Providing Location-privacy in Opportunity Mobile Social Network

by

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Abstract

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# Introduction

# Background and Related Work

# Multi-Hop Location-Privacy Protection

# Appointment Card Protocol

## System Model

The network architecture consists of two main entities: Users and Location-Based Service Providers (LBSPs). Users are mobile and communicate with others in a certain range, e.g., the communication range of their portable devices. For a user, other ones in the social network are either strangers or his friends whom he can detect when they are in his communication range. Let *RSi,j* denotes the relationship strength between user *i* and *j*, if *RSi,j* is larger than a specific friend threshold *FTmin*, user *i* and *j* (*Ui* and *Uj*) are friends. LBSPs are fixed and not connected in a social network, which provides Location-Based Services (LBSs) for users. We assume that the only information which is necessary for the LBS is the location of the original requester (*U*0), but he should still give an identity to the LBSP so that the LBSP can reply to that identity.

Attackers can access the database of LBSPs, so that they can learn everything recorded in LBSPs, including users’ identities and locations. If *U*0 sends queries with his own identity and accurate location, his will be located by attackers easily, so the key is to hide his identity and location. We propose Appointment Card Protocol (ACP) to protect the identity and location-privacy of the original requester by providing a fake identity which disenable attackers to locate the original requester with records in the LBSP database.

## Appointment Card Protocol Overview

We propose an Appointment Card Protocol (ACP) to protect original requesters when they are served by LBSPs. A user (*Agc*1) generates his own Appointment Cards (ACs) containing his identity, a unique Appointment Number (AN) and a location which are be exchanged when he encounters a friend. When *U*0 sends a query, he chooses an AC and sends the query with the identity (*Agc*1), AN and location in the AC. A LBSP replies to *Agc*0 when it receives the query. Since *Agc*1 is actually the one who generates the AC, he re-transmits it to the user (*Agc*2) whom he exchanged the AC to, and so do the following users until the reply reach *U*0.



AC

AC

AC

Reply

Query

Reply

Reply

Reply

Figure 4.1 Process example

## Appointment Card

Any user in the network can generate his own Appointment Cards (ACs) and he is called the creator of those ACs. The creator should write down his identity (*Cid*), a unique Creator Appointment Number (*Capt*) and his current location (*Clct*) on his AC. Entries of AC are shown in Table 1.

The (ready) AC is an authorization that anyone holds it can send a query to LBSPs using its creator’s identity (*Cid*) and location (*Clct*). When LBSPs receives the query, it replies to the creator (*Cid*) and attaches the *Capt* in reply, which enable the creator to determine who is the next one to transmit the reply.

Table 1 Appointment Card

|  |  |  |
| --- | --- | --- |
| ***entry*** | ***explanation*** | |
| *Cid* | The identity of the creator who generates the AC. |
| *Capt* | A unique number that distinguishes an AC from other ones generated by the same creator. |
| *Clct* | The location where the creator generates the AC. |
| *Aid* | The identity of an agency who gives the AC to the recent holder. (The previous hop of the AC) |
| *Aapt* | A unique number that distinguishes an AC from other ones transmitted by the same agency. |
| *time-out* | The time when the AC expires. |
| *EQ* | A queue (Exchange Queue) which records users who exchange the AC in order. Its length is *EQL*. |
| *dc* | A counter which is used to distribute ACs of the same creator to various users. |
| *Psd* | A pseudonym or a fake identity for the original requester receiving a reply. |

To guarantee the security, the distance between the original requester and the AC creator should be far enough, an AC must be exchanged for a specific *k* (see Table 2) times before it can be used for a requester to send a query. The value of *k* should neither be too small nor too large. If *k* is small, e.g. *k* = 1, attackers can infer the original requester from the records of his friends in the database easily. If *k* is large, replies from LBSPs can hardly reach the original requester, because replies must go through all users in their EQs.

Both *Capt* and *Aapt* are appointment numbers, which are unique random integers. When we mention “unique”, it means that there are not an identical (same value and type) existing appointment number generated by the same user in the network. Existing means that the CA which is relevant to the appointment number does not expires. For example, if a user (*Ui*) generates a *Aapt* “123456” and it does not expire, he cannot generate another *Aapt* “123456”, but a *Capt* “123456” is acceptable. He may also generate another *Aapt* “123456” after the previous one expires. It would not prevent another user (*Uj*, *j*≠*i*) generating an *Aapt* “123456” no matter whether *Ui* generates one or not.

