STOR 415, Fall 2019
Homework Assignment No. 7

For each problem that requires Jupyter-GAMS coding:

- Create an ipynb file with exactly the same name as required in the problem. In the GAMS code, declare variables with names given in the problem. Then, in the last cell of your notebook, write the following codes to display values of all variables (replace "var1", "var2" and "var3" with names of variables in the problem):

  %gams display var1.l, var2.l, var3.l;

  %gams_lst -e

- Submit your ipynb files on Sakai as attachments to this assignment.

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1. Consider the following LP.

$$\begin{array}{llrrrrrcr}
\max & z = & 3x_1 & +7x_2 & +5x_3 & & & & \\
\text{s.t} & & x_1 & +x_2 & +x_3 & +s_1 & & = & 50, \\
 & & 2x_1 & +3x_2 & +x_3 & & +s_2 & = & 100, \\
 & & x_1, & x_2, & x_3, & s_1, & s_2 & \geq & 0.
\end{array}$$

   After using the simplex method to solve it, we obtain the following tableau, in which x_3 and x_2 are basic variables.

z	x_1	x_2	x_3	s_1	s_2	rhs	Basic var
1	3	0	0	4	1	300	$z = 300$
0	0.5	0	1	1.5	-0.5	25	$x_3 = 25$
0	0.5	1	0	-0.5	0.5	25	$x_2 = 25$

   $\max z; x, s \geq 0$

   (a) What is the optimal solution for this LP, and what is the optimal value?

   (b) Suppose that the objective function changes to $\max z = (3+\Delta)x_1 + 7x_2 + 5x_3$. How does this change affect the simplex tableau in which x_3 and x_2 are basic variables? For what range of Δ does that tableau show an optimal solution? Write down an optimal solution and the optimal value for all Δ belonging to that range. Does $\Delta = 4$ belong to that range? Find an optimal solution and the optimal value for $\Delta = 4$.

   (c) Suppose that the objective function changes to $\max z = 3x_1 + (7+\Delta)x_2 + 5x_3$. Write down the new simplex tableau in which x_3 and x_2 are basic variables. For what range of Δ does that tableau show an optimal solution? Write down an optimal solution and a formula for the optimal value for Δ in that range. Does $\Delta = 4$ belong to that range? Find an optimal solution and the optimal value for $\Delta = 4$.

(d) Suppose that the objective function changes to $\max z = 3x_1 + 7x_2 + (5+\Delta)x_3 + \Theta s_1$. Write down the new simplex tableau in which x_3 and x_2 are basic variables. For which Δ and Θ does that tableau show an optimal solution? Write down an optimal solution and a formula for the optimal value for Δ belonging to that range. Does $(\Delta, \Theta) = (2,2)$ belong to that range?

(e) Let the objective function be the original function. Suppose that the right hand side constant of the first constraint changes from 50 to 50+Δ. For what range of Δ does the basis $\{x_3, x_2\}$ continue to be optimal? What is the optimal value when Δ belongs to this range?

(f) Let the objective function be the original function. Suppose that the right hand side constant of the second constraint changes from 100 to 100+Δ. For what range of Δ does the basis $\{x_3, x_2\}$ continue to be optimal? What is the optimal value when Δ belongs to this range?

2. Consider the following linear program:

$$
\begin{array}{rrcrcrcrcl}
\max z = & 3x_1 & + & x_2 & + & 4x_3 & + & x_4 & & \\
& 6x_1 & + & 3x_2 & + & 5x_3 & + & 4x_4 & \leq & 25 \\
& 3x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & \leq & 15 \\
& 3x_1 & + & 4x_2 & + & 5x_3 & + & 2x_4 & \leq & 20 \\
& x_1 \geq 0 & & x_2 \geq 0 & & x_3 \geq 0 & & x_4 \geq 0 & &
\end{array}
$$

By adding three slack variables x_5, x_6 and x_7 to the first, second, and third constraints, we convert the above LP into canonical form. Then we apply the simplex method to it to the following simplex tableau, in which the value in the rhs entry of row 0 is missing.

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	rhs	Basic Var
1	0	2	0	1	$1/5$	0	$3/5$	?	$z=?$
0	1	$-1/3$	0	$2/3$	$1/3$	0	$-1/3$	$5/3$	$x_1 = 5/3$
0	0	0	0	-1	$-2/5$	1	$-1/5$	1	$x_6 = 1$
0	0	1	1	0	$-1/5$	0	$2/5$	3	$x_3 = 3$

(a) What is the value of the rhs entry in row 0 of the above tableau? Justify your answer.

(b) Consider the LP with all seven variables $x_1, \cdots, x_7$ as the primal LP. Write down the dual LP, and give the optimal solution for the dual LP.

(c) Suppose that the objective coefficient of x_3 (currently 4) changes to $4+\Delta$. For what range of Δ does the current optimal basis $\{x_1, x_6, x_3\}$ continue to be optimal? What is the optimal value for Δ in this range?

(d) Let the objective function be the original function. Suppose that the right hand side constants for the first and second constraints change to $25+\Delta$ and $15+\Delta$ simultaneously (for example if $\Delta = 1$ then the rhs constants in the first two constraints become 26 and 16 respectively). For what range of Δ does the current optimal basis $\{x_1, x_6, x_3\}$ continue to be optimal? What is the optimal value for Δ in this range?

3. Young MBA Erica Cudahy may invest up to \$1000. She can invest her money in stocks and loans. Each dollar invested in stocks yields 10 cents of profit, and each dollar invested in a loan yields 15 cents of profit. At least 30% of all money invested must be in stocks, and at least \$400 must be in loans. Formulate an LP to maximize total profit earned from Erica's investment.

   Create a Jupyter notebook named *Erica.ipynb*. In the GAMS code, declare two positive variables, *stock* and *loan*, to denote the amount of money to put in stock and loan respectively. Also declare a free variable called *profit* to denote the total profit. Model and solve as an LP.