Solutions to STOR 415, Spring 2013, Exam 2

1. C.

2. B

3. C

4. D

5. (a) Dual is:

$$\begin{array}{rrrrrr} \min & 20y_1 & +5y_2 & +5y_3 & & \\ \text{s.t} & y_1 & +y_2 & +y_3 & \geq & 1, \\ & 2y_1 & +y_2 & & \geq & -1, \\ & y_1 \geq 0, & y_2 \leq 0, & y_3 \geq 0. & & \end{array}$$

   (b) By complementarity slackness conditions, an optimal solution y for the dual needs to satisfy $y_1 = 0$, $y_2 \leq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 = 1$ and $2y_1 + y_2 \geq -1$. The set of y's that satisfies all these is $\{(0, t, 1-t) \mid t \in [-1, 0]\}$.

   (c) The primal LP has a unique optimal solution, as can be seen by complementary slackness conditions (based on a dual optimal solution), or by graphical solution.

   (d) After the change, the conditions for y are $y_1 = 0$, $y_2 \leq 0$, $y_3 \geq 0$, $y_1 + y_2 + y_3 = 1$ and $2y_1 + y_2 \geq 0$. The only y's that satisfies all these is $(0, 0, 1)$. So the dual LP has a unique optimal solution $(0, 0, 1)$.

6. (a) Optimal solution: $x = (3, 4, 1, 0, 0)$; optimal value 17.

   (b) Range of Δ: [-7/3, 1]. The optimal value on this range is $17 + 3\Delta$.

   (c) Range of Δ: [-5, 15]. The optimal value on this range is $17 + 0.2\Delta$.

   (d) The rhs column changes to $(17, 3, 1, 4) + (0, 0, 1, 0)\Delta + (0.2, -0.2, 0.2, 0.4)5\Delta = (17+\Delta, 3-\Delta, 1+2\Delta, 4+2\Delta)$. The range of Δ is $[-1/2, 3]$ and the optimal value on this range is $17 + \Delta$.