Name: **PID:**

STOR 415 Practice Final Exam, Fall 2019

Multiple Choice Questions (4 points each).

1. Which statement is true about the following system of linear equations (represented in augmented matrix form)?

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

   (a) The system has no solution.
   (b) The vector $(3, 0, 2, 0)$ is the unique solution to this system.
   (c) The vector $(3, 1, 1, 1)$ is a solution to this system.
   (d) None of the above.

2. Consider the linear program

$$\begin{aligned} \max \quad & z = 3x + 2y \\ \text{s.t.} \quad & x + y \le 10 \\ & 2x + y \le 16 \\ & x, y \ge 0. \end{aligned}$$

   Which of the following statements is true?

   (a) $(x, y) = (0, 16)$ is a corner point of the feasible set.
   (b) $(x, y) = (5, 5)$ is a corner point of the feasible set.
   (c) The optimal solution is $(x, y) = (8, 0)$.
   (d) The optimal value is 26.

3. Which of the following is true about **any** $n \times n$ matrices A, B and C?

   (a) $ABC = CBA$.
   (b) $(ABC)^{-1} = A^{-1}B^{-1}C^{-1}$.
   (c) $(A + B)^T C = A^T C + B^T C$.
   (d) $\det(A + B + C) = \det(A) + \det(B) + \det(C)$.

4. Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student population of grade g is S_{ig}. Finally, the distance from school j to neighborhood i is d_{ij}. Formulate an LP whose objective is to assign all students to schools, while minimizing the total distance traveled by all students. Students from the same neighborhood can be assigned to different schools. You may ignore the fact that numbers of students must be integer. Let x_{ijg} be the number of students in grade g, from neighborhood i and assigned to school j.

   Which of the following should NOT be included in the LP?

   (a) $\min \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{g=1}^{G} d_{ij} x_{ijg}$

   (b) $\sum_{j=1}^{J} x_{ijg} = S_{ig},\ i = 1, \cdots, I,\ g = 1, \cdots, G$

   (c) $\sum_{i=1}^{I} x_{ijg} \leq C_{jg},\ j = 1, \cdots, J,\ g = 1, \cdots, G$

   (d) $\sum_{g=1}^{G} x_{ijg} = S_{ig},\ i = 1, \cdots, I,\ j = 1, \cdots, J$

5. Let A be an $n \times n$ square matrix, and $b \in \mathbb{R}^n$. Which of the following statements is true?

   (a) The system of equations $Ax = 0$ always has a unique solution.

   (b) The system of equations $Ax = 0$ always has at least a solution.

   (c) The system of equations $Ax = b$ always has at least a solution.

   (d) The system of equations $Ax = b$ always has a unique solution.

6. Which of the following is true about any $n \times n$ matrix A and any column vector $x \in \mathbb{R}^n$?

   (a) Ax is a linear combination of rows of A.

   (b) $x^T Ax = x^T A^T x$.

   (c) $x^T A$ is a linear combination of columns of A.

   (d) $x^T Ax \geq 0$.

7. Consider the LP

$$
\begin{array}{lrlcl}
\max & z = & 2x_1 - x_2 & & \\
\text{s.t} & & x_1 + 2x_2 + x_3 - 3x_4 & = & 4 \\
 & & x_1 - x_2 + x_3 + 2x_4 & = & 1 \\
 & & x \geq 0. & &
\end{array}
\tag{1}
$$

   Which of the following is correct?

   (a) $x = (2, 1, 0, 0)$ is a basic feasible solution.

   (b) $x = (\frac{11}{5},\ 0,\ 0,\ -\frac{3}{5})$ is a basic feasible solution.

   (c) $x = (1, 1, 1, 0)$ is a basic feasible solution.

   (d) $x = (2, 0, 2, 0)$ is a basic feasible solution.

8. Consider the following linear program (LP1) and its standard form (LP1′)

$$(LP1) \quad \begin{aligned} \max\ & z = 3x_1 + 2x_2 \\ \text{s.t. } & x_1 - 2x_2 \geq 0 \\ & x_1 + x_2 \quad \leq 20 \\ & x_1, x_2 \geq 0 \end{aligned} \qquad \begin{aligned} \max\ & z = 3x_1 + 2x_2 \\ \text{s.t. } & x_1 - 2x_2 - s_1 = 0 \\ & x_1 + x_2 \qquad + s_2 = 20 \\ & x_1, x_2, s_1, s_2 \geq 0. \end{aligned} \quad (LP1')$$

Which of the following is true?

(a) Given a feasible solution (x_1, x_2) for (LP1), the corresponding feasible solution for (LP1′) is $(x_1, x_2, x_1 - 2x_2, x_1 + x_2)$.

