STOR 415, Fall 2019
Solutions to Homework Assignment No. 4

1. (a)Choose x_3 as the entering variable, and x_1 as the leaving variable. Pivot on the entry a_{33}.

   (b)Unbounded.

   (c)Stop with the current BFS being an optimal solution. (In fact, it is the unique optimal solution: the equation $z = d - c_3x_3 - c_4x_4$ holds for all feasible solutions, so $z \le d$ on the entire feasible set and $z = d$ only when $x_3 = x_4 = 0$.)

   (d)Stop with the current BFS being an optimal solution. (There might exist multiple optimal solutions, which can be found by increasing x_4.)

   (e)Choose x_4 as the entering variable, and conduct a ratio test to compare b_1/a_{14} and b_2/a_{24}. If the former is smaller then x2 leaves and pivot on the entry a_{14}. If the latter is smaller then x_5 leaves and pivot on a_{24}. If the two equal then either a_{14} or a_{24} can be chosen as the pivoting element.

2. (a) Last simpelx tableau:

z	x_1	x_2	s_1	s_2	rhs	Basic var
1	0	0	1.33	0.33	10.67	$z = 10.67$
0	0	1	0.67	-0.33	1.33	$x_2 = 1.33$
0	1	0	-0.33	0.67	3.33	$x_1 = 3.33$

max z; $x, s \ge 0$

The LP has a unique optimal solution $(x, s) = (3.33, 1.33, 0, 0)$ and optimal value $z = 10.67$. Alternatively: the original LP has a unique optimal solution $x = (10/3, 4/3)$ and optimal value $z = 32/3$.

(b) Last simplex tableau:

z	x_1	x_2	s_1	s_2	rhs	Basic var
1	0	0	0	3	6	$z = 6$
0	8.5	0	1	-3.5	28	$s_1 = 28$
0	-0.5	1	0	0.5	1	$x_2 = 1$

max z; $x, s \ge 0$

The LP has multiple optimal solutions with the optimal value $z = 6$. To provide three optimal solutions: $(x, s) = (0, 1, 28, 0)$, $(x, s) = (3.29, 2.65, 0, 0)$, and $(x, s) = (1.65, 1.82, 14, 0)$. Optional: the set of all optimal solutions is given by

$$\left\{ \lambda \begin{bmatrix} 0 \\ 1 \\ 28 \\ 0 \end{bmatrix} + (1-\lambda) \begin{bmatrix} 3.29 \\ 2.65 \\ 0 \\ 0 \end{bmatrix} \middle| \, 0 \le \lambda \le 1 \right\}.$$

It is OK to write down the optimal solutions of the original LP (without the slack variables).

(c) Last simplex tableau.

z'	x_1	x_2	s_1	s_2	rhs	Basic var
1	-4	0	0	3	9	$z = 9$
0	-1	0	1	2	10	$s_1 = 10$
0	-1	1	0	1	3	$x_2 = 3$

max z'; $x, s \geq 0$

The LP is unbounded. Starting from the point $(x, s) = (0, 3, 10, 0)$, one can travel along the direction $(1, 1, 1, 0)$, to keep increasing z' while staying feasible. In other words for each $t \geq 0$ the point $x = (t, 3 + t, 10 + t, 0)$ is feasible with $z' = 9 + 4t$. Alternatively, in terms of the original minimization LP the starting point is $x = (0, 3)$ and the direction of unboundedness is $(1, 1)$. Along this direction the objective function z keeps decreasing.

3. (a) Unbounded. Starting point: (0, 2, 10, 0); direction: (0, 0, 1, 1)

   (b) Unique optimal solution (8, 3, 0, 0, 0)

   (c) Multiple optimal solutions. Three optimal solutions can be chosen as (2, 4, 0, 0, 0), (3, 5, 0, 0, 1), (4, 6, 0, 0, 2). It is OK to write other optimal solutions. Optional: the set of all optimal solutions is

$$\{(2 + t, 4 + t, 0, 0, t) \mid t \geq 0\}.$$