

STOR 415, Fall 2019
Homework Assignment No. 6

1. Determine whether each statement below is true or false; justify your answer.
 - (a) If the primal LP is feasible, then the dual LP must be feasible.
 - (b) If the primal LP is infeasible, then the dual LP must be unbounded.
2. For each of the following linear programs, write down the dual LP and graphically solve the dual LP, and then use complementary slackness conditions to find the set of optimal solutions of the primal LP.

(a)

$$\begin{array}{ll} \max & z = 5x_1 + 3x_2 + x_3 \\ \text{s.t.} & 2x_1 + x_2 + x_3 \leq 6 \\ & x_1 + 2x_2 + x_3 \leq 7 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

(b)

$$\begin{array}{ll} \max & z = 5x_1 - 7x_2 + 5x_3 \\ \text{s.t.} & 2x_1 + x_2 + x_3 \leq 5 \\ & x_1 - x_2 + x_3 = 3 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

3. Consider the following pair of primal and dual LPs.

$$\begin{array}{llllllll}
 \text{(Primal)} & \min & 35x_1 & +30x_2 & +60x_3 & +50x_4 & +27x_5 & +22x_6 \\
 & \text{s.t.} & x_1 & & +2x_3 & +2x_4 & +x_5 & +2x_6 \geq 9, \\
 & & & x_2 & +3x_3 & +x_4 & +3x_5 & +2x_6 \geq 19, \\
 & & & & & & x & \geq 0.
 \end{array}$$

$$\begin{array}{llll}
 \text{(Dual)} & \max & 9\pi_1 & +19\pi_2 \\
 & \text{s.t.} & \pi_1 & \leq 35, \\
 & & & \pi_2 \leq 30, \\
 & & 2\pi_1 & +3\pi_2 \leq 60, \\
 & & 2\pi_1 & +\pi_2 \leq 50, \\
 & & \pi_1 & +3\pi_2 \leq 27, \\
 & & 2\pi_1 & +2\pi_2 \leq 22, \\
 & & & \pi \geq 0.
 \end{array}$$

- (a) Suppose that $x^* = (0, 0, 0, 0, 5, 2)$ is an optimal solution to the primal LP. Use complementary slackness conditions to find the set of all optimal solutions of the dual LP. Does the primal LP have a unique optimal solution, or multiple optimal solutions?
- (b) Now suppose that the right hand side constants of the first two primal constraints are changed to 11 and 23 respectively (from the current values of 9 and 19). Does the dual optimal solution(s) you found in (a) remain optimal after the change? If so, use it to find the set of all optimal solutions to the primal LP.