STOR 415, Fall 2019
Homework Assignment No. 2

1. Let $A = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 2 \end{bmatrix}$.

   (a) Compute the product AB using the definition of matrix multiplication.

   (b) Partition A into 2 blocks by a vertical line between its 2nd and 3rd columns, and partition B into 2 blocks by a horizontal line between its 2nd and 3rd rows. Compute AB using block operations.

2. Determine if each of the following statements is true or false. Justify your answer by a proof or a counter example. Proofs should be based on material covered in this course.

   (a) The equality $x^T y = y^T x$ holds for any two n-dimensional column vectors x and y.

   (b) If A, B and C are invertible matrices of the same dimensions, then the product of their transposes, $A^T B^T C^T$, is invertible.

   (c) If A, B and C are invertible matrices of the same dimensions, then their sum $A + B + C$ is invertible.

   (d) Suppose that A is an invertible matrix. Switch its top two rows to obtain a new matrix B. The matrix B must be invertible as well.

   (e) Let $v_1, v_2, \cdots, v_k$ be column vectors of the same dimension, and let $v = v_1 + v_2 + \cdots + v_k$. The vectors $v_1, v_2, \cdots, v_k, v$ are linearly dependent.

   (f) The system $Ax = b$ has a solution, if b is a linear combination of columns of A.

   (g) For a system of linear equations that contains 2 equations and 3 variables (i.e., $A \in \mathbb{R}^{2\times 3}$), there are always infinitely many solutions.

3. Use the Gauss-Jordan method to solve the following linear systems.

(a)

$$\begin{array}{rcrcrcr} & & x_2 & + & 2x_3 & = & 3 \\ x_1 & + & 2x_2 & + & x_3 & = & 4 \\ x_1 & + & x_2 & - & 2x_3 & = & 0 \end{array}$$

(b)

$$\begin{array}{rcrcrcr} x_1 & - & 4x_2 & + & 2x_3 & = & -4 \\ & & 2x_2 & - & x_3 & = & 1 \\ -x_1 & + & 2x_2 & - & x_3 & = & 3 \\ -2x_1 & + & 6x_2 & - & 3x_3 & = & 7 \end{array}$$

(c)

$$\begin{array}{rrrcrcr} x_1 & & & + & x_4 & = & 5 \\ & x_2 & & + & 2x_4 & = & 5 \\ & & x_3 & + & 0.5x_4 & = & 1 \\ & & 2x_3 & + & x_4 & = & 3 \end{array}$$

4. For each of the following linear systems in augmented form, determine if it has unique solution, multiple solutions, or no solution. Write down the solution sets when they exist.

(a)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 4 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 2 \end{array}\right]$$

(b)

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 9 & 3 \\ 0 & 1 & -1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(c)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

(d)

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 9 & 0 \\ 0 & -1 & 1 & -3 & 1 \\ 1 & -1 & 2 & 6 & -2 \end{array}\right]$$

5. Determine if each group of vectors is linearly independent or dependent. Justify your answer.

   (a) $v_1 = \begin{bmatrix} 3 & 1 & 0 & 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 & 2 & 1 & -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 4 & 3 & 1 & 1 \end{bmatrix}$.

   (b) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

6. Let

$$A = \begin{bmatrix} -1 & 2 & 3 & 4 \\ 5 & 0 & -1 & -1 \\ 8 & -6 & -10 & -13 \end{bmatrix}.$$

   (a) Show that the last row of A is a linear combination of the first two rows.

   (b) Let B be a 3×3 matrix formed by three columns of A. Is B invertible? Does your answer depend on which three columns of A are included in B?