Binomial example: μ_i mean (prob) $\quad \eta_i = \log\left(\frac{\mu_i}{1-\mu_i}\right) \quad V_i(\mu_i) = \frac{\mu_i(1-\mu_i)}{w_i}$

y_i: proportion $\quad w_i = n_i$

$$\frac{d\eta}{d\mu} = \frac{1}{\mu} + \frac{1}{1-\mu} = \frac{1}{\mu(1-\mu)}$$

$\boxed{\text{STOR 590} \quad 9/14/2020}$

$\hat\mu^{(0)}$: sample proportions

$$z^{(h)} = \hat\eta^{(h)} + (y-\hat\mu^{(h)})\frac{d\eta}{d\mu} = \hat\eta^{(h)} + \frac{y-\hat\mu^{(h)}}{\hat\mu^{(h)}(1-\hat\mu^{(h)})}$$

$$\frac{1}{w_i^{(h)}} = \left(\frac{d\eta_i}{d\mu_i}\right)^2 \cdot V_i(\hat\mu_i^{(h)}) =$$

$$= \left[\frac{1}{\mu_i(1-\mu_i)}\right]^2 \cdot \frac{\mu_i(1-\mu_i)}{w_i}$$

$$w_i^{(h)} = n_i\mu_i^{(h)}(1-\mu_i^{(h)})$$

$$\cancel{\mathrm{var}(\hat\beta) = (X^TW^*X)^{-1}}$$

$$\mathrm{var}(\hat\beta) = (X^TWX)^{-1}$$ F. p.155 middle

estimate of σ^2 in the LM

Compare standard LM: (with weights) $\quad \mathrm{var}\,\hat\beta = (X^TWX)^{-1}\hat\sigma^2$

Standard errors of GLM: $\dfrac{\text{standard errors of LM}}{\hat\sigma}$

Deviances $\qquad$ Residuals

Deviances $\quad D = \text{deviance} \qquad \dfrac{D}{\phi}$: scaled deviance

Recall: $\quad l_i(y_i \mid \phi, \mu_i) = \dfrac{y_i\theta_i - b(\theta_i)}{a_i(\phi)} + \dots \qquad a_i(\phi) = \dfrac{\phi}{w_i}$

Fitted model: defines $\hat{\mu}_i, \hat{\theta}_i$

Saturated model: typically $\mu_i = \tilde{\mu}_i = y_i$

$$\tilde{\theta}_i = \log \frac{\tilde{\mu}_i}{1-\tilde{\mu}_i}$$

Define $\cancel{\dfrac{D}{\phi} = \dfrac{2}{\phi}\sum_i \{ l_i(y_i \mid \phi, \tilde{\mu}_i) - l_i(y_i \mid \phi, \hat{\mu}_i) \}}$

$$\frac{D}{\phi} = \frac{2}{\phi}\sum_i \{ l_i(y_i \mid \phi, \tilde{\mu}_i) - l_i(y_i \mid \phi, \hat{\mu}_i) \}$$

$$= \frac{2}{\phi}\sum_i w_i \{ y_i\tilde{\theta}_i - b(\tilde{\theta}_i) - y_i\hat{\theta}_i + b(\hat{\theta}_i) \}$$

Binomial $\quad \theta = \log \dfrac{\mu}{1-\mu} \qquad b(\theta) = \log(1+e^{\theta}) = -\log(1-\mu)$

[p. 4 of GLM handout]

$\phi = 1$

$$D = 2\sum_i n_i \left\{ y_i \log \frac{y_i}{1-y_i} - y_i \log \frac{\hat{\mu}_i}{1-\hat{\mu}_i} + \log(1-y_i) - \log(1-\hat{\mu}_i) \right\}$$

$$= 2\sum_i n_i \left\{ y_i \log \frac{y_i}{\hat{\mu}_i} + (1-y_i)\log \frac{1-y_i}{1-\hat{\mu}_i} \right\}$$

Residuals   Standard LM: $r_i = y_i - \hat{\mu}_i$

"Response Residual"

not often used

"Pearson Residual" $\dfrac{y_i - \hat{\mu}_i}{\sqrt{V_i(\hat{\mu}_i)}}$

"Deviance Residual" $D = \sum_i d_i^2$

$$d_i = \sqrt{d_i^2} \cdot \text{sign}(y_i - \hat{\mu}_i)$$

"Working Residual": the residual that comes from the LM fit inside the iteration