

## APPENDIX A PROOF OF LEMMA 1

**Lemma 1.** *Given any feasible task replication and power allocation, the Lyapunov drift-plus-penalty function  $B$  in our formulation is upper bounded by the following expression.*

$$B \geq \frac{1}{2}E \left[ (Q(t))^2 + \omega^2 \right] + \frac{1}{2} \sum_{i=1}^I E \left[ \left( \frac{1}{T} \sum_{t=0}^{T-1} D_{i,b}(t) \right)^2 + \epsilon^2 \right],$$

where  $\omega$  is the restricted queue length in the cloud, and  $R_{i,b}(t)$  is denoted as the total amount of computing from TaV  $i$  in the time slot  $t$ ,  $\epsilon$  is the average delay constraint.

*Proof.* By squaring both sides of the equation, we can get the expression below.

$$\begin{aligned} H(t+1)^2 &= (\max \{H(t) + Q(t) - \omega, 0\})^2 \\ &\leq H(t)^2 + \omega^2 + Q(t)^2 + 2H(t)Q(t) \\ &\quad - 2\omega Q(t) - 2\omega H(t). \end{aligned}$$

The following inequality can be obtained as follows.

$$\begin{aligned} \frac{1}{2}[H(t+1)^2 - H(t)^2] &\leq \frac{1}{2}(\omega^2 + Q(t)^2 \\ &\quad + 2Q(t)H(t) - 2\omega Q(t) - 2\omega H(t)) \\ &\leq \frac{1}{2}[Q(t)^2 + \omega^2 + 2H(t)(Q(t) - \omega)]. \end{aligned}$$

Similarly, we can obtain the following expression.

$$\begin{aligned} \frac{1}{2} \sum_{i=1}^I [G_i(t+1)^2 - G_i(t)^2] &\leq \frac{1}{2} \sum_{i=1}^I \left[ \left( \sum_{t=0}^{T-1} D_{i,b}(t) \right)^2 \right. \\ &\quad \left. + \epsilon^2 + 2G_i(t) \left( \sum_{t=0}^{T-1} D_{i,b}(t) - \epsilon \right) \right]. \end{aligned}$$

According to the definition of Lyapunov drift function, we can get the expression below.

$$\begin{aligned} L(\Theta(t)) &= \frac{1}{2} \left( (H(t))^2 + \sum_{i=1}^I (G_i(t))^2 \right), \\ \Delta L(t) &= E[L(\Theta(t+1)) - L(\Theta(t))]. \end{aligned}$$

Then, we can get a bound on  $\Delta L(t)$ .

$$\begin{aligned} \Delta L(t) &\leq B + [H(t)(Q(t) - \omega) \\ &\quad + \sum_{i=1}^I G_i(t) \left( \sum_{t=0}^{T-1} D_{i,b}(t) - \epsilon \right)], \end{aligned}$$

where  $B$  is upper bounded by the expression as below.

$$B \geq \frac{1}{2}E \left[ (Q(t))^2 + \omega^2 \right] + \frac{1}{2} \sum_{i=1}^I E \left[ \left( \frac{1}{T} \sum_{t=0}^{T-1} D_{i,b}(t) \right)^2 + \epsilon^2 \right].$$

□

## APPENDIX B PROOF OF LEMMA 2

**Lemma 2.** *The BER  $\Omega_{i,b}(t)$  between TaV  $i$  and the edge cloud and the BER  $\Omega_{i,j}(t)$  between TaV  $i$  and SeV  $j$  under Rayleigh fading conditions have the approximation as below.*

$$\begin{aligned} \Omega_{i,b}(t) &\approx \frac{(e^{R_{i,b}(t)} - 1) \left( \sum_{k=i+1}^I |h_{k,b}(t)|^2 \alpha_i^n(t) p_{k,b}(t) + \sigma^2 \right)}{p_{i,b}(t) |h_{i,b}(t)|^2}, \\ \Omega_{i,j}(t) &\approx \frac{(e^{\frac{K_{i,j}}{L_{i,j}}} - 1) \sigma^2}{p_{i,j}(t) |h_{i,j}(t)|^2}. \end{aligned}$$

*Proof.* According to Taylor's Expanded Form, we can get the approximations of BER as below.

