int\frac{1}{4}\text{sen}(4t)\,dt = \left[\frac{t}{4}\text{sen}(4t) + \frac{1}{16}\cos(4t)\right]_0^{\pi} = 0$$

$$a_2 \rightarrow = \frac{2}{\pi^2}\times 0 = 0$$

$$\boxed{b_0} \quad \frac{2}{\pi}\int_0^{\pi} \frac{t}{\pi}\cdot \underbrace{\text{sen}(n\omega_0 t)}_{\text{sen } 0 = 0}\,dt = \frac{2}{\pi^2}\int_0^{\pi} t\cdot 0\,dt = \frac{2}{\pi^2}\times 0 = 0$$

$$\boxed{b_1} \quad \frac{2}{\pi}\cdot\int_0^{\pi} \frac{t}{\pi}\cdot \text{sen}(2t)\,dt = \frac{2}{\pi^2}\cdot\int_0^{\pi} t\cdot \text{sen}(2t)\,dt = / \; \frac{2}{\pi^2}\cdot\left(-\frac{\pi}{2}\right) = -\frac{1}{\pi}$$

$$/\int_0^{\pi} \underset{u}{t}\cdot \underset{dv}{\text{sen}(2t)dt} = t\cdot\left(-\frac{1}{2}\cos(2t)\right) - \int_0^{\pi} -\frac{1}{2}\cos(2t)\cdot 1\,dt =$$

$$= \left[-\frac{t}{2}\cos(2t) + \underbrace{\frac{1}{2}\cdot\frac{1}{2}}_{4}\text{sen}(2t)\right]_0^{\pi} = \left[\left(-\frac{\pi}{2}+0\right) - (0+0)\right] = -\frac{\pi}{2}$$

$$\boxed{b_2} \quad \frac{2}{\pi}\cdot\int_0^{\pi} \frac{t}{\pi}\cdot \text{sen}(4t)\,dt = \frac{2}{\pi^2}\int_0^{\pi} t\cdot \text{sen}(4t)\,dt = / \; \frac{2}{\pi^2}\cdot\left(-\frac{\pi}{4}\right) = -\frac{1}{2\pi}$$

$$/\int_0^{\pi} t\cdot \text{sen}(4t)\,dt = t\cdot\left(-\frac{1}{4}\cos(4t)\right) - \int_0^{\pi}\left(-\frac{1}{4}\cos(4t)\right)\cdot 1\,dt =$$

$$= \left[-\frac{t}{4}\cos(4t) + \underbrace{\frac{1}{4}\cdot\frac{1}{4}}_{16}\text{sen}(4t)\right]_0^{\pi} = \left[\left(-\frac{\pi}{4}+0\right) - (0+0)\right] = -\frac{\pi}{4}$$

$$\boxed{b_{20}} \quad \frac{2}{\pi}\cdot\int_0^{\pi} \frac{t}{\pi}\cdot \text{sen}(40t)\,dt = \frac{2}{\pi^2}\int_0^{\pi} t\cdot \text{sen}(40t)\,dt = / \; \frac{2}{\pi^2}\cdot\left(-\frac{\pi}{40}\right) = -\frac{1}{20\pi}$$

$$/\int_0^{\pi} t\cdot \text{sen}(40t)\,dt = t\cdot\left(-\frac{1}{40}\cos(40t)\right) - \int -\frac{1}{40}\cos(40t)\,dt =$$

$$= \left[-\frac{t}{40}\cos(40t) + \frac{1}{40}\times\frac{1}{40}\text{sen}(40t)\,dt\right]_0^{\pi} = \left[\left(-\frac{\pi}{40}+0\right) - (0+0)\right] = -\frac{\pi}{40}$$

—— // —— // ——

$$C_0 = \sqrt{a_0^2 + b_0^2} = \sqrt{1^2 + 0^2} = 1$$

$$C_1 = \sqrt{a_1^2 + b_1^2} = \sqrt{0^2 + \left(-\frac{1}{\pi}\right)^2} = \frac{1}{\pi}$$

$$C_2 = \sqrt{a_2^2 + b_2^2} = \sqrt{0^2 + \left(-\frac{1}{2\pi}\right)^2} = \frac{1}{2\pi}$$

$$C_{20} = \sqrt{a_{20}^2 + b_{20}^2} = \sqrt{0^2 + \left(-\frac{1}{20\pi}\right)^2} = \frac{1}{20\pi}$$

Gráficos *MatLab*

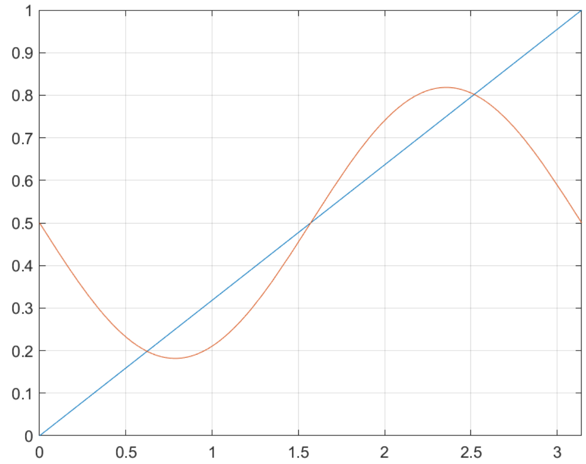


Figura 1 - Aproximação com 1 termo

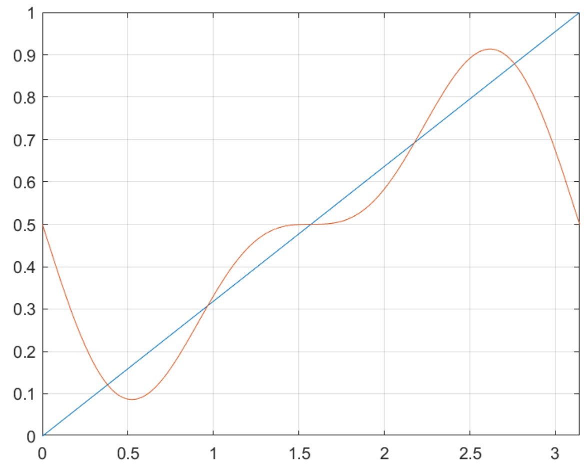


Figura 2 - Aproximação com 2 termos

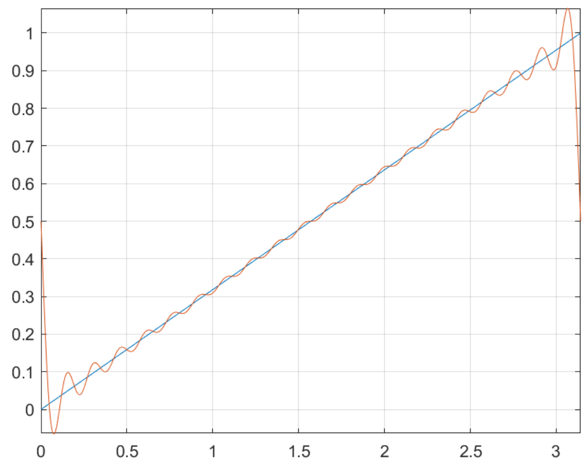
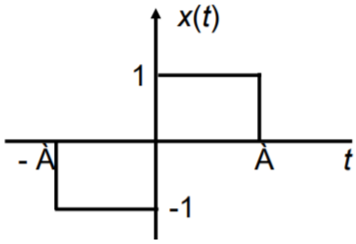
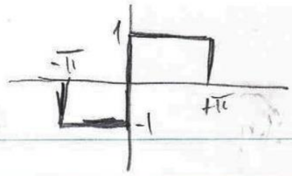


Figura 3 - Aproximação com 20 termos

Slide 6 – Gráfico 2





n	a_n	b_n	c_n
0	0	0	0
1	0	$\frac{4}{\pi}$	$\frac{4}{\pi}$
2	0	0	0
3	0	$\frac{4}{3\pi}$	$\frac{4}{3\pi}$
⋮	⋮	⋮	⋮
20	0	0	0

$w_0 = \frac{2\pi}{2\pi} = 1$

$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t) \cdot \cos(n w_0 t)\, dt$

$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t) \cdot \text{sen}(n w_0 t)\, dt$

$c_n = \sqrt{a_n^2 + b_n^2}$

$\boxed{a_0}$

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} f(t) \cdot \cos(n t)\, dt = \frac{1}{\pi}\int_{-\pi}^{0} -1 \cdot \cos(0t)\, dt + \frac{1}{\pi}\int_{0}^{\pi} 1 \cdot \cos(0t)\, dt$$

$$= \frac{1}{\pi}\left[-t\right]_{-\pi}^{0} + \frac{1}{\pi}\left[t\right]_{0}^{\pi} = -1 + 1 = 0$$