Table 2 AC system parameters

|  |  |  |
| --- | --- | --- |
| **Parameters** | **explanation** | |
| *k* | The minimum length of the exchange record for an AC eligible to be used in a query. |
| *Seg* | dispensed ACs are exchanged when two friends encounter each other. |
| *DN* | The minimum number of ACs that a user hands out in anytime. |
| *SU* | The time unit |

The parameter *k* and the *EQL* also divides ACs into 2 states: dispensed and ready (see Table 3). Ready ACs can be used when their holder sends a query. Users still exchange ready ACs among their friends, but they do not add themselves in ACs’ *EQ* any more. Dispensed ones are not qualified to be used by current holder. Therefore, in order to make an AC useful, it should be exchanged for *k* times as fast as possible.

Table 3 AC states

|  |  |
| --- | --- |
| **State** | **Explanation** |
| *Dispensed* | *EQL* < *k* |
| *Ready* | *EQL* = *k* |

In order to manage ACs, each user maintains two lists: the *dispense-list* and the *ready-list*. It is obviously that *dispense-list* contains dispensed ACs and *ready-list* contains ready ones.

Table 4 AC storages

|  |  |
| --- | --- |
| *List Name* | *Explanation* |
| *ready-list* | It contains ready ACs. |
| *dispense-list* | It contains dispensed ACs. |

To avoid ACs generated by a creator simultaneously being dispensed to the same *k*th user, we employ the parameter *Seg* and *dc*. The *dc* is a counter which is initialized to a random integer in the interval [1, *Seg*] when a user received an AC. The AC should not be sent to the next friend until *dc* reaches 0, which enable users to split these ACs into about *Seg* batches.

## AC life cycle

The life cycle of an AC starts when it is generated by its creator. The creator initializes the *Cid*, *Capt* and *Capt* when he generates it. These three entries remain constant in the whole life cycle. In the first *k* exchanges, the user who is sending it modify its entries *Aid* to his own identity. The Aapt is also replaced by a new appointment number generated by him. he adds his identity in its *EQ* before sending it out. When the AC’s ER length reaches *k*, it is eligible to be used in a query and called a ready AC. A ready AC can be exchanged to other friends either, but senders never modify previous entries, i.e., *Aid*, *Aapt* and *EQ*. Each of ACs can be used for no more than one time. In another word, an AC expires, when reaching its time-out or it is used in a query.



Figure 4.2 AC's life cycle

As shown in the Figure 4.2, an AC starts at a generated state. When it leaves the creator, it switches to a dispensed state. If the length of its ER reaches k, it changes to the ready state. It can be time-out in either the dispensed state or the ready state. If the holder uses it in any queries, it expires, too.

## Exchange sending process

Each AC is unique and hold by only one user at one time. When a user exchanges an AC to a friend, he loses the control of that AC and cannot give it to another one. We discuss the process that users exchange ACs with their friends. The process that a user (*Ui*) exchanges ACs to his friend is shown in Figure 4.3 AC exchange process (send). When *Ui* encounters a friend (*Uj*), he would send 3 kinds of ACs to the friend, which include dispensed ACs, ready ACs and new ACs. Even though new ACs must be dispensed ACs, we discuss them separately. During the exchange process, *Ui* generates the specified number of ACs and selects a part of dispensed ACs and ready ACs, then he sends them to his friend. He also remembers whom he sends them to, which enables him to transmit replies to the friend.



Figure 4.3 AC exchange process (send)

### Generate new ACs

Each user (*Ui*) has an obligation to generate proper number of ACs for their friends, so that he guarantees that the number of ACs created by him in the network is larger than *DN* after he exchanges ACs with a friend (*Uj*). We assume that he must give *Uj* *Sg* new ACs to achieve that goal. *Ui* initializes entries for his new ACs, i.e., *Cid* (his identity), *Capt* (a new appointment number), *Clct* (his current location), *Aid* (A default value *Aiddefault*), *Aapt* (The same value as the *Cid*), *time-out*. The *EQ* in a new AC is initialized to empty, and *dc* is 0. We assume that the number of *Ui*’s ACs that are remained in the network is equal to *RNi*. The method of determining *RNi* is discussed in 4.5.4.1. Then .

### Pick dispensed ACs

*Ui* decreases *dc*s of all his dispensed ACs, so that he gets *Sd* ACs whose *dc* values are equal to zero. They should be sent to his friend. Since the parameter dc is initialed as an integer between 1 and *Seg* randomly, there are about  dispensed ACs eligible to be sent to the friend. That enables us to separate ACs which come from a common creator.

