STOR 415, Fall 2019
Homework Assignment No. 4

1. Suppose that, after applying the simplex method to solve a maximization LP, you obtain the tableau below, where $a_{13}, a_{14}, a_{23}, a_{24}, a_{33}, a_{34}, b_1, b_2, b_3, c_3, c_4, d$ are real numbers.

z	x_1	x_2	x_3	x_4	x_5	rhs	Basic var
1	0	0	c_3	c_4	0	d	$z = d$
0	0	1	a_{13}	a_{14}	0	b_1	$x_2 = b_1$
0	0	0	a_{23}	a_{24}	1	b_2	$x_5 = b_2$
0	1	0	a_{33}	a_{34}	0	b_3	$x_1 = b_3$

max z; $\quad x \geq 0$

Suppose $b_1 \geq 0$, $b_2 \geq 0$ and $b_3 \geq 0$. For each of the scenarios below, describe the next action to take in applying the simplex method. If you conclude that an optimal solution is found, write down an optimal solution. If you conclude that the LP is unbounded, it suffices to state that. If the next action is to conduct another iteration, describe the entering variable and the way to choose the leaving variable, but you do not have to conduct the actual row operations to write down the next tableau.

   (a) $c_3 < 0$, $c_4 > 0$, $a_{13} < 0$, $a_{23} < 0$ and $a_{33} > 0$.

   (b) $c_3 < 0$, $c_4 > 0$, $a_{13} < 0$, $a_{23} < 0$ and $a_{33} = 0$.

   (c) $c_3 > 0$, $c_4 > 0$.

   (d) $c_3 > 0$, $c_4 = 0$.

   (e) $c_3 > 0$, $c_4 < 0$, $a_{14} > 0$, $a_{24} > 0$ and $a_{34} < 0$.

2. Follow the following steps for each of the problems below. First, convert any minimization problem into a maximization problem by changing the signs of coefficients in the objective function. Next, convert the problem into standard form; you will find the resulted LP to be in canonical form. Then, apply the simplex method, and decide if the problem is unbounded or has unique or multiple optimal solutions. Provide three optimal solutions for each problem identified as having multiple optimal solutions. For each problem identified as unbounded, provide a starting point and a direction, so that the objective value can be increased indefinitely without violating any constraint by moving from the starting point along that direction.

(a)

$$\begin{array}{llll} \max & z = & 2x_1 + 3x_2 & \\ \text{s.t} & & x_1 + 2x_2 & \leq 6, \\ & & 2x_1 + x_2 & \leq 8, \\ & & x_1, x_2 & \geq 0 \end{array}$$

(b)

$$\begin{array}{llll} \max & z = & -3x_1 + 6x_2 & \\ \text{s.t} & & 5x_1 + 7x_2 & \leq 35, \\ & & -x_1 + 2x_2 & \leq 2, \\ & & x_1, x_2 & \geq 0 \end{array}$$

(c)

$$\begin{array}{llll} \min & z = & -x_1 - 3x_2 & \\ \text{s.t} & & x_1 - 2x_2 & \leq 4, \\ & & -x_1 + x_2 & \leq 3, \\ & & x_1, x_2 & \geq 0 \end{array}$$

3. For each of the simplex tableaus below, determine if the LP is unbounded or has unique or multiple optimal solutions. For each LP identified as having multiple optimal solutions, provide three optimal solutions. For each LP identified as being unbounded, provide a starting point and a direction, such that the halfline from the starting point along the direction is entirely contained in the feasible set and the objective value increases along the direction.

(a)

z	x_1	x_2	x_3	x_4	rhs	basic var
1	0	0	0	-1	5	$z = 5$
0	2	0	1	-1	10	$x_3 = 10$
0	1	1	0	0	2	$x_2 = 2$

max z; $x \geq 0$

(b)

z	x_1	x_2	x_3	x_4	x_5	rhs	basic var
1	0	0	2	2	3	4	$z = 4$
0	1	0	1	2	-2	8	$x_1 = 8$
0	0	1	0	1	1	3	$x_2 = 3$

max z; $x \geq 0$

(c)

z	x_1	x_2	x_3	x_4	x_5	rhs	basic var
1	0	0	2	1	0	12	$z = 12$
0	1	0	1	2	-1	2	$x_1 = 2$
0	0	1	0	1	-1	4	$x_2 = 4$

max z; $x \geq 0$