$\boxed{a_1}$

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} f(t) \cdot \cos(t)\, dt = \frac{1}{\pi}\int_{-\pi}^{0} -1 \cdot \cos(t)\, dt + \frac{1}{\pi}\int_{0}^{\pi} 1 \cdot \cos(t)\, dt$$

$$= -\frac{1}{\pi} \cdot \left[\text{sen}\, t\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\text{sen}\, t\right]_{0}^{\pi} = 0 + 0 = 0$$

$\boxed{\text{Qualquer que seja o } n,\ a_n = 0}$

$\boxed{b_0}$

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} f(t) \cdot \text{sen}(0t)\, dt = \frac{1}{\pi}\int_{-\pi}^{0} -1\ \text{sen}(0)\, dt + \frac{1}{\pi}\int_{0}^{\pi} 1 \cdot \text{sen}(0)\, dt =$$

$$= 0 + 0 = 0$$

$\boxed{b_1}$

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} f(t) \cdot \text{sen}(t)\, dt = \frac{1}{\pi}\int_{-\pi}^{0} -1\ \text{sen}(t)\, dt + \frac{1}{\pi}\int_{0}^{\pi} 1 \cdot \text{sen}(t)\, dt =$$

$$= -\frac{1}{\pi}\left[-\cos t\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\cos t\right]_{0}^{\pi} = -\frac{1}{\pi}(-1-1) + \frac{1}{\pi}(1-(-1)) = \frac{2}{\pi} + \frac{2}{\pi} = \frac{4}{\pi}$$

$\boxed{b_2}$

$$\frac{1}{\pi}\int_{-\pi}^{0} -1\ \text{sen}(2t)\, dt + \frac{1}{\pi}\int_{0}^{\pi} 1\ \text{sen}(2t)\, dt = -\frac{1}{\pi}\left[-\frac{1}{2}\cos(2t)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\frac{1}{2}\cos(2t)\right]_{0}^{\pi} = 0$$

$\boxed{b_{20}}$

$$\frac{1}{\pi}\int_{-\pi}^{0} -1\ \text{sen}(20t)\, dt + \frac{1}{\pi}\int_{0}^{\pi} 1\ \text{sen}(20t)\, dt = -\frac{1}{\pi}\left[-\frac{1}{20}\cos(20t)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\frac{1}{20}\cos(20t)\right]_{0}^{\pi} = 0$$