$$\begin{aligned} \Omega_{i,b}(t) &\approx 1 - k_1 \gamma_{i,b}(t) \left( 1 + \frac{k_2}{\gamma_{i,b}(t)} + \frac{k_2^2}{2\gamma_{i,b}^2(t)} \right. \\ &\quad \left. - 1 - \frac{k_3}{\gamma_{i,b}(t)} - \frac{k_3^2}{2\gamma_{i,b}^2(t)} \right), \end{aligned}$$

where  $k_1 = \sqrt{\frac{L_{i,b}}{2\pi(e^{\frac{K_{i,b}}{L_{i,b}}} - 1)}}$ ,  $k_2 = 1 - e^{\frac{K_{i,b}}{L_{i,b}}} + \sqrt{\frac{\pi(e^{\frac{K_{i,b}}{L_{i,b}}} - 1)}{2L_{i,b}}}$ ,  
and  $k_3 = 1 - e^{\frac{K_{i,b}}{L_{i,b}}} - \sqrt{\frac{\pi(e^{\frac{K_{i,b}}{L_{i,b}}} - 1)}{2L_{i,b}}}$ .

$$\begin{aligned} \Omega_{i,j}(t) &\approx 1 - g_1 \gamma_{i,j}(t) \left( 1 + \frac{g_2}{\gamma_{i,j}(t)} + \frac{g_2^2}{2\gamma_{i,j}^2(t)} \right. \\ &\quad \left. - 1 - \frac{g_3}{\gamma_{i,j}(t)} - \frac{g_3^2}{2\gamma_{i,j}^2(t)} \right), \end{aligned}$$

where  $g_1 = \sqrt{\frac{L_{i,j}}{2\pi(e^{\frac{K_{i,j}}{L_{i,j}}} - 1)}}$ ,  $g_2 = 1 - e^{\frac{K_{i,j}}{L_{i,j}}} + \sqrt{\frac{\pi(e^{\frac{K_{i,j}}{L_{i,j}}} - 1)}{2L_{i,j}}}$ ,  
and  $g_3 = 1 - e^{\frac{K_{i,j}}{L_{i,j}}} - \sqrt{\frac{\pi(e^{\frac{K_{i,j}}{L_{i,j}}} - 1)}{2L_{i,j}}}$ .

After simplification, we can obtain the below expressions.

$$\begin{aligned} \Omega_{i,b}(t) &\approx \frac{(e^{\frac{K_{i,b}}{L_{i,b}}} - 1) \left( \sum_{k=i+1}^I |h_{k,b}(t)|^2 p_{k,b}(t) + \sigma^2 \right)}{p_{i,b}(t) |h_{i,b}(t)|^2}, \\ \Omega_{i,j}(t) &\approx \frac{(e^{\frac{K_{i,j}}{L_{i,j}}} - 1) \sigma^2}{p_{i,j}(t) |h_{i,j}(t)|^2}. \end{aligned}$$

□

## APPENDIX C PROOF OF LEMMA 3

**Lemma 3.** *The expected service reliability incurred by P-ATRLF could achieve the effect that has a gap with the maximal service reliability, which can be described below.*

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} E\{\Upsilon(t) | \Theta(t)\} \leq \Upsilon^{\sup}(t) - \frac{B}{V},$$

□

where  $\Upsilon^{\sup}(t)$  is the supremum of the average service reliability achieved by any joint strategy under the required constraints, and  $B$  is mentioned in **Lemma 1**.

*Proof.* From **Lemma 2**, we can get the below expression.

$$\begin{aligned} \Delta L(t) - VE\{\Upsilon(t)|\Theta(t)\} &\leq B + H(t)E\{Q(t) - \omega|\Theta(t)\} \\ &+ \sum_i^I G_i(t)E\left\{\frac{1}{T} \sum_{t=0}^{T-1} D_{i,b}(t) - \epsilon|\Theta(t)\right\} - VE\{\Upsilon(t)|\Theta(t)\}. \end{aligned}$$

$B$  is the upper bound, which is mentioned in **Lemma 1**. From **Section 3.1.4** in the study [16], we have the following inequality.

$$B - V\Upsilon^{\sup}(t) \leq \Delta L(t) - VE\{\Upsilon(t)|\Theta(t)\}.$$

By adding  $t = 0, 1, \dots, T-1$  cumulatively, we get:

$$\begin{aligned} TB - TV\Upsilon^{\sup}(t) &\leq E\{L(\Theta(t)) - L(\Theta(0))\} \\ &- V \sum_{t=0}^{T-1} E\{\Upsilon(t)|\Theta(t)\}. \end{aligned}$$

Since  $L(\Theta(t)) \geq 0, L(\Theta(0)) = 0$ , both sides divide by the same  $-TV$ , when  $t \rightarrow +\infty$ , we can obtain that:

$$\lim_{t \rightarrow +\infty} \sup \frac{1}{t} \sum_{t=0}^{T-1} E\{\Upsilon(t)\} \leq \Upsilon^{\sup}(t) - \frac{B}{V}.$$

With Jensen's inequality, we can get:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} E(\Upsilon(t)) \leq \Upsilon^{\sup}(t) - \frac{B}{V}.$$

□

#### APPENDIX D PROOF OF LEMMA 4

**Lemma 4.** The virtual task queue in the edge cloud and the virtual delay queue for all TaVs in NOMA-enabled VEC are stable, which can be presented as follows.

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \left( E\{H(t)\} + \sum_{i=1}^I E\{G_i(t)\} \right) \\ \leq \frac{V(\hat{\Upsilon}(t) - \check{\Upsilon}(t)) - B}{\varsigma}, \end{aligned}$$

where  $\hat{\Upsilon}(t)$  and  $\check{\Upsilon}(t)$  are defined as the maximum and minimum service reliability among all strategies.  $B$  is mentioned in **Lemma 1**.

*Proof.* We now prove that  $Q(t)$  and  $G_i(t)$  are mean rate stable under Algorithm 3.

Denote  $\Phi(t) = TV\{\hat{\Upsilon}(t)\} - V \sum_{t=0}^{T-1} E\{\Upsilon(\bar{t})|\Theta(t)\} - TB + E\{L(\Theta(0))\}$ .

Based on **Lemma 3**, we can get  $E\{L(\Theta(t))\} \leq \Phi(t)$ , i.e.,  $E\{H(t)^2\} \leq 2\Phi(t)$  and  $E\{\sum_{i=1}^I G_i(t)^2\} \leq 2\Phi(t)$ . Thus, we have the following inequality.

$$\begin{aligned} 0 &\leq E\{H(t)\} \leq \sqrt{2\Phi(t)}. \\ 0 &\leq \sum_{i=1}^I E\{G_i(t)\} \leq \sqrt{2\Phi(t)}. \end{aligned}$$

Since  $\lim_{T \rightarrow \infty} \frac{\sqrt{2\Phi(t)}}{T} = 0$ , we can also acknowledge that

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{E\{H(t)\}}{T} &= 0. \\ \lim_{T \rightarrow \infty} \sum_{i=1}^I \frac{E\{G_i(t)\}}{T} &= 0. \end{aligned}$$

To find the upper bound of the average sum queue length of  $E\{H(t)\}$  and  $\sum_{i=1}^I E\{G_i(t)\}$ , we can obtain the below expression in the proof of **Lemma 3**,

$$\begin{aligned} L(\Theta(t+1)) - L(\Theta(t)) - VE\{\Upsilon(t)|\Theta(t)\} &\geq \\ \left[ \sum_{i=1}^I G_i(t) \left( \frac{1}{T} \sum_{t=0}^{T-1} D_{i,b}(t) - \epsilon \right) + H(t)(Q(t) - \omega) \right] \\ &+ B - V\Upsilon^{\sup}(t), \end{aligned}$$

where  $\Upsilon^{\sup}(t)$  is the supremum of the average service reliability achieved by any joint strategy under the required constraints.

Then, summing over  $t \in \{0, 1, 2, \dots, T-1\}$ , we can obtain the expression below.

$$\begin{aligned} L(\Theta(T)) - L(\Theta(0)) - \sum_{t=0}^{T-1} VE\{\Upsilon(t)|\Theta(t)\} &\geq \\ TB - TV\Upsilon^{\sup}(t) + \varsigma \left( E\{H(t)\} + \sum_{i=1}^I E\{G_i(t)\} \right), \end{aligned}$$

where  $\varsigma > 0$ , and after dividing  $T$  and  $\varsigma$  by both sides of the above inequality, we can finally get.

$$\begin{aligned} \frac{1}{T} \left( E\{H(t)\} + \sum_{i=1}^I E\{G_i(t)\} \right) &\leq \frac{E(L(T))}{\varsigma T} - \frac{E(L(0))}{\varsigma T} \\ &- \frac{B - V\Upsilon^{\sup}(t) + \frac{1}{T} \sum_{t=0}^{T-1} VE\{\Upsilon(t)|\Theta(t)\}}{\varsigma}, \end{aligned}$$

$$\Upsilon^{\sup}(t) - \frac{1}{T} \sum_{t=0}^{T-1} E\{\Upsilon(t)|\Theta(t)\} \leq \hat{\Upsilon}(t) - \check{\Upsilon}(t).$$

From the predefined value  $L(0)$ , we can get the conclusion.

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \left( E\{H(t)\} + \sum_{i=1}^I E\{G_i(t)\} \right) \\ \leq \frac{V(\hat{\Upsilon}(t) - \check{\Upsilon}(t)) - B}{\varsigma}. \end{aligned}$$

□