### Pick ready ACs

There are two ways for a user to provide ready ACs: 1. give his own ready ACs to friends directly; 2. dispensed ACs whose *EQL* values are equal to *k*-1. Since ready ACs are precious for every user, the strategy that a user shares ready ACs with his friends should save his own ready ACs. the number of ready ACs which would be sent to the friend is



, where *Sd*,*k*-1 denotes the number of dispensed ACs whose *EQL* values are equal to *k*-1. We assume that *Uj* has no ACs at all when they encounter. After their exchange, if *Uj* has more ready ACs than *Ui*, *Ui* must have all his old ready ACs with him; if *Ui* loses some of his ready ACs, *Uj* does not have more ready ACs than *Ui*. In another word, *Ui* should do his best to help his friend *Uj*, but *Ui* is not in a dilemma worse than his friends. That strategy makes users who sends more queries than friends supplement their AC consumption quickly, while protecting their friends’ profits. Then *Ui* picks *Sr* ready ACs randomly and sets their *dc*s to 0.

### Send ACs

An AC should be sent to *Uj*, if and only if its *dc* is equal to 0. Before it is sent to *Uj*, *Ui* adds its identity to the its *EQ*. Besides, *Ui* replaces its *Aid* and *Aapt* with his own identity and a new appointment number. The old *Aid* is recorded in his *agency-list*, so do both old and new *Aapt*s. The structure of the *agency-list* is shown in Table 5. When the AC comes from his *dispense-list*, he checks its *EQL*. If it is equal to k and the last entry of *EQ* is himself, he generates a unique pseudonym for the *AC* (*Psd*) and records the pseudonym in the *agency-list* (*Nxt*). Otherwise, he records the identity of his friend in *Nxt*. Since items which is relevant to an expired ACs would be deleted, each item in the *agency-list* is related to a distinct existing AC. Information in it can be used for statistics showing in the following sections.

Table 5 Agency\_list entries

|  |  |
| --- | --- |
| *List Name* | *Explanation* |
| *Aidold* | The previous agency identity. |
| *Aaptold* | The previous agency appointment number |
| *Aaptnew* | The current agency appointment number |
| *Nxt* | The identity of the next friend. |
| *time-out* | The time when the AC expires |

#### Get the number of ACs

*Ui* can get the number of existing ACs which is generated by him based on the information in his *agency-list*. He counts the items in the list whose *Aidold* value is equal to *Aiddefault*, that is the *RNi*.

#### Check duplicated appointment numbers

All existing appointment numbers generated by a user are recorded in his *agency-list*. If *Aidold* value is equal to *Aiddefault*, the *Aaptold* is a *Capt*; Otherwise, the *Aaptnew* is a *Aapt*. When the user generates a new appointment number, he must check the *agency-list* to guarantee it is unique.

## Exchange receiving process

The process that *Ui* receives ACs from his friend is shown in Figure 4.4 AC exchange process (receive). ACs are divided into two groups based on their *EQL* values, which are dispensed ACs and ready ACs. The *dc* value of a dispensed AC is assigned to a random integer in the interval [1, *Seg*], then the AC is moved into his *dispense-list*. Ready ACs are moved into his *ready-list*, and their dc values are set to -1.



Figure 4.4 AC exchange process (receive)

## Sending queries

When a user wants to send a query, he picks an AC from his *ready-list*. He gives preference to the AC that has the earliest time-out. As a result, he sorts ACs in the *ready-list* in ascending order based on their remaining time and selects the first AC (e.g. *ac*) to send his query. That means the query is sent under the name of *ac*’s creator (i.e., *Cid*), and the location of the query is also *ac*’ location (i.e., *Clct*). Besides, *ac*’ *Capt* is also included in the query. The query can be transmitted to LBSPs by an arbitrary DTN routing protocol (e.g., Spray and Wait). When a LBSP receives the query, it provides services based on ac’s location and reply to ac’s creator (ac’s *Cid*). Users can also use a general DTN routing protocol to deliver the reply.

## Transmit replies

When a user receives a reply, he checks his *agency\_list*. If the sender identity is the LBSP, he searches for a record in his agency\_list, whose *Aidold* is *Aiddefault* and *Aaptold* is equal to the *Aapt* in the reply. If the sender identity is a user, he looks up for an item whose *Aidold* is equal to the sender identity and *Aaptold* is equal to the *Aapt* in the reply.