Gráficos *MatLab*

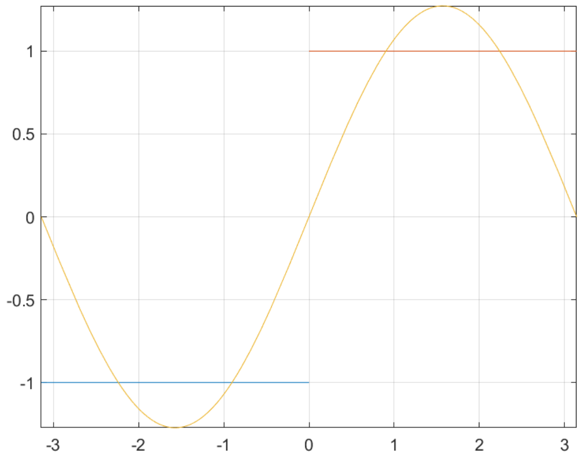


Figura 4 - Aproximação com 1 termo

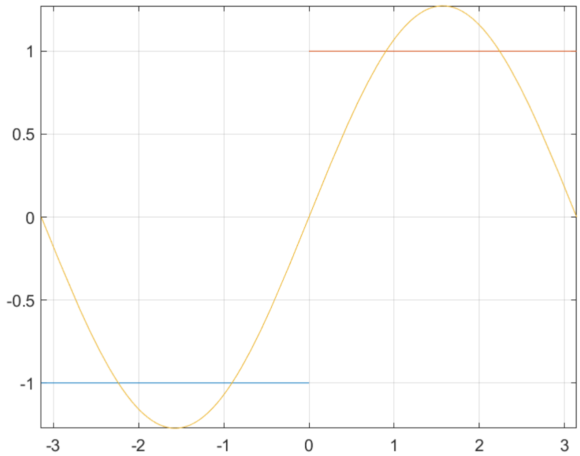


Figura 5 - Aproximação com 2 termos

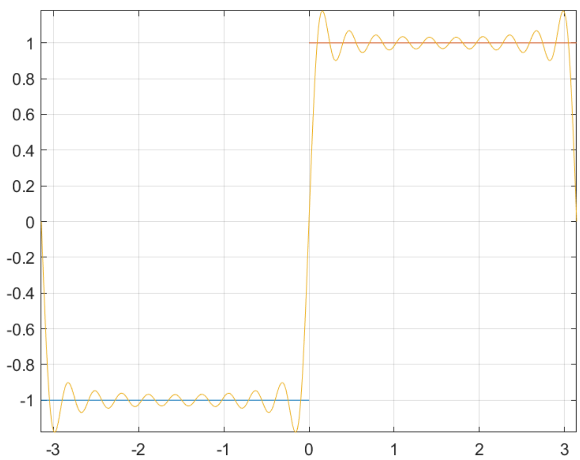
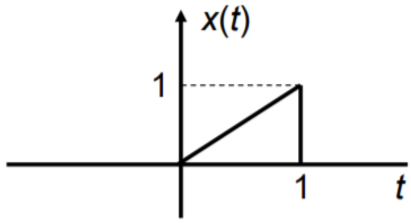


Figura 6 - Aproximação com 20 termos

Slide 11 – Gráfico 1




```
syms t  %variavel t
sum=0;  %variável somatório
y=t;    %x(t)

for n=1:20
    f=factorial(n);
    g=((t^(2))-1)^(n);
    p=(1/((2^(n))*f))*(diff(g,n));    %cálculo dos polinómios
    fprintf("%s\n", p);               %print dos polinómios
    c=(((2*n)+1)/2)*(int(y*p,t,-1,1));
    sum=sum+(p*c);                    %cálculo do somatório
end

fplot(t,y,[0,1]);                     %gráfico de x(t)
grid on;hold on;
fplot(t,(sum),[0,1]);                 %gráfico da aproximação a x(t)
```

Gráficos *MatLab*

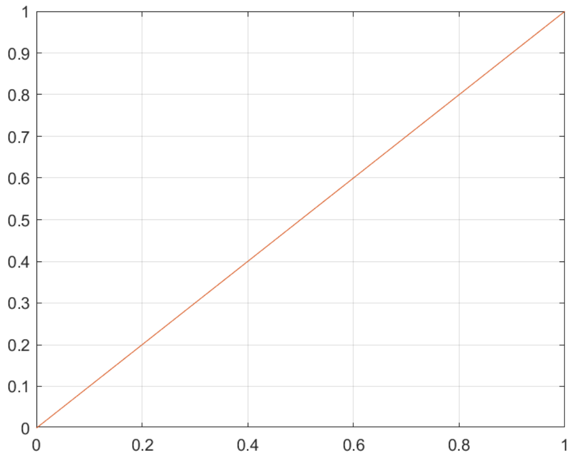


Figura 7 - Aproximação com 1 termo

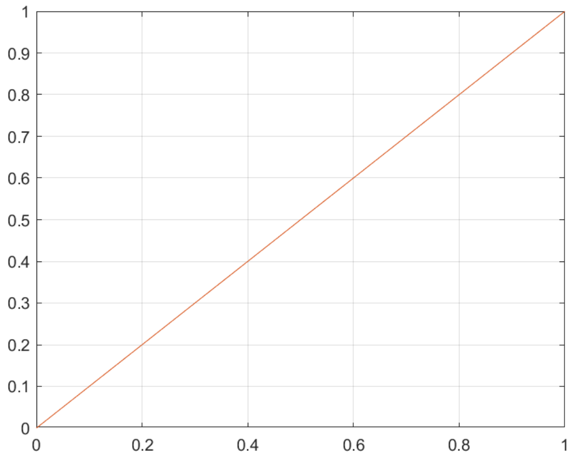


Figura 8 - Aproximação com 2 termos

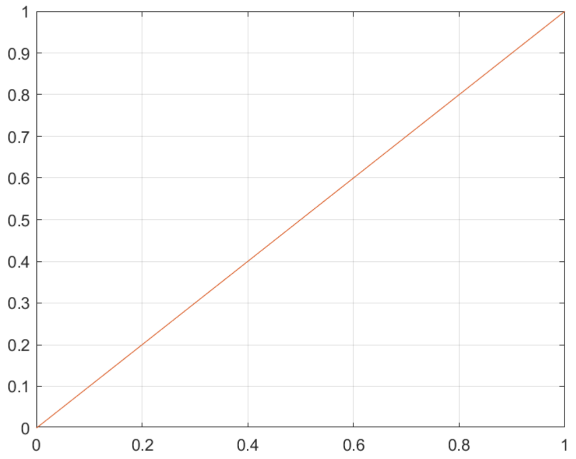
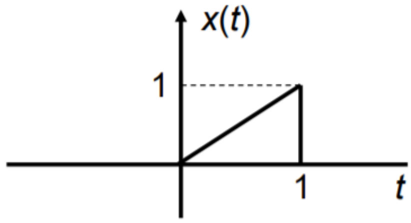


