# APPENDIX A
# PROOF OF LEMMA 1

**Lemma 1.** *Given any feasible task replication and power allocation, the Lyapunov drift-plus-penalty function $B$ in our formulation* (32) is upper bounded by the following expression.

$$B \geq \frac{1}{2}E\left[(Q(t))^2+\omega^2\right] + \frac{1}{2}\sum_{i=1}^{I}E\left[\left(\frac{1}{T}\sum_{t=0}^{T-1}D_{i,b}(t)\right)^2+\epsilon^2\right],$$

where $\omega$ is the restricted queue length in the cloud, and $R_{i,b}(t)$ is denoted as the total amount of computing from TaV $i$ in the time slot $t$, $\epsilon$ is the average delay constraint.

*Proof.* By squaring both sides of the equation (12), we can get the expression below.

$$\begin{aligned}&H(t+1)^2\\&=(\max\{H(t)+Q(t)-\omega,0\})^2\\&\leq H(t)^2+\omega^2+Q(t)^2+2H(t)Q(t)\\&-2\omega Q(t)-2\omega H(t).\end{aligned}$$

The following inequality can be obtained as follows.

$$\begin{aligned}&\frac{1}{2}[H(t+1)^2-H(t)^2]\\&\leq\frac{1}{2}\left(\omega^2+Q(t)^2+2Q(t)H(t)\right.\\&\left.-2\omega Q(t)-2\omega H(t)\right)\\&\leq\frac{1}{2}\left[Q(t)^2+\omega^2+2H(t)\left(Q(t)-\omega\right)\right].\end{aligned}$$

Similarly, we can obtain the following expression.

$$\begin{aligned}&\frac{1}{2}[G_i(t+1)^2-G_i(t)^2]\\&\leq\frac{1}{2}\left[\epsilon^2+\left(\frac{1}{T}\sum_{t=0}^{T-1}D_{i,b}(t)\right)^2-2\epsilon G_i(t)-2\epsilon\sum_{t=0}^{T-1}D_{i,b}(t)\right.\\&\left.+2G_i(t)\sum_{t=0}^{T-1}D_{i,b}(t)\right]\\&\leq\frac{1}{2}\left[\left(\sum_{t=0}^{T-1}D_{i,b}(t)\right)^2+\epsilon^2+2G_i(t)\left(\sum_{t=0}^{T-1}D_{i,b}(t)-\epsilon\right)\right].\end{aligned}$$

Summing over all the above inequality when $i=1,2,\ldots,I$, the following inequality can be obtained as below.

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{I}[G_i(t+1)^2-G_i(t)^2]\\&\leq\frac{1}{2}\sum_{i=1}^{I}\left[\left(\sum_{t=0}^{T-1}D_{i,b}(t)\right)^2+\epsilon^2+2G_i(t)\left(\sum_{t=0}^{T-1}D_{i,b}(t)-\epsilon\right)\right].\end{aligned}$$

According to the definition of Lyapunov drift function, we can get the expression below.

$$L(\Theta(t))=\frac{1}{2}\left((H(t))^2+\sum_{i=1}^{I}(G_i(t))^2\right),$$

$$\triangle L(t)=E[L(\Theta(t+1))-L(\Theta(t))].$$

Then, we can get a bound on $\triangle L(t)$.

$$\triangle L(t)\leq B+\left[H(t)\left(Q(t)-\omega\right)+\sum_{i=1}^{I}G_i(t)\left(\sum_{t=0}^{T-1}D_{i,b}(t)-\epsilon\right)\right],$$

where $B$ is upper bounded by the expression as below.

$$B \geq \frac{1}{2}E\left[(Q(t))^2+\omega^2\right] + \frac{1}{2}\sum_{i=1}^{I}E\left[\left(\frac{1}{T}\sum_{t=0}^{T-1}D_{i,b}(t)\right)^2+\epsilon^2\right].$$

□

# APPENDIX B
# PROOF OF LEMMA 2

**Lemma 2.** *The BER $\Omega_{i,b}(t)$ between TaV $i$ and the edge cloud and the BER $\Omega_{i,j}(t)$ between TaV $i$ and SeV $j$ under Rayleigh fading conditions have the approximation as below.*

$$\Omega_{i,b}(t)\approx\frac{(e^{R_{i,b}(t)}-1)\left(\sum_{k=i+1}^{I}|h_{k,b}(t)|^2\alpha_i^n(t)p_{k,b}(t)+\sigma^2\right)}{p_{i,b}(t)\,|h_{i,b}(t)|^2},$$

$$\Omega_{i,j}(t)\approx\frac{(e^{\frac{K_{i,j}}{L_{i,j}}}-1)\sigma^2}{p_{i,j}(t)\,|h_{i,j}(t)|^2}.$$

*Proof.* According to Taylor's Expanded Form, we can get the approximations of (7) and (8) as below.

