STOR 415, Fall 2019
Homework Assignment No. 5

1. Determine the feasibility of each LP below by solving the Phase-1 LP: First, if an LP is not in standard form, convert it into standard form. Then, add artificial variables and define the Phase-1 LP; to reduce computation, do not define artificial variables for equations that already isolate variables (see Example 4.8 in the book).

   For each LP determined as feasible, write down its feasible solution obtained from the last tableau of the Phase-1 LP, but you do not need to find its optimal solution.

   (a)

$$
\begin{array}{llrcr}
\max & z = & 2x_1 + 3x_2 - x_4 & & \\
\text{s.t} & & 2x_1 + x_2 + 2x_3 & \leq & 16, \\
 & & x_1 + x_2 - x_3 & \geq & 15, \\
 & & x_1 \qquad + x_3 - x_4 & = & -10, \\
 & & x_i \geq 0 \text{ for all } i & &
\end{array}
$$

   (b)

$$
\begin{array}{ll}
\max & x_8 \\
\text{s.t.} & -x_1 - x_3 + x_4 + x_5 = -2, \\
 & x_1 - 2x_3 - 3x_4 + x_6 - x_8 = 4, \\
 & x_1 + x_3 - 0.5x_4 + x_7 = 1, \\
 & 2x_1 + x_2 - 5x_4 - x_8 = 6, \\
 & x_i \geq 0 \text{ for all } i
\end{array}
$$

2. Consider the following LP

$$
\begin{array}{ll}
\min & z = y_1 + y_2 + \cdots + y_m \\
\text{s.t.} & Ax + y = b \\
 & x \geq 0, \quad y \geq 0,
\end{array}
$$

   where A is some $m \times n$ matrix, b is some vector in $\mathbb{R}^m$, and $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ are the variables. Suppose $b \geq 0$.

   (a) Is this LP always feasible? Why?

   (b) Is it possible for this LP to be unbounded? Why?

3. For each LP below, write down the dual LP.

(a)

$$
\begin{array}{llllllll}
\min & x_1 & +2x_2 & -3x_3 & +x_4 & & \\
\text{s.t.} & x_1 & -2x_2 & +3x_3 & +x_4 & \leq & 3, \\
 & & x_2 & +2x_3 & +2x_4 & \geq & -5, \\
 & 2x_1 & -3x_2 & -7x_3 & -4x_4 & = & 2, \\
 & \multicolumn{6}{l}{x_1 \geq 0, \quad x_4 \leq 0.}
\end{array}
$$

(b)

$$
\begin{array}{llllll}
\max & -x_1 & & +2x_3 & & \\
\text{s.t.} & x_1 & +x_2 & & \leq & 1, \\
 & -x_1 & & +x_3 & = & 2, \\
 & \multicolumn{5}{l}{x_1 \leq 0, \quad x_2 \geq 0}
\end{array}
$$

(c)

$$
\begin{array}{ll}
\max & c^T x \\
\text{s.t.} & Ax = b, \\
 & Bx \geq d, \\
 & x \geq 0,
\end{array}
$$

where $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{p \times n}$, $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ and $d \in \mathbb{R}^p$.