Figura 9 - Aproximação com 20 termos

Slide 11 – Gráfico 2




```
syms t  %variavel t
sum=0;  %variável somatório
y1=-1;  %x(t) , -1<=x<=0
y2=1;   %x(t) ,  0<=x<=1

for n=1:20
    f=factorial(n);
    g=((t^(2))-1)^(n);
    p=(1/((2^(n))*f))*(diff(g,n));   %cálculo dos polinómios
    fprintf("%s\n", p);              %print dos polinómios
    c=(((2*n)+1)/2)*((int(y1*p,t,-1,0))+(int(y2*p,t,0,1)));
    sum=sum+(p*c);                   %cálculo do somatório
end

fplot(-1,[-1,0]);       %gráfico de x(t) negativo
grid on;hold on;
fplot(1,[0,1]);         %gráfico de x(t) positivo
grid on;hold on;
fplot(t,(sum),[-1,1]);  %gráfico da aproximação a x(t)
```

Gráficos *MatLab*

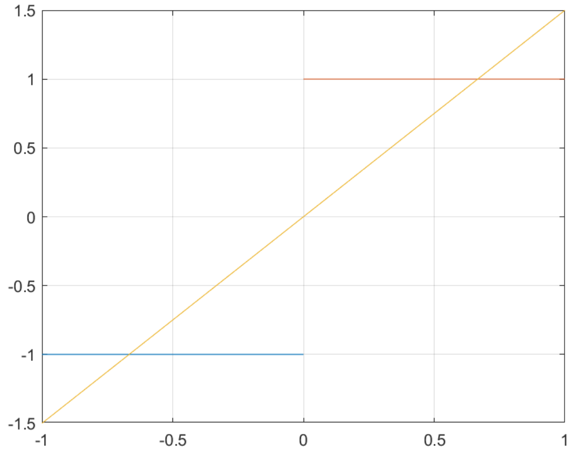


Figura 10 - Aproximação com 1 termo

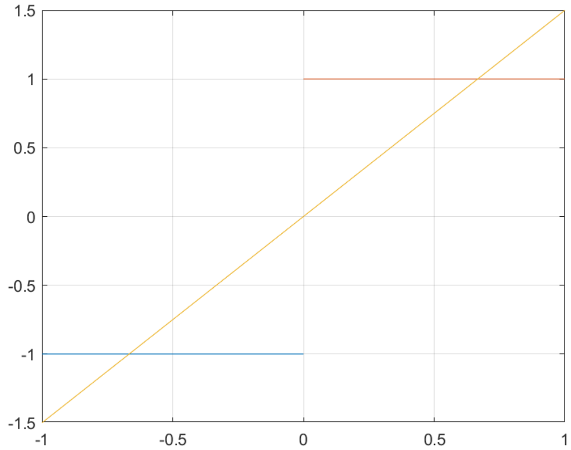


Figura 11 - Aproximação com 2 termos

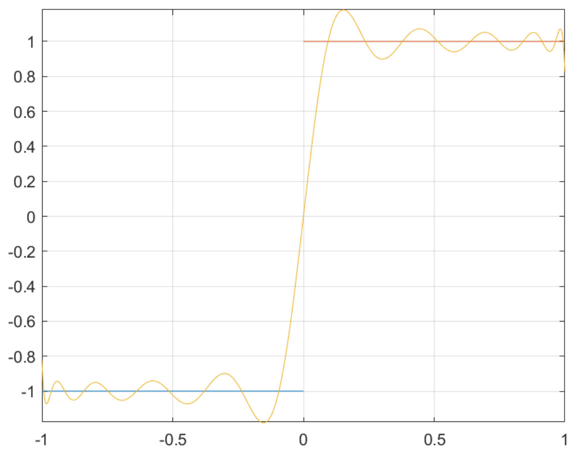


Figura 12 - Aproximação com 20 termos