Name: **PID:**

STOR 415 Midterm, Fall 2018

The exam starts at 10:10am and ends at 11:00am. It contains 5 pages. Please choose only one answer for each multiple choice question.

The exam is closed book/notes. Calculators are allowed, and computers are not. Write all answers on the exam sheets.

Multiple Choice Questions (6 points each).

1. Which of the following statements is true for all $n \times n$ matrices A, B and C?

   (a) $ABC = BAC$.

   (b) $(A+B)C = BC + AC$.

   (c) $A^T B^T C^T = (ABC)^T$.

   (d) $(A+B+C)^{-1} = A^{-1} + B^{-1} + C^{-1}$.

2. After some elementary row operations on a matrix $A \in \mathbb{R}^{3\times 4}$, we obtain a new matrix B given by $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Which of the following must be true about the matrix A?

   (a) The third row of A is a linear combination of its first two rows.

   (b) The third column of A is a linear combination of its first two columns.

   (c) The rows of A are linearly independent.

   (d) The columns of A are linearly independent.

3. A matrix $M \in \mathbb{R}^{3\times 4}$ is given by $M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & c \end{bmatrix}$, where c is a constant. Which of the following statements is always true, no matter what value is chosen for c?

   (a) Columns of M are linearly independent.

   (b) For any vector $b \in \mathbb{R}^3$, the equation $Mx = b$ has a unique solution.

   (c) For any vector $b \in \mathbb{R}^3$, the equation $Mx = b$ has no solution.

   (d) For any vector $b \in \mathbb{R}^3$, the equation $Mx = b$ has multiple solutions.

4. Which of the following statements is true?

   (a) An LP in canonical form must have at least a feasible solution.

   (b) An LP in standard form must have at least a feasible solution.

   (c) An LP in general form can always be converted into an equivalent LP that is in canonical form.

   (d) None of the above.

5. After applying the Gauss-Jordan method to a system of linear equations, we obtain the following augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

Which of the following is true?

(a) $(4,-2,0,2)$ is a solution to the system.
(b) The system has no solution.
(c) The system has a unique solution.
(d) The system has infinitely many solutions.

6. Consider the following linear program in which x_1 and x_2 are the two variables, and c_1 and c_2 are constants:

$$\begin{aligned} \min\ & z = c_1x_1 + c_2x_2 \\ \text{s.t. } & x_1 + x_2 \geq 1, \\ & x_1 \geq 0,\ \ x_2 \geq 0. \end{aligned}$$

Which of the following statements is true?

(a) If $c_1 = 1$ and $c_2 = 0$ then the LP has optimal value 1.
(b) If $c_1 = 1$ and $c_2 = 0$ then the LP has a unique optimal solution.
(c) If $c_1 = 1$ and $c_2 = 1$ then the LP has optimal value 1.
(d) If $c_1 = 1$ and $c_2 = 1$ then the LP has a unique optimal solution.

7. You obtain the following simplex tableau in using the simplex method to solve a maximization LP. From this tableau, what is the entering variable to choose?

	z	x_1	x_2	x_3	x_4	x_5	x_6	rhs	Basic var
(row 0)	1	0	0	-1	0	2	0	25	$z = 25$
(row 1)	0	1	0	4	0	1	0	2	$x_1 = 2$
(row 2)	0	0	0	-9	0	1	1	6	$x_6 = 6$
(row 3)	0	0	0	-3	1	-1	0	5	$x_4 = 5$
(row 4)	0	0	1	2	0	-1	0	8	$x_2 = 8$
max z;	$x \geq 0$								

(a) x_1 (b) x_2 (c) x_3 (d) x_5

8. In the simplex tableau in Question (7), what is the leaving variable to choose?

(a) x_1 (b) x_2 (c) x_3 (d) x_5

Free Response Questions.

9. (10 points for each part) You obtain the following simplex tableaus in using the simplex method to solve maximization linear programs. For each of the tableaus, determine if the LP represented by it is unbounded, has a unique optimal solution, or has multiple optimal solutions.

   For each LP identified as having a unique optimal solution, write down the optimal solution. For each LP identified as having multiple optimal solutions, provide three different optimal solutions. For each LP identified as unbounded, provide a starting point and a direction, so that one can "travel" from the starting point along the direction to increase the objective value without ever violating any constraint.

(a)

z	x_1	x_2	x_3	x_4	rhs	basic var
1	0	0	2	3	5	$z=5$
0	1	0	3	-1	0	$x_1=0$
0	0	1	1	0	2	$x_2=2$

max z; $x \geq 0$

(b)

z	x_1	x_2	x_3	x_4	x_5	rhs	basic var
1	0	0	2	1	-1	8	$z=8$
0	1	0	1	2	0	2	$x_1=2$
0	0	1	0	1	-2	4	$x_2=4$

max z; $x \geq 0$

10. A company produces three types of products, denoted by product i, $i = 1, 2, 3$. The per-unit profit, labor usage, and pollution produced per unit are given below.

Product	Per-unit profit ($)	Labor usage per unit (Hours)	Pollution per unit (Lb)
1	6	4	0.000003
2	4	3	0.000002
3	3	2	0.000001

At most 3 million labor hours can be used to produce the three products, and government regulations require that the company produce at most 2 lb of pollution.

(a) (10 points) Formulate the problem of determining how many products of each type to make in order to maximize the total profit. To receive full credits, clearly state the variables, the objective function and all constraints.

(b) (8 points) Convert the formulation you obtain in part (a) into standard form.

(c) (8 points) Write down the initial simplex tableau in applying the simplex method to the problem you obtain in part (b).

(d) (6 points) What is the basic feasible solution associated with the simplex tableau in part (c)?