Name: Solution **PID:**

STOR 415, Fall 2019, Exam 1

This 7-page exam starts at 12:20pm and ends at 1:10pm. It is closed book/notes; calculators are allowed, and laptops/ipads/phones are not. Write all answers on the exam sheets.

Multiple Choice Questions (5 points each). Choose one answer for each question.

1. Which of the following statements is true? Here A, B and C are $n \times n$ matrices.

   (a) $ABC = BAC$.

   (b) If A, B and C are all invertible, then their product ABC is invertible.

   (c) If A, B and C are all invertible, then their sum $A+B+C$ is invertible.

   (d) $(AB)^{-1} = A^{-1}B^{-1}$.

2. After applying the Gauss-Jordan method to a system of linear equations, we obtain the following augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 3 \\ 0 & 1 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

   Which of the following is true?

   (a) $(0,0,-1,1)$ is a solution to the system.

   (b) $(0,2,1,1)$ is a solution to the system.

   (c) The system has no solution.

   (d) The system has a unique solution.

3. A matrix $M \in \mathbb{R}^{4\times 4}$ is given by $M = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & a_2 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, where a_1, a_2, a_3 are constants.

   Which of the following statements is true?

   (a) Columns of M are always linearly dependent, regardless of values of a_1, a_2, a_3.

   (b) Columns of M are always linearly independent, regardless of values of a_1, a_2, a_3.

   (c) For any vector $b \in \mathbb{R}^4$, the equation $Mx = b$ always has a unique solution regardless of values of a_1, a_2, a_3.

   (d) For any vector $b \in \mathbb{R}^4$, the equation $Mx = b$ always has no solution regardless of values of a_1, a_2, a_3.

4. Which of the following operations on a system of m linear equations with n variables is possible to change the set of solution(s) of this system?

   (a) In matrix form, premultiply both sides of the system by an invertible $m \times m$ matrix.

   (b) Add a multiple of the first equation to the mth equation.

   (c) Multiply both sides of the first equation by a real number.

   (d) Switch the first two equations.

5. Which of the following statements is true?

   (a) An LP in standard form must have at least a feasible solution.

   (b) An LP in general form can always be converted into an equivalent LP that is in canonical form.

   (c) An LP in canonical form must have at least an optimal solution.

   (d) None of the above.

6. Consider the following linear program in which x_1, x_2 and x_3 are the variables, and c_1 and c_2 are constants:

$$\begin{aligned} \min\ & z = c_1 x_1 + c_2 x_2 \\ \text{s.t. } & x_1 + x_2 + x_3 = 1, \\ & x_1 \geq 0,\ \ x_2 \geq 0,\ \ x_3 \geq 0. \end{aligned}$$

   Which of the following statements is true?

   (a) If $c_1 = 1$ and $c_2 = 0$ then the LP has optimal value 1.

   (b) If $c_1 = 1$ and $c_2 = 0$ then the LP has a unique optimal solution.

   (c) If $c_1 = 1$ and $c_2 = 1$ then the LP has optimal value 1.

   (d) If $c_1 = 1$ and $c_2 = 1$ then the LP has a unique optimal solution.

7. Consider the LP in question 6. How many basic feasible solutions does it have?

   (a) 1 (b) 2 (c) 3 (d) 4

Free Response Questions.

8. You obtain the following simplex tableau in using the simplex method to solve a maximization LP.

	z	x_1	x_2	x_3	x_4	x_5	x_6	rhs
(row 0)	1	0	0	3	0	-1	0	20
(row 1)	0	1	0	4	0	2	0	3
(row 2)	0	0	0	-2	0	1	1	2
(row 3)	0	0	0	0	1	-1	0	4
(row 4)	0	0	1	2	0	-1	0	10

max z; $x \geq 0$

(a) (5 points) What is the basic feasible solution associated with this simplex tableau?

$$x = \begin{bmatrix} 3 \\ 10 \\ 0 \\ 4 \\ 0 \\ 2 \end{bmatrix}$$

(b) (5 points) From this simplex tableau, what is the entering variable to choose?

x_5

(c) (5 points) What is the leaving variable? Do not carry out the elementary row operations; you do not need to provide the next simplex tableau.

x_1

9. The Candid Camera Company manufactures three types of cameras: the Cub, the Quickiematic and the VIP, which yield profits of \$3, \$9 and \$25 per unit, respectively. The distribution center requires that at least 250 Cubs, 375 Quickiematics and 150 VIPs be produced each week.

