STOR 415, Fall 2019
Solutions to Homework Assignment No. 10

1. (a) The node set is $N = \{A, B, C, X, Y, Z\}$, the arc set is

$$A = \{(A,X), (A,Y), (A,Z), (B,X), (B,Y), (B,Z), (C,X), (C,Y), (C,Z)\},$$

and the net supplies are given by $s(i) = 1$ for $i = A, B, C$ and $s(i) = -1$ for $i = X, Y, Z$. The cost c_{ij} on arc (i, j) is given by the negative of the value of putting course i in classroom j. Let x_{ij} be the amount of flow on arc (i, j). We can then use the general MCNFP formulation:

$$\begin{aligned} \min \quad & \sum_{(i,j)\in A} c_{ij}x_{ij}, \\ \text{s.t.} \quad & \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = s_i, \quad i \in N, \\ & x_{ij} \geq 0, \quad (i, j) \in A. \end{aligned}$$

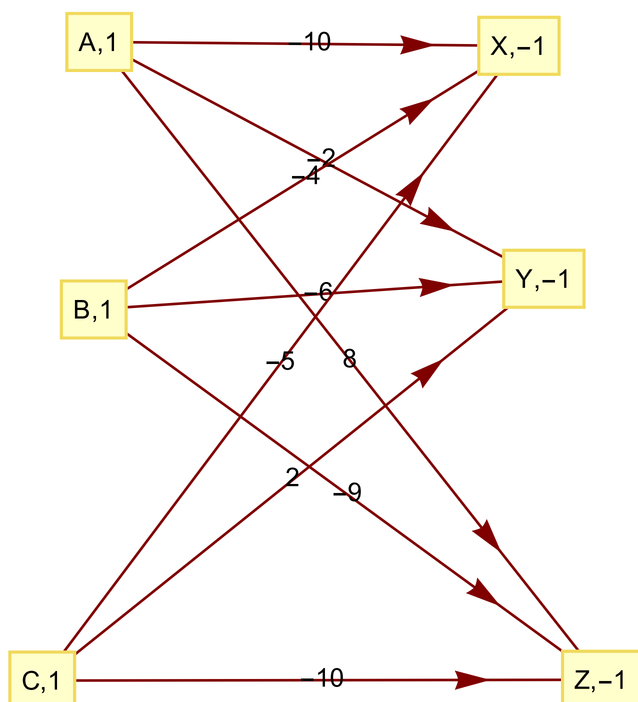
Equivalently, this can be formulated as a balanced transportation problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^{3}\sum_{j=1}^{3} c_{ij}x_{ij}, \\ \text{s.t.} \quad & \sum_{j=1}^{3} x_{ij} = 1, \quad i = 1,2,3 \\ & \sum_{i=1}^{4} x_{ij} = 1, \quad j = 1,2,3, \\ & x_{ij} \geq 0, \quad i = 1,2,3, \ j = 1,2,3, \end{aligned}$$

which is a special MCNFP.

In the following graph, numbers next to the node labels are the net supplies and numbers on arcs are unit costs. The network simplex method needs to be used to guarantee finding an integer valued optimal solution.

(b) The optimal solution is to assign A to X, B to Y, and C to Z, respectively. The optimal value is 26 (or -26 for the MCNFP).


A,1
X,−1
B,1
Y,−1
C,1
Z,−1
−10
−2
−4
−6
−5
8
2
−9
−10

2. (a) In the MCNFP, the node set is $N = \{1, ..., 6\}$, the arc set is $A = \{$(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (3, 6), (4, 1), (4, 5), (4, 6), (5, 1), (5, 2), (5, 6), (6, 1), (6, 2), (6, 3)$\}$ with corresponding costs as in the given table. The net supplier of node 1 is 5 while that of other nodes is -1. The lower bound for flow on each arc is 0 and the upper bound is ∞.

(c) The graph of the shortest paths tree with minimal total distance 23: