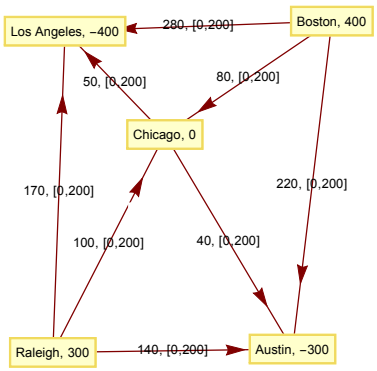
STOR 415, Fall 2019
Solutions to Homework Assignment No. 9

2. (a) Let x_{ij} denote the amount to ship from city i to j, $(i,j) \in A$, where $A = \{$(B, C), (B, A), (B, L), (R, C), (R, A), (R, L), (C, A), (C, L)$\} \subset N \times N$, and $N = \{B, R, C, A, L\}$. Let c_{ij} be the corresponding shipping cost. The net supply is $s = (400, 300, 0, -300, -400)$. The mathematical formulation is:

$$\begin{aligned} \min \quad & \sum_{(i,j)\in A} c_{ij} x_{ij}, \\ \text{s.t.} \quad & \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = s_i, \quad i \in N, \\ & 0 \le x_{ij} \le 200, \quad (i,j) \in A. \end{aligned}$$

The graph is:

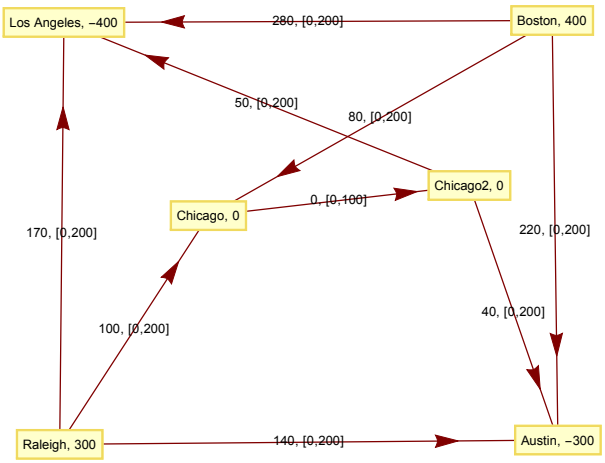


2. (b) The optimal solution is

	To		
From	Chicago	Austin	Los Angeles
Boston	200	200	0
Raleigh	0	100	200
Chicago	-	0	200

The minimal cost: $\$118,000$.

2. (c) Graph:



2. (d) The optimal solution:

	To			
From	Chicago	Chicago2	Austin	Los Angeles
Boston	100	-	200	100
Raleigh	0	-	100	200
Chicago	-	100	-	-
Chicago2	-	-	0	100

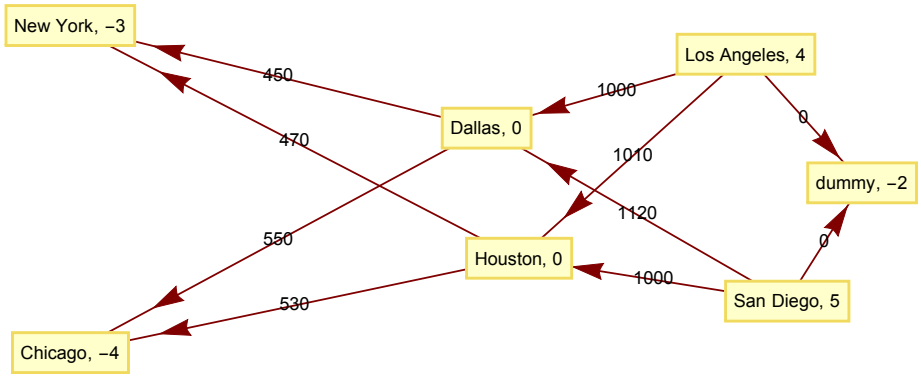
The minimal cost: $133000.

3. (a) Add a dummy city with demand 2 (or equivalently, net supply -2). Let x_{ij} denote the amount to ship from city i to j, $(i,j) \in A$, where $A = \{(L,D),(L,H),(S,D),(S,H),(D,N),(D,C),(H,N),(H,C),(L,\text{dummy}),(S,\text{dummy})\} \subset N \times N$, and $N = \{L,S,D,H,N,C\}$. Let c_{ij} be the corresponding unit shipping cost. If j has a refinery, then c_{ij} includes the refining cost. The net supply $s = (4,5,0,0,-4,-3,-2)$.

The mathematical formulation is:

$$\begin{aligned} &\min \sum_{(i,j)\in A} c_{ij}x_{ij}, \\ \text{s.t.} \sum_{j:(i,j)\in A} x_{ij} - &\sum_{j:(j,i)\in A} x_{ji} = s_i, \quad i \in N, \\ &x_{ij} \geq 0, \quad (i,j) \in A. \end{aligned}$$

The graph is:



3. (a) The optimal solution is:

	To				
From	Dallas	Houston	New York	Chicago	dummy
Los Angeles	3	0	-	-	1
San Diego	0	4	-	-	1
Dallas	-	-	3	0	-
Houston	-	-	0	4	-

The optimal value: $10470.