HOMEWORK 1 - Proportion Not Returned

STOR 590, FALL 2020

Rui Li

8/20/2020

Import and Summarize the Dataset

```
#Import Proportion Not Returned.csv
library(readr)
dataset <- read_csv("ProportionNotReturned.csv")
```

```
## Parsed with column specification:
## cols(
##   County = col_character(),
##   PNR = col_double(),
##   Pop = col_double(),
##   Rural = col_double(),
##   MedAge = col_double(),
##   Travel = col_double(),
##   Hsgrad = col_double(),
##   Collgrad = col_double(),
##   MedInc = col_double(),
##   Black = col_double(),
##   Hisp = col_double(),
##   AbsBal = col_double()
## )
```

```
#Have a general idea about the dataset
head(dataset)
```

```
## # A tibble: 6 x 12
##   County    PNR    Pop Rural MedAge Travel Hsgrad Collgrad MedInc Black  H
isp
##   <chr>   <dbl>  <dbl> <dbl>  <dbl>  <dbl>  <dbl>    <dbl>  <dbl> <dbl> <d
bl>
## 1 ALAMA~ 0.0116 163339  28.6     40   23.5   84.7     22.1  54263    19
 11
## 2 ALEXA~ 0.0044  38206  72.8     42   25.3   80.6     13.3  51893     5
  4
## 3 ALLEG~ 0.0082  11387 100       49   26.1   81       18.6  45244     1
  9
## 4 ANSON  0.0357  25460  78.5     40   28     80.4      9.5  42500    48
  3
## 5 ASHE   0.0082  27418  84.9     47   26.8   83.3     19.5  47509     1
```

```
  5
## 6 AVERY  0.0162  17953  88.8     44    22.3    79       19.8  46516     4
  4
## # ... with 1 more variable: AbsBal <dbl>
```

```
print(summary(dataset))
```

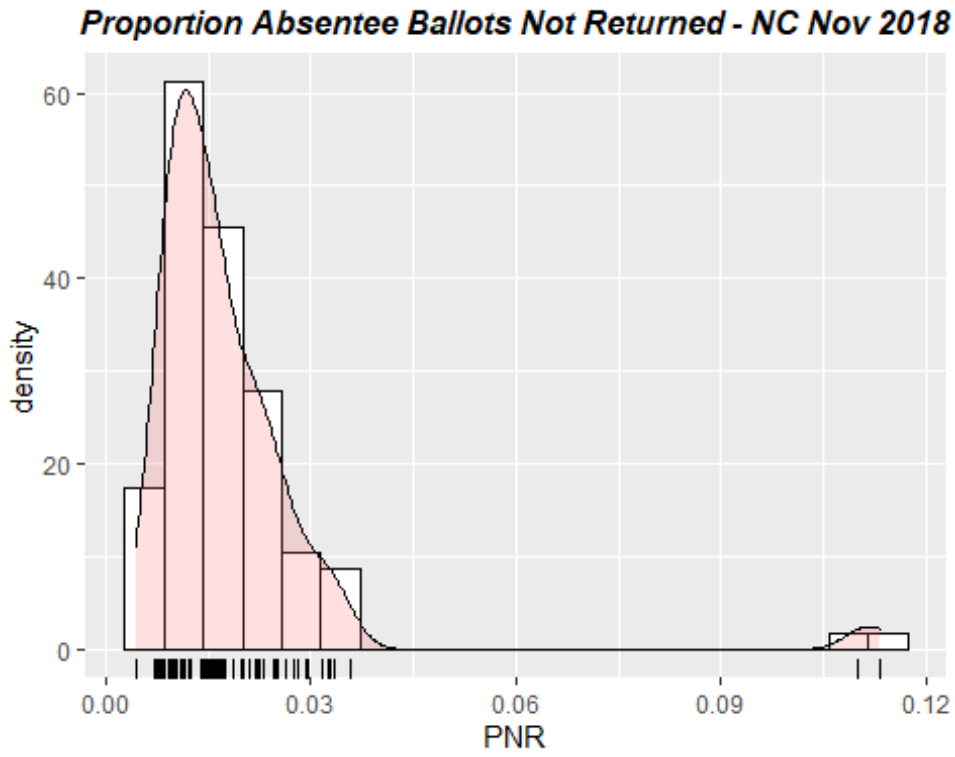
```
##     County               PNR                Pop               Rural        
##  Length:100         Min.   :0.00440   Min.   :   4310   Min.   :  1.10  
##  Class :character   1st Qu.:0.01105   1st Qu.:  25102   1st Qu.: 42.35  
##  Mode  :character   Median :0.01495   Median :  56534   Median : 62.25  
##                     Mean   :0.01815   Mean   : 102833   Mean   : 61.21  
##                     3rd Qu.:0.02133   3rd Qu.: 117801   3rd Qu.: 84.92  
##                     Max.   :0.11310   Max.   :1074596   Max.   :100.00  
##      MedAge          Travel          Hsgrad         Collgrad     
##  Min.   :26.00   Min.   :19.30   Min.   :72.00   Min.   : 8.20  
##  1st Qu.:40.00   1st Qu.:22.80   1st Qu.:80.28   1st Qu.:14.45  
##  Median :42.00   Median :24.35   Median :83.40   Median :18.80  
##  Mean   :41.96   Mean   :24.94   Mean   :83.30   Mean   :20.68  
##  3rd Qu.:45.00   3rd Qu.:26.80   3rd Qu.:87.22   3rd Qu.:23.65  
##  Max.   :51.00   Max.   :36.70   Max.   :92.50   Max.   :57.70  
##      MedInc          Black            Hisp           AbsBal      
##  Min.   :36958   Min.   : 0.00   Min.   : 1.00   Min.   :   532  
##  1st Qu.:46459   1st Qu.: 5.00   1st Qu.: 3.75   1st Qu.:  4284  
##  Median :51774   Median :18.50   Median : 6.00   Median :  9710  
##  Mean   :53305   Mean   :20.43   Mean   : 6.45   Mean   : 21118  
##  3rd Qu.:58652   3rd Qu.:32.25   3rd Qu.: 8.25   3rd Qu.: 21208  
##  Max.   :88887   Max.   :62.00   Max.   :21.00   Max.   :225409
```

Find the optimal model to predict PNR

```
#Summarize the PNR variable
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.6.3
```

```
ggplot(dataset, aes(x=PNR)) +
  geom_histogram(aes(y=..density..), bins = 20,fill = "white", col = "black")
 +
  geom_density(alpha=.2, fill="#FF6666") +
  geom_rug() +
  labs(title = 'Proportion Absentee Ballots Not Returned - NC Nov 2018') +
  theme(plot.title = element_text(hjust = 0.5, size=12, face="bold.