STOR 415, Fall 2019
Solutions to HW 7

Note: In general, finding answers for questions on sensitivity analysis do not require much computational work.

1. (a) Optimal sol: $x = (0, 25, 25, 0, 0)$, optimal value: 300.

   (b) We know that the only difference between the new simplex tableau and the given tableau is that the reduced cost of x_1 changes from 3 to $3 - \Delta$. If $\Delta \leq 3$ then the new tableau shows an optimal solution $(0, 25, 25, 0, 0)$, with the optimal value being 300. When $\Delta = 4$, an extra simplex iteration is needed to find the optimal solution to be (50,0,0,0,0); optimal value: 350.

   (c) In the new simplex tableau the reduced costs of x_1, x_2, x_3, s_1, s_2 are $3 + 0.5\Delta$, 0, 0, $4 - 0.5\Delta$, $1 + 0.5\Delta$ respectively, and the rhs entry in row 0 is $300 + 25\Delta$. The range of Δ is $-2 \leq \Delta \leq 8$. When Δ belongs to this range, the tableau shows an optimal solution $(0, 25, 25, 0, 0)$, and the optimal value is $300 + 25\Delta$. $\Delta = 4$ belongs to this range. When $\Delta = 4$, an optimal solution is $(0, 25, 25, 0, 0)$, and the optimal value is 400.

   (d) The reduced costs of x_1, x_2, x_3, s_1, s_2 in the new simplex tableau are $3 + 0.5\Delta$, 0, 0, $4 + 1.5\Delta - \Theta$, $1 - 0.5\Delta$, and the rhs entry in row 0 is $300 + 25\Delta$. The range of (Δ, Θ) for this tableau to show an optimal solution is $-6 \leq \Delta \leq 2$ and $\Theta \leq 4 + 1.5\Delta$. When (Δ, Θ) belongs to this range, the tableau shows an optimal solution $(0, 25, 25, 0, 0)$, and the optimal value is $300 + 25\Delta$. $(\Delta, \Theta) = (2, 2)$ belongs to that range.

   (e) The range of Δ is $-\frac{50}{3} \leq \Delta \leq 50$, obtained from the requirement that the rhs entries $25 + 1.5\Delta$ and $25 - 0.5\Delta$ be nonnegative. When Δ belongs to this range, the tableau shows an optimal solution $(0, 25 - 0.5\Delta, 25 + 1.5\Delta, 0, 0)$, and the optimal value is $300 + 4\Delta$.

   (f) The range of Δ is $-50 \leq \Delta \leq 50$, obtained from the requirement that the rhs entries $25 - 0.5\Delta$ and $25 + 0.5\Delta$ be nonnegative. When Δ belongs to this range, the optimal value is $300 + \Delta$.

2. (a) The rhs entry in row 0 is 17, because the basic feasible solution shown in this tableau is (5/3, 0, 3, 0, 0, 1, 0), whose objective value is 17. This can also be seen from the dual solution: since x_5, x_6, x_7 are the isolated variables in the original canonical form LP, their reduced costs in the optimal tableau give a dual optimal solution $y^* = (1/5, 0, 3/5)$. The dual optimal value is therefore $25(1/5) + 15(0) + 20(3/5) = 17$, which is also the primal optimal value.

   (b) The dual is

$$\begin{array}{llllll} \min & 25y_1 & +15y_2 & +20y_3 & & \\ \text{s.t} & 6y_1 & +3y_2 & +3y_3 & \geq & 3, \\ & 3y_1 & +2y_2 & +4y_3 & \geq & 1, \\ & 5y_1 & +3y_2 & +5y_3 & \geq & 4, \\ & 4y_1 & +y_2 & +2y_3 & \geq & 1, \\ & y \geq 0. & & & & \end{array}$$

   The dual optimal solution is given by $y^* = (1, 1/5, 0, 3/5)$.

   (c) For the basis to remain optimal, the reduced costs for x_2, x_5 and x_7 given by $2 + \Delta$, $1/5 - 1/5\Delta$ and $3/5 + 2/5\Delta$ need to be nonnegative. The corresponding range of Δ is [-3/2,1]. For Δ in this range the optimal value is 17+3Δ.

   (d) Entries in the RHS column (from row 0 to row 3) in the new tableau are $17 + 1/5\Delta$, $5/3 + 1/3\Delta$, $1 + 3/5\Delta$ and $3 - 1/5\Delta$. For the basis to remain optimal, the latter three entries need to be nonnegative. The corresponding range of Δ is [-5/3,15]. For Δ in this range the optimal value is $17 + 1/5\Delta$.

3. See Erica.ipynb.