STOR 415, Fall 2019
Homework Assignment No. 8

For each problem that requires Jupyter-GAMS coding:

- Create an ipynb file with exactly the same name as required in the problem. In the GAMS code, declare variables with names given in the problem. Then, in the last cell of your notebook, write the following codes to display values of all variables (replace "var1", "var2" and "var3" with names of variables in the problem):

  %gams display var1.l, var2.l, var3.l;
  %gams_lst -e

- Download your ipynb file(s) to your local computer, and then submit them on Sakai as attachments to this assignment.

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1. A company supplies goods to three customers, who require 40, 50 and 40 units respectively. The company has three warehouses, each of which has 30 units available. The costs of shipping 1 unit from each warehouse to each customer are shown in the table below.

		To	
From	Customer 1	Customer 2	Customer 3
Warehouse 1	\$15	\$35	\$25
Warehouse 2	\$10	\$50	\$40
Warehouse 3	\$20	\$40	\$30

   There is a penalty for unmet demand: With customer 1, a penalty cost of \$70 per unit is incurred; with customer 2, \$75 per unit; and with customer 3, \$65 per unit. The company needs to minimize the total cost.

   (a) **Non-coding.** Formulate the problem as a balanced transportation problem by adding a dummy warehouse. Write down the mathematical formulation, including the definition of variables, the objective function and all constraints.

   (b) **Coding.** Create a Jupyter notebook named *company.ipynb* to solve the problem. Display values of all variables. (The optimal value is \$4,950.)

2. The height distributions of 20-year old and 30-year old males in a county are given in the following table.

Intervals	Height (inches)	percent in 20-years old (%)	percent in 30-years old (%)
1	≤ 64	4	4
2	65-66	8	9
3	67-68	22	19
4	69-70	25	26
5	71-72	21	20
6	73-74	14	16
7	≥ 75	6	6

Let $i, j \in \{1, \cdots, 7\}$ be indices of the intervals, p_i be the percentage of 20-years old with height in the ith interval, and q_j be the percentage of 30-years old with height in the jth interval. For example, $p_1 = 4$, $p_2 = 8$, $q_1 = 4$ and $q_2 = 9$. Let $c_{ij} = |i - j|$ be the distance between the ith and jth intervals. For example, $c_{13} = |1 - 3| = 2$.

The *Kantorovich's optimal transport problem* is to find values of π_{ij}, $i, j = 1, \cdots, 7$, that solve the following problem:

$$\begin{aligned} \min \quad & \sum_{i,j=1,\cdots,7} c_{ij}\pi_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{7} \pi_{ij} = p_i, \ i = 1, \cdots, 7, \\ & \sum_{i=1}^{7} \pi_{ij} = q_j, \ j = 1, \cdots, 7, \\ & \pi_{ij} \geq 0, \ i, j = 1, \cdots, 7. \end{aligned}$$

Create a Jupyter notebook named *height.ipynb* to solve the problem. Display values of all variables. (The optimal value is 6.)