Based on the record he finds, he modifies information in the reply. The sender identity is set to his own identity; the destination is set to the user whose identity is *Nxt*; *Aapt* is assigned to *Aaptnew*. The item related to that AC can be deleted after the replacement, then he replies to the next friend (*Aidnew*).

After the original requester sends the query out, he claims himself as the one whose identity is *Psd*. That identity is equal to the *Nxt* in the *k*th user’s *agency\_list*. As a result, nobody knows the identity of the original requester. The process is shown in Figure 4.5.



Figure 4.5 Transmit reply

## Example

We assume there are four users in our social network: Alice, Bob, Charlie and David and there are 3 pairs of friends (i.e. Alice and Bob, Bob and Charlie, Charlie and David) in our example. We assign the system parameter *k* to 3, so that each Appointment Card must be exchanged for three times before it can be used. The system parameter *DN* is equal to 100, so that each user guarantees there are more than 100 ACs generated by him in the network.

When Alice encounters Bob, both of them generate 100 new ACs and give all the new ACs to the other one. Table 6 shows the number of ACs hold by Alice and Bob after the exchange where ACAlice,1 denotes the ACs generated by Alice and with *EQL* equal to 1.

Table 6 Alice and Bob exchange ACs

|  |  |  |
| --- | --- | --- |
| Users | Alice | Bob |
| Dispensed ACs | 100 ACBob, 1 | 100 ACAlice, 1 |

Then Bob encounters a friend Charlie who is a new user. Since Bob has already generated 100 ACs and none of them are used, he generates no AC for Charlie. But, Bob should give half of his dispensed ACs to Charlie while Charlie generates 100 new ACs and give them all to Bob.

Table 7 Bob and Charlie exchange ACs

|  |  |  |
| --- | --- | --- |
| Users | Bob | Charlie |
| Dispensed ACs | 100 ACCharlie, 1  50 ACAlice, 1 | 50 ACAlice, 2 |

When Charlie encounters David, Charlie gets 100 new ACs from David. David get 25 ACs which are generated by Alice from Charlie. The 25 ready-ACs hold by David can also be exchanged to other users, as shown in Table 8.

Table 8 Charlie and David exchange ACs

|  |  |  |
| --- | --- | --- |
| Users | Charlie | David |
| Dispensed ACs | 100 ACDavid, 1  25 ACAlice, 2 |  |
| Ready ACs |  | 25 ACAlice, 3 |

The route of one of the ready ACs (for example, the ACAlice, 3) after it is received by David is shown as follow. For convenience, we donate it by AC*x*. David gives a pseudonym (Say *p*) to AC*x* before he exchanges it to another user. We assume that David gives it to Elizabeth and Elizabeth gives it to Franklin, and Franklin would use the AC*x* to send the query. Franklin sends the query pretending he is Alice and receives the reply pretending he is *p.* More specifically, in the replying phase, LBSP replies to Alice and then the reply would be transmitted by Bob, Charlie and received by David. After that, David broadcast the reply to *p*, other users would ignore that reply while Franklin realized it is the reply of his query since he uses the AC with pseudonym *p.*

Table 9 The exchange history of ACx

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| User name | *Cid* | *Capt* | *Aid* | *Aapt* | *Pseudonym* |
| Alice | Alice | 233 | Alice | 557 | -- |
| Bob | Alice | 233 | Bob | 743 | -- |
| Charlie | Alice | 233 | Charlie | 691 | -- |
| David | Alice | 233 | David | 283 | *p* |
| Elizabeth | Alice | 233 | David | 283 | *p* |
| Franklin | Alice | 233 | -- | -- | -- |

When Franklin wants to send a query to a LBSP, he uses Alice’s identity and attaches the *Capt* (i.e. 233) in the query. Then he uses the pseudonym *p* as his identity when he receives a reply.

The LBSP replies to Alice, and the *Capt* (i.e. 233) is included in the message. When Alice receives the message, she checks her *agency\_list* and finds the entry whose *Aidold* is equal to *Aiddefault* and *Aaptold* is equal to 233. Based on the entry, she attaches the *Aaptnew* (i.e. 557) in the reply and sends it to *Nxt* (i.e. Bob). We collect entries which are related to the AC*x* in Table 10. Then Alice remove the item from her *agency\_list*. Also, the number of ACs generated by her decreases to 99 (=100-1), so that she should generate a new AC when she encounters a friend.