(b) Given a feasible solution (x_1, x_2) for (LP1), the corresponding feasible solution for (LP1′) is $(x_1, x_2, x_1 - 2x_2, 20 - x_1 - x_2)$.

(c) (LP1′) is in canonical form.

(d) (LP1) and (LP1′) have the same feasible set, which is a subset in $\mathbb{R}^2$.

9. Consider the following primal LP

$$\begin{array}{llrl} \max & z = & 3x_1 + 4x_2 - x_3 & \\ \text{s. t.} & & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 & \leq 2, \\ & & a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 & \leq 5. \end{array}$$

Suppose that $x = (2, 1, 0, 4)$ is a feasible solution. Let y_1 and y_2 be the dual variables corresponding to the two primal constraints respectively. Which of the following is possible to be a feasible solution of the dual LP?

(a) $y_1 = -2,\ y_2 = 3.$

(b) $y_1 = 1,\ y_2 = 3.$

(c) $y_1 = 2,\ y_2 = 1.$

(d) $y_1 = 0,\ y_2 = 1.$

10. You obtain the following simplex tableau at some iteration for a maximization problem, in which all variables are required to be nonnegative.

	z	x_1	x_2	x_3	x_4	x_5	x_6	rhs	Basic var
(row 0)	1	0	0	1	0	-3	0	30	$z = 30$
(row 1)	0	1	0	2	0	-4	0	2	$x_1 = 2$
(row 2)	0	0	0	3	0	-1	1	3	$x_6 = 3$
(row 3)	0	0	0	0	1	-2	0	5	$x_4 = 5$
(row 4)	0	0	1	2	0	0	0	10	$x_2 = 10$

Which of the following is the next action you take?

(a) Let x_3 enter, and x_1 leave.

(b) Let x_3 enter, and x_6 leave.

(c) Let x_5 enter, and x_2 leave.

(d) Stop the algorithm.

11. If an LP has at least one feasible solution, then:

(a) The dual may be infeasible.

(b) The dual may be unbounded.

(c) The dual must have an optimal solution.

(d) The dual has at least one feasible solution.

12. You are formulating an LP of maximizing profit. One of the constraints in this LP is a $\leq$ inequality. You have also written the dual of this LP. Now you decide to change that primal constraint from $\leq$ to $=$. The dual variable corresponding to this constraint:

(a) Was formerly nonnegative, and now will be free.

(b) Was formerly nonpositive, and now will be nonnegative.

(c) Was formerly nonpositive, and now will be free.

(d) Was formerly nonnegative, and now will be nonpositive.

13. Which of the following statements is true?

(a) Every LP in general form can be converted into an equivalent LP in canonical form.

(b) Every LP in standard form is feasible.

(c) Every phase-1 LP constructed in the two phase simplex algorithm has an optimal solution.

(d) Every LP in canonical form has an optimal solution.

14. Which of the following statement is **NOT** true?

    (a) A balanced transportation problem can be formulated as an MCNFP.

    (b) An MCNFP is a linear programming problem.

    (c) A shortest path problem can be formulated as a balanced transportation problem.

    (d) An assignment problem can be formulated as an MCNFP.

15. Which of the following algorithms is specially designed to solve the MCNFP?

    (a) Network simplex method. (b) Two-phase simplex method.
    (c) Branch and bound method. (d) Interior point method.

16. Consider the functions $f(x_1, x_2, x_3, x_4) = x_1 - x_2 + x_3 + 2x_4 + 10$ and $g(x_1, x_2) = x_1^2 - 2x_1x_2 + 4x_2^2$. Which of the following statements is true?

    (a) f and g are both convex functions.

    (b) f is a convex function, and g is not a convex function.

    (c) f is not a convex function, and g is a convex function.

    (d) Neither f nor g is a convex function.

17. Suppose the basic feasible solution shown in a simplex tableau is nondegenerate. Which of the following statements is always true?

    (a) The reduced costs of nonbasic variables in this tableau are all strictly positive.

    (b) The entries in the right hand side column are all strictly positive (excluding the one in row 0).

    (c) The basic feasible solution shown in this tableau must be an optimal solution.

    (d) It is impossible for this basic feasible solution to be an optimal solution.

18. You have an optimization problem of minimizing a convex function $f(x)$ subject to some constraints. You have found a local optimal solution x^*, with $f(x^*) = 25$. Which of the following statements must be true?

    (a) If the feasible set is convex, then there is no feasible solution x' with $f(x') > 25$.

    (b) If the feasible set is convex, then there is no feasible solution x' with $f(x') < 25$.

    (c) If the feasible set is not convex, then there is no feasible solution x' with $f(x') > 25$.

