STOR 415, Fall 2019
Homework Assignment No. 3

1. Determine if each of the following linear programs is in standard form or canonical form. For linear programs in canonical form, specify the isolated variables. For linear programs not in standard form, convert them into standard form.

   (a)
$$
\begin{array}{lll}
\min & z = & 3x_1 + x_2 \\
\text{s.t} & & x_1 \geq 3, \\
& & x_1 + x_2 \leq 20, \\
& & 2x_1 - x_2 = 3, \\
& & x_1, x_2 \geq 0
\end{array}
$$

   (b)
$$
\begin{array}{lll}
\max & z = & 3x_1 + x_2 + x_3 + x_4 \\
\text{s.t} & & x_1 + 2x_2 + x_3 + x_4 \geq 10, \\
& & x_1 + x_2 \leq 4, \\
& & x_1 \geq 0, \quad x_2 \leq 0, \quad x_4 \geq 0
\end{array}
$$

   (c)
$$
\begin{array}{lll}
\max & z = & x_1 + x_2 - x_3 \\
\text{s.t} & & x_1 + x_3 + x_4 = 10, \\
& & x_1 + x_2 + x_3 = 6, \\
& & x_1, x_2, x_3, x_4 \geq 0
\end{array}
$$

   (d)
$$
\begin{array}{lll}
\max & z = & x_1 - 2x_2 - x_3 + x_4 \\
\text{s.t} & & x_1 + x_3 + x_4 + s_1 = 10, \\
& & x_1 + x_2 + x_3 + s_2 = 6, \\
& & x_1, x_2, x_3, x_4, s_1, s_2 \geq 0
\end{array}
$$

2. Consider the LP

$$\begin{array}{llrcl}
\max & z = & 2x_1 + 3x_2 & & \\
\text{s.t} & & x_1 + x_2 + x_3 - x_4 & = & 2 \\
& & x_1 - x_2 \qquad + x_4 & = & 0 \\
& & x \geq 0 & &
\end{array} \tag{1}$$

   (a) List all the basic solutions. Identify all the nondegnerate basic feasible solutions and degenerate basic feasible solutions.

   (b) Project the feasible set onto the (x_1, x_2) plane: first express x_3 and x_4 as functions of x_1 and x_2, then convert the nonnegativity constraints on x_3 and x_4 into constraints on x_1 and x_2, and finally write down the set of (x_1, x_2) that satisfies the nonnegativity constraints on themselves and the constraints transformed from the nonnegativity of x_3 and x_4.

   (c) Graph the set you obtain in part (b) in the (x_1, x_2) plane, and label all of its vertices.

   (d) Does the LP in (1) have an optimal solution?

3. Consider the following LP in standard form, with a single equality constraint:

$$\begin{aligned}
\max \; & \sum_{i=1}^{n} c_i x_i \\
\text{s.t.} \; & \sum_{i=1}^{n} a_i x_i = b \\
& x_i \geq 0, \;\; i = 1, \cdots, n
\end{aligned} \tag{2}$$

   Suppose $b > 0$, and $c_i > 0$ and $a_i > 0$ for each $i = 1, \cdots, n$.

   (a) Write down all the basic feasible solutions.

   (b) For each $i = 1, \cdots, n$, find a number M_i, which may depend on b and a_i, such that any feasible solution x satisfies $x_i \leq M_i$.

   (c) What is the optimal value of the LP in (2)?