When Bob receives the reply from Alice, he also checks his *agency\_list* and learns that the reply is related to the AC*x* based on the sender identity (i.e. Alice) and the *Aapt* (i.e. 557). He modifies the *Aapt* to 743 in the reply message and relays it to Charlie. Charlie also follows the same process. Different from the previous 3 friends, David relays the message to a pseudonym *p*, because the *Nxt* value is equal to *p* in his *agency\_list*. Since Franklin is using the pseudonym *p* as his identity, he can get the reply message at last.

Table 10 Users' agency\_list

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| User Identity | *Aidold* | *Aaptold* | *Aaptnew* | *Nxt* |
| Alice | *Aiddefault* (LBSP) | 233 | 557 | Bob |
| Bob | Alice | 557 | 743 | Charlie |
| Charlie | Bob | 743 | 691 | David |
| David | Charlie | 691 | 283 | *p* |

Table 11 Important information in the reply messages

|  |  |  |  |
| --- | --- | --- | --- |
| Transmit | Sender | Receiver | *Aapt* |
| LBSP → Alice | LBSP | Alice | 233 |
| Alice → Bob | Alice | Bob | 557 |
| Bob → Charlie | Bob | Charlie | 743 |
| Charlie → David | Charlie | David | 691 |
| David → *p* | David | *p* | 283 |

In a nutshell, the *Aapt* in the reply is the key for each user (friend) to relay the reply message to the next one. After a user relays the reply message to the next one, he removes the entry from his *agency\_list*.

# Vertex reduce algorithm

Movement of network nodes is a significant factor for the performance of DTNs. To evaluate the performance of different DTN protocols, researchers presented different kinds of movement models, like in [2]. Since shortest path algorithm is an essential and time-cost part of creating many movement models, it is an important topic to optimize the time-cost for simulators calculating the all-pairs shortest path problem in the entire map.

The best known non-negative edge weight undirected all pairs shortest path algorithm [3] has a complexity , which is still expensive when *n* is huge. For most real-world city map, there are thousands of points in a single square kilometer. Since a tens square kilometers map and thousands of repeats are reasonable for a DTN protocol evaluation, a simulator must cut down the time-cost of the calculation of the all-pairs shortest path.

Real-world map developers often use short straight lines to present curves, like in [4], so that a several-meters curves may contain tens of points. It is obviously that we do not need to calculate shortest path for each of these points. In this paper, we present an Vertex Reduce Algorithm (VRA) to reduce the number of points which are used as vertex in the shortest path algorithm. The basic idea is that VRA removes all nodes that have less than 3 degrees from the map, while keeps the result of all-pairs shortest path algorithm correct.

The rest of this paper is organized as follows: (-) presents some basic lemmas and definitions. The process of VRA is described in (-).

## Ignorable vertex and reserved vertex

Vertex are considered as ignorable and reserved ones. In each iteration of VRA, we remove the ignorable vertex from the graph, while the reserved vertices compose a new graph.

Definition: the vertex whose degree is larger than 2 is called the reserved vertex (R).

Definition: the vertex whose degree is smaller than or equal to 2 is called the ignorable vertex (G).

Definition: if two reserved vertices are connected by a sequence of ignorable vertices, the sequence from one reserved vertex to the other one going through those ignorable vertices is called line segment (LS). It is denoted by .

Definition: both the reserved vertex and ignorable vertex are called vertices (V)

Definition: the shortest path between two vertices inside a line segment is called the inner shortest path (SPI), and its length is the inner distance (DI). Let denote the inner shortest path between two vertices (i.e. ) inside a line segment, and its length is .

Lemma1: If two vertices (i.e. , ; ) are on the same LS, the short path between them is

Since an all-pairs SPI has a complexity equal to or smaller than and the *n* is much fewer than the number of points on the entire map, the time-cost for SPI is ignorable comparing to the entire map all-pairs shortest path calculation.

Lemma2: We assume that there are two different line-segments (i.e., and ). We pick a vertex from and a vertex from , then the shortest path between and is

where . and are shortest path inside line-segments so that we can use lemma1 to calculate their value. The is the only part we need to calculate using all-pair shortest path algorithms.

Therefore, we can get the all-pair shortest path of the entire map, if we get an all-pair shortest path of all reserved vertices.

## Vertex reduce iteration

The vertex reduce algorithm can be implement with several iterations and ends when no ignorable vertices is found. In each iteration, we remove all ignorable vertices from the graph and connect neighbour reserved vertices directly with a line. The line’s weight is the sum of all deleted lines between that two reserved vertices. For example, there is a line-segment (i.e., ). we remove and connect and . The weight of the line which connects and is

where is the weight of the line between and . The algorithm is shown in Algorithm 51.

