Name:

STOR 415, Spring 2013, Exam 2

The exam starts at 2:00pm and ends at 2:50pm. It is closed book/notes. You can use calculators, but not laptops. Write all answers in the blue book.

Multiple Choice Questions (make one choice for each question; 5 points each).

1. Which of the following statements is true about a pair of primal and dual LP's?

   (a) If the primal LP is feasible, then the dual LP must be infeasible.

   (b) If the primal LP is infeasible, then the dual LP must be unbounded.

   (c) If the primal LP is unbounded, then the dual LP must be infeasible.

   (d) None of the above.

2. You declared two positive variables x and y and a free variable z in GAMS. You need to model the constraints $5x + y \leq 100$ and $x - 2y \geq 8$, and the objective function $z = 2x + 3y$. Which of the following GAMS statements is correct?

   (a) *equation constraint1;*
   *constraint1.. 5*x + y =e= 100;*

   (b) *equation constraint2;*
   *constraint2.. x − 2*y =g= 8;*

   (c) *equation obj;*
   obj.. z = 2x+3y;

   (d) None of the above.

3. You are formulating an LP of minimizing cost. One of the variables in this LP is a nonnegative variable. You have also written the dual of this LP. Now you decide to change that primal variable from being nonnegative to being free. The dual constraint corresponding to this variable:

   (a) Was formerly of type $\geq$, and will now be of type $\leq$.

   (b) Was formerly of type $\geq$, and will now be of type $=$.

   (c) Was formerly of type $\leq$, and will now be of type $=$.

   (d) Was formerly of type $\leq$, and will now be of type $\geq$.

4. Consider the following two LPs.

$$\text{(LP1)} \quad \begin{array}{rl} \max\limits_{x} & z = c^T x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \end{array} \qquad \text{and} \qquad \begin{array}{rl} \max\limits_{x,y} & t = -y_1 - y_2 - \cdots - y_m \\ \text{s.t.} & Ax + y = b \\ & x \geq 0,\ y \geq 0 \end{array} \quad \text{(LP2)}$$

   where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ and $b \geq 0$. Which of the following is true?

   (a) (LP1) is always feasible.

   (b) If (LP1) is feasible, then (LP2) is unbounded.

   (c) If (LP1) is infeasible, then (LP2) is unbounded.

   (d) None of the above.

Free Response Questions.

5. Consider the following primal LP.

$$\begin{array}{rllll} \max & x_1 & - & x_2 & \\ \text{s.t} & x_1 & +\ 2\ x_2 & \leq & 20, \\ & x_1 & + & x_2 & \geq \ \ 5, \\ & x_1 & & & \leq \ \ 5, \\ & \multicolumn{4}{l}{x_1 \geq 0,\ x_2 \geq 0.} \end{array}$$

(a) (8 points) Write down its dual LP.

(b) (12 points) Suppose we know that $x^* = (5, 0)$ is an optimal solution to the primal LP. Use the complementarity slackness conditions, to find the set of all optimal solutions to the dual LP.

(c) (10 points) Does the primal LP have a unique optimal solution, or multiple optimal solutions?

(d) (10 points) Now change the objective function of the primal LP to be max x_1. Suppose we know that $x^* = (5, 0)$ continues to be an optimal solution to the new primal LP. Use the complementarity slackness conditions, to find the set of all optimal solutions to the new dual LP.

6. Consider the following LP.

$$\begin{array}{llllllll} \max & z = & 3x_1 & +2x_2 & & & & \\ \text{s.t} & & x_1 & & +x_3 & & & = \ \ 4, \\ & & x_1 & +3x_2 & & +x_4 & & = 15, \\ & & 2x_1 & +x_2 & & & +x_5 & = 10, \\ & & x \geq 0. & & & & & \end{array}$$

After using simplex method to solve it, we obtain the following tableau.

z	x_1	x_2	x_3	x_4	x_5	rhs	Basic var
1	0	0	0	0.2	1.4	17	$z = 17$
0	1	0	0	-0.2	0.6	3	$x_1 = 3$
0	0	0	1	0.2	-0.6	1	$x_3 = 1$
0	0	1	0	0.4	-0.2	4	$x_2 = 4$

$\max z;\ \ x \geq 0$

(a) (6 points) Write down an optimal solution of this LP, and its optimal value.

(b) (12 points) Suppose that the objective function changes from $3x_1 + 2x_2$ to $(3 + \Delta)x_1 + 2x_2$. For what range of Δ does the current optimal basis $\{x_1, x_3, x_2\}$ continue to be optimal? What is the optimal value when Δ belongs to this range?

(c) (12 points) Suppose that the objective function is the original function, and the right hand side constant of the second constraint changes from 15 to $15 + \Delta$. For what range of Δ does the current optimal basis $\{x_1, x_3, x_2\}$ continue to be optimal? What is the optimal value when Δ belongs to this range?

(d) (10 points) Suppose that the objective function is the original function, and the right hand side constants of the first and second constraints change from 4 and 15 to $4 + \Delta$ and $15 + 5\Delta$ respectively. For what range of Δ does the current optimal basis $\{x_1, x_3, x_2\}$ continue to be optimal? What is the optimal value when Δ belongs to this range?