$$\begin{aligned}\Omega_{i,b}(t)\approx&1-k_1\gamma_{i,b}(t)\left(1+\frac{k_2}{\gamma_{i,b}(t)}+\frac{k_2^2}{2{\gamma_{i,b}}^2(t)}\right.\\&\left.-1-\frac{k_3}{\gamma_{i,b}(t)}-\frac{k_3^2}{2{\gamma_{i,b}}^2(t)}\right),\end{aligned}$$

where $k_1=\sqrt{\frac{L_{i,b}}{2\pi(e^{2\frac{K_{i,b}}{L_{i,b}}}-1)}}$, $k_2=1-e^{\frac{K_{i,b}}{L_{i,b}}}+\sqrt{\frac{\pi(e^{2\frac{K_{i,b}}{L_{i,b}}}-1)}{2L_{i,b}}}$, and $k_3=1-e^{\frac{K_{i,b}}{L_{i,b}}}-\sqrt{\frac{\pi(e^{2\frac{K_{i,b}}{L_{i,b}}}-1)}{2L_{i,b}}}$.

$$\begin{aligned}\Omega_{i,j}(t)\approx&1-g_1\gamma_{i,j}(t)\left(1+\frac{g_2}{\gamma_{i,j}(t)}+\frac{g_2^2}{2\gamma_{i,j}^2(t)}\right.\\&\left.-1-\frac{g_3}{\gamma_{i,j}(t)}-\frac{g_3^2}{2{\gamma_{i,j}}^2(t)}\right),\end{aligned}$$

where $g_1=\sqrt{\frac{L_{i,j}}{2\pi(e^{2\frac{K_{i,j}}{L_{i,j}}}-1)}}$, $g_2=1-e^{\frac{K_{i,j}}{L_{i,j}}}+\sqrt{\frac{\pi(e^{2\frac{K_{i,j}}{L_{i,j}}}-1)}{2L_{i,j}}}$, and $g_3=1-e^{\frac{K_{i,j}}{L_{i,j}}}-\sqrt{\frac{\pi(e^{2\frac{K_{i,j}}{L_{i,j}}}-1)}{2L_{i,j}}}$.

After simplification, we can obtain the below expressions.

$$\Omega_{i,b}(t) \approx \frac{(e^{\frac{K_{i,b}}{L_{i,b}}}-1)\left(\sum_{k=i+1}^{I}|h_{k,b}(t)|^2 p_{k,b}(t)+\sigma^2\right)}{p_{i,b}(t)\,|h_{i,b}(t)|^2},$$

$$\Omega_{i,j}(t) \approx \frac{(e^{\frac{K_{i,j}}{L_{i,j}}}-1)\sigma^2}{p_{i,j}(t)\,|h_{i,j}(t)|^2}.$$

□

## Appendix C
## PROOF OF LEMMA 3

**Lemma 3.** *The expected service reliability incurred by PA-TRLF could achieve the effect that has a gap with the maximal service reliability, which can be described below.*

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}E\{\Upsilon(t)|\Theta(t)\} \leq \Upsilon^{\sup}(t)-\frac{B}{V},$$

*where $\Upsilon^{\sup}(t)$ is the supremum of the average service reliability achieved by any joint strategy under the required constraints, and $B$ is mentioned in **Lemma 1**.*

*Proof.* From **Lemma 2**, we can get the below expression.

$$\begin{aligned}\triangle L(t)-VE\{\Upsilon(t)|\Theta(t)\} \leq B-VE\{\Upsilon(t)|\Theta(t)\}\\ +\sum_{i}^{I}G_i(t)E\{\frac{1}{T}\sum_{t=0}^{T-1}D_{i,b}(t)-\epsilon|\Theta(t)\}\\ +H(t)E\{Q(t)-\omega|\Theta(t)\}.\end{aligned}$$

$B$ is the upper bound, which follows the below inequality.

$$\begin{aligned}B \geq& \frac{1}{2}E\left[(Q(t))^2+\omega^2\right]\\ &+\frac{1}{2}\sum_{i=1}^{I}E\left[\left(\frac{1}{T}\sum_{t=0}^{T-1}D_{i,b}(t)\right)^2+\epsilon^2\right].\end{aligned}$$

From **Section 3.1.4** in the study [16], we have the following inequality.

$$B-V\Upsilon^{\sup}(t) \leq \triangle L(t)-VE\{\Upsilon(t)|\Theta(t)\}.$$

By adding $t=0,1,\ldots,T-1$ cumulatively, we get:

$$\begin{aligned}TB-TV\Upsilon^{\sup}(t) \leq& E\left\{L(\Theta(t))-L(\Theta(0))\right\}\\ &-V\sum_{t=0}^{T-1}E\left\{\Upsilon(t)|\Theta(t)\right\}.\end{aligned}$$

Since $L(\Theta(t))\geq 0, L(\Theta(0))=0$, both sides divide by the same $-TV$, when $t\to+\infty$, we can obtain that:

$$\limsup_{t\to+\infty}\frac{1}{t}\sum_{t=0}^{T-1}E\left\{\Upsilon(t)\right\} \leq \Upsilon^{\sup}(t)-\frac{B}{V}.$$

With Jensen's inequality, we can get:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}E\left(\Upsilon(t)\right) \leq \Upsilon^{\sup}(t)-\frac{B}{V}.$$

□

## Appendix D
## PROOF OF LEMMA 4

**Lemma 4.** *The virtual task queue in the edge cloud and the virtual delay queue for all TaVs in NOMA-enabled VEC are stable, which can be presented as follows.*

$$\begin{aligned}&\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\left(E\{H(t)\}+\sum_{i=1}^{I}E\{G_i(t)\}\right)\\ &\leq \frac{V(\hat{\Upsilon}(t)-\check{\Upsilon}(t))-B}{\varsigma},\end{aligned}$$

*where $\hat{\Upsilon}(t)$ and $\check{\Upsilon}(t)$ are defined as the maximum and minimum service reliability among all strategies. $B$ is mentioned in **Lemma 1**.*

*Proof.* We now prove that $Q(t)$ and $G_i(t)$ are mean rate stable under Algorithm 3.

Denote $\Phi(t)TV\{\hat{\Upsilon}(t)\}-V\sum_{t=0}^{T-1}E\left\{\Upsilon(\bar{t})|\Theta(t)\right\}-TB+E\left\{L(\Theta(0))\right\}$.

Based on **Lemma 3**, we can get $E\{L(\Theta(t))\}\leq\Phi(t)$, i.e., $E\{H(t)^2\}\leq 2\Phi(t)$ and $E\{\sum_{i=1}^{I}G_i(t)^2\}\leq 2\Phi(t)$. Thus, we have the following inequality.

$$0 \leq E\{H(t)\} \leq \sqrt{2\Phi(t)}.$$
$$0 \leq \sum_{i=1}^{I}E\{G_i(t)\} \leq \sqrt{2\Phi(t)}.$$

Since $\lim_{T\to\infty}\frac{\sqrt{2\Phi(t)}}{T}=0$, we can also acknowledge that

$$\lim_{T\to\infty}\frac{E\{H(t)\}}{T}=0.$$
$$\lim_{T\to\infty}\sum_{i=1}^{I}\frac{E\{G_i(t)\}}{T}=0.$$

To find the upper bound of the average sum queue length of $E\{H(t)\}$ and $\sum_{i=1}^{I}E\{G_i(t)\}$, we can obtain the below expression in the proof of **Lemma 3**,

$$\begin{aligned}&L(\Theta(t+1))-L(\Theta(t))-VE\{\Upsilon(t)|\Theta(t)\} \geq\\ &\left[\sum_{i=1}^{I}G_i(t)\left(\frac{1}{T}\sum_{t=0}^{T-1}D_{i,b}(t)-\epsilon\right)+H(t)\left(Q(t)-\omega\right)\right]\\ &+B-V\Upsilon^{\sup}(t),\end{aligned}$$

where $\Upsilon^{\sup}(t)$ is the supremum of the average service reliability achieved by any joint strategy under the required constraints.

Then, summing over $t\in\{0,1,2,\cdots,T-1\}$, we can obtain the expression below.

$$\begin{aligned}&L(\Theta(T))-L(\Theta(0))-\sum_{t=0}^{T-1}VE\{\Upsilon(t)|\Theta(t)\} \geq\\ &TB-TV\Upsilon^{\sup}(t)+\varsigma\left(E\{H(t)\}+\sum_{i=1}^{I}E\{G_i(t)\}\right),\end{aligned}$$

where $\varsigma>0$.

After dividing $T$ and $\varsigma$ by both sides of the above inequality, we can finally get.

$$\frac{1}{T}\left(E\{H(t)\}+\sum_{i=1}^{I}E\{G_i(t)\}\right)\leq\frac{E(L(T))}{\varsigma T}-\frac{E(L(0))}{\varsigma T}$$
$$-\frac{B-V\Upsilon^{\sup}(t)+\frac{1}{T}\sum_{t=0}^{T-1}VE\{\Upsilon(t)|\Theta(t)\}}{\varsigma},$$

$$\Upsilon^{\sup}(t)-\frac{1}{T}\sum_{t=0}^{T-1}E\{\Upsilon(t)|\Theta(t)\}\leq\hat{\Upsilon}(t)-\check{\Upsilon}(t).$$

From the predefined value $L(0)$, we can get the conclusion.

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\left(E\{H(t)\}+\sum_{i=1}^{I}E\{G_i(t)\}\right)$$
$$\leq\frac{V(\hat{\Upsilon}(t)-\check{\Upsilon}(t))-B}{\varsigma}.$$

□