    (d) If the feasible set is not convex, then there is no feasible solution x' with $f(x') < 25$.

19. Which of the following statements is true?

   (a) In a general nonlinear programming problem, any local solution is also a global solution.

   (b) Any function of x in the form $ax^2 + bx + c$ is a convex function.

   (c) The set in $\mathbb{R}^2$ defined by the constraint $x_1 + x_2 \leq 1$ is a convex set.

   (d) None of the above.

20. Let $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$, and let $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ be two vectors. Which of the following is true?

   (a) If $Ax \leq b$ and $x \geq 0$, then $y^T Ax \leq y^T b$.

   (b) If $Ax \geq b$ and $x \leq 0$, then $y^T Ax \leq y^T b$.

   (c) If $A^T y \geq c$ and $x \leq 0$, then $x^T A^T y \leq x^T c$.

   (d) If $A^T y \geq c$ and $x \geq 0$, then $x^T A^T y \leq x^T c$.

21. Consider the following primal LP in canonical form

$$
\begin{array}{llrcl}
\max & z = & x_1 + 3x_2 + 2x_5 + x_7 & & \\
\text{s. t.} & & x_1 - 2x_2 + x_3 - x_5 + x_7 & = & 10, \\
 & & 2x_1 + 3x_2 + x_4 + x_5 + 2x_7 & = & 40, \\
 & & x_1 + x_2 + 2x_5 + x_6 + 3x_7 & = & 30, \\
 & & x \geq 0. & &
\end{array}
\qquad \text{(LP1)}
$$

An application of the simplex method to the above LP ends with a simplex tableau whose row 0 is given below, in which the rhs entry is replaced with $\times$.

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	rhs	Basic var
1	1.2	0	0	0.8	0	0.6	2.4	$\times$	$z = \times$

What is the correct value of the rhs entry currently marked with $\times$?

(a) 50 (b) 62 (c) 96 (d) None of the above.

22. Consider (LP1) in question 20. Suppose its objective function changes to

$$z = (1 + \delta)x_1 + 3x_2 + 2x_5 + (1 + \theta)x_7$$

and its constraints remain the same. What is the range of (δ, θ) for the basis $\{x_2, x_3, x_5\}$ to remain optimal?

   (a) $\delta \geq 1.2$ and $\theta \geq 2.4$.

   (b) $\delta \leq 1.2$ and $\theta \leq 2.4$.

   (c) $\delta \leq -1.2$ and $\theta \leq -2.4$.

   (d) $\delta \geq -1.2$ and $\theta \geq -2.4$.

23. You are writing a GAMS code to minimize the total cost of shipping electricity from 5 plants to 10 cities. You have defined two sets as follows

*Sets i "plants" / p1*p5 /,*
*j "cities" / c1*c10 /;*

and have declared parameters s over i and d over j, such that $s(i)$ represents the supply at plant i and $d(j)$ represents the demand at city j. You declared a positive variable x over (i, j), such that $x(i, j)$ denotes the amount to ship from plant i to city j. Each city has to satisfy its demand, and each plant cannot exceed its supply. Which of the following GAMS statements is correct?

(a) *equation obSupply(i);*
obSupply(i) .. sum(j, x(i,j)) =l= s(i) ;

(b) *equation obSupply;*
obSupply .. sum(i, x(i,j)) =l= s(i) ;

(c) *equation meetDem(j);*
meetDem(j) .. sum(i, x(i,j)) =l= d(j) ;

(d) *equation meetDem(i);*
meetDem(i) .. sum(i, x(i,j)) =g= d(j) ;

24. In the above problem, suppose that the amount to ship from each individual plant to city c1 cannot exceed 100 units. Which of the following GAMS statements correctly models this?

(a) *equation bound;*
bound .. x(i, "c1") =g= 100;

(b) *equation bound(i);*
bound(i) .. x(i, "c1") =l= 100;

(c) *equation bound(i);*
bound(i) .. sum(i, x(i, "c1")) =g= 100;

(d) *equation bound(i);*
bound(i) .. sum(i, x(i, "c1")) =l= 100;

25. In the above problem, suppose that each city cannot receive more than 40 percent of its total amount of electricity from plant p1. Which of the following GAMS statements correctly models this?

(a) *equation percentP1;*
percentP1 .. x("p1", j) =l= 0.4 sum(i, x(i,j)) ;*

(b) *equation percentP1(j);*
percentP1(j) .. x(i,j) =l= 0.4 x("p1", j) ;*

(c) *equation percentP1(j);*
percentP1(j) .. x("p1", j) =l= 0.4 sum(i, x(i,j)) ;*

(d) *equation percentP1(j);*
percentP1(j) .. x(i,j) =l= 0.4 sum(i, x(i,j)) ;*