The input of the algorithm is the original entire map where there are a lot of ignorable vertices, which is called it *Graph*0. In each iteration, VRA makes a copy of *Graphi* in the *i*th iteration. If , the *Graphi* is the output of the previous iteration. In other words, *Graphi* is smaller and smaller as *i* increases. If *Graphi* is equal to *Graphi*+1, it means that the *i*th iteration makes no modification on the graph, which is the ending criteria.

At the beginning of the *i*th iteration, VRA copies the map *Graphi* to *Graphi*+1 and modifies the *Graphi*+1. It removes all ignorable vertices while leaving the connections between reserved vertices. As shown in Figure 5.1, two reserved vertices are connected by two line-segments. We remove all the ignorable vertices and leave their lines, then we notice that the two reserved vertices are connected by two lines Figure 5.1 b. Both of the two lines will be removed, and a new line (i.e., the bold one) is inserted between that two nodes. The weight of the new line is equal to the smallest weight of deleted lines. At the end of the iteration, we get a new graph (i.e. *Graphi*+1) consisted of reserved vertices.

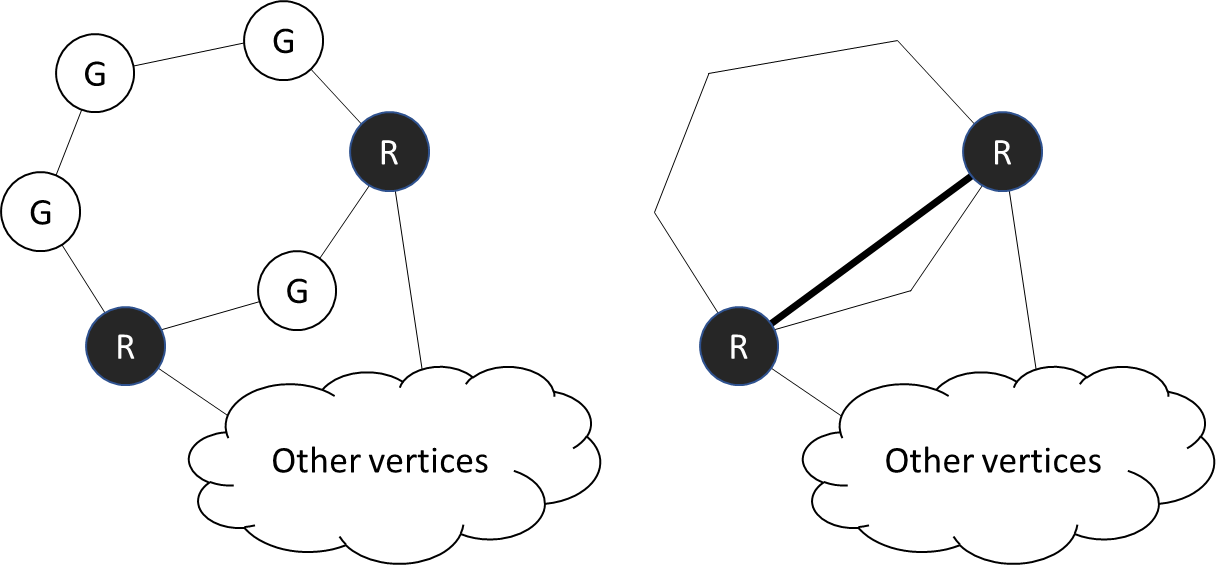


Figure 5.1 remove ignorable vertices

Algorithm 51

function remove(*Gu*)

Connect two vertices (*Vu*-1, *Vu*+1) next to *Gu*.

Set the weight of the line between *Vu*-1 and *Vu*+1 as *W*(*Vu*-1, *Gu*)+ *W*(*Gu* ,*Vu*+1)

Remove *Gu*

end function

function reduce(*Graphi*)

Copy *Graphi* to *Graphi*+1

for every ignorable vertex *Gu* in *Graphi*+1 do call remove(*Gu*)

for every pair of reserved vertices do

create a new connection between the two reserved vertices

Set its weight equal to the smallest weight of their connections

remove all old connections

end

end function

function VRA(*Graph*0)

i=0

while i=0 or there are any differences between *Graph*i-1 and *Graph*i

do

call reduce(*Graph*i); i=i+1;

end

end function

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