   Each camera requires a certain amount of time in order to (1) manufacture the body parts; (2) assemble the parts; (3) inspect, test and package the final product. Each Cub takes 0.1 hours to manufacture, 0.2 hours to assemble, and 0.1 hours to inspect, test and package. Each Quickiematic needs 0.2 hours to manufacture, 0.35 hours to assemble, and 0.2 hours to inspect, test and package. Each VIP requires 0.7, 0.1 and 0.3 hours, respectively. There are 250 hours per week of manufacturing time available, 350 hours of assembly, and 150 hours total to inspect, test and package.

   (a) (10 points) Formulate the problem of determining how many cameras of each type to make, in order to maximize the total profit, as an LP. To receive full credits, clearly state the variables, the objective function and all constraints.

Let x_1 be the number of Cubs to make

x_2 Quickimatic

x_3 VIP.

Objective function: Max $3x_1 + 9x_2 + 25x_3$.

s.t.

$x_1 \geq 250$

$x_2 \geq 375$

$x_3 \geq 150$

$0.1x_1 + 0.2x_2 + 0.7x_3 \leq 250$ (Manufacturing time)

$0.2x_1 + 0.35x_2 + 0.1x_3 \leq 350$ (Assembly time)

$0.1x_1 + 0.2x_2 + 0.3x_3 \leq 150$ (Inspect).

(b) (10 points) Convert the formulation you obtain in part (a) into standard form.

$$\max \quad 3x_1 + 8x_2 + 25x_3$$

s.t. $x_1 - s_1 = 250$

$x_2 - s_2 = 375$

$x_3 - s_3 = 150$

$0.1x_1 + 0.2x_2 + 0.7x_3 + s_4 = 250$

$0.2x_1 + 0.35x_2 + 0.1x_3 + s_5 = 350$

$0.1x_1 + 0.2x_2 + 0.3x_3 + s_6 = 150$

$x, s \geq 0$

Or

$$\max \quad 3(s_1+250) + 8(s_2+375) + 25(150+s_3)$$

s.t.

$0.1(s_1+250) + 0.2(s_2+375) + 0.1(s_3+150) + s_4 = 250$

$0.2(s_1+250) + 0.35(s_2+375) + 0.1(s_3+150) + s_5 = 350$

$0.1(s_1+150) + 0.2(s_2+375) + 0.3(s_3+150) + s_6 = 150$

$s \geq 0.$

10. (10 points for each part) You obtain the following simplex tableaus in using the simplex method to solve maximization linear programs. For each of the tableaus, determine if the LP represented by it is unbounded, has a unique optimal solution, or has multiple optimal solutions.

For each LP identified as having a unique optimal solution, write down the optimal solution. For each LP identified as having multiple optimal solutions, provide three different optimal solutions. For each LP identified as unbounded, provide a starting point and a direction, such that the halfline from the starting point along the direction is entirely contained in the feasible set and the objective value increases along the direction.

(a)

z	x_1	x_2	x_3	x_4	rhs	basic var
1	0	0	0	1	$\frac{15}{2}$	$z = \frac{15}{2}$
0	1	0	2	-2	4	$x_1 = 4$
0	0	1	1	0	2	$x_2 = 2$

max z; $x \geq 0$

Multiple opt sols.

Three opt sols

$$x = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Solution set:

$$x = \begin{bmatrix} 4-2t \\ 2-t \\ t \\ 0 \end{bmatrix} \quad \text{for } 0 \leq t \leq 2.$$

	z	x_1	x_2	x_3	x_4	x_5	rhs	basic var
	1	0	0	3	5	-2	0	$z = 0$
(b)	0	1	0	2	4	0	20	$x_1 = 20$
	0	0	1	-1	1	-2	10	$x_2 = 10$

max z; $x \geq 0$

Unbounded.

$$x = \begin{bmatrix} 20 \\ 10+2t \\ 0 \\ 0 \\ t \end{bmatrix} = \underbrace{\begin{bmatrix} 20 \\ 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{Starting point}} + t \underbrace{\begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{Direction}}$$