STOR 415, Fall 2019
Homework Assignment No. 11

1. Decide whether each of the following statements is true or false.

   (a) An affine function of the form $f(x) = c^T x + d$, where $c \in \mathbb{R}^n$ and $d \in \mathbb{R}$, is always a convex function.

   (b) The feasible set of a linear programming problem is always a convex set.

   (c) A quadratic function of the form $f(x) = x^T M x + c^T x + d$, where $M \in \mathbb{R}^{n \times n}$, $c \in \mathbb{R}^n$, $d \in \mathbb{R}$ is always a convex function.

   (d) The function
   $$f(x_1, x_2) = x_1^2 - 3x_1 x_2 + 2x_2^2$$
   is a convex function.

2. You have an optimization problem of minimizing a nonlinear function $f(x)$ subject to some constraints. You have found a local solution x^*, with $f(x^*) = 25$.

   (a) Suppose that f is a convex function, and the feasible set is a convex set. Is it possible to find a feasible solution x' with $f(x') < 25$?

   (b) Now suppose that f is a convex function, and the feasible set is a nonconvex set. Is it possible to find a feasible solution x' with $f(x') < 25$?

3. Let f be a convex function from $\mathbb{R}^m$ to $\mathbb{R}$, and let g be an affine function from $\mathbb{R}^n$ to $\mathbb{R}^m$. (A function g is said to be affine, if it is of the form $g(x) = Ax + b$.) Prove the function F defined by $F(x) = f(g(x))$ is a convex function on $\mathbb{R}^n$.

4. Show that the function
   $$F(x) = (-x_1 + 3x_2 + 5x_3 - 6x_4)^2 - 2(x_1 - x_3)$$
   a convex function. (Hint: You can either write down the Hession matrix of f and show that it is positive semidefinite, or use the fact proved in the previous question and write $F(x) = f(g(x))$ with a convex function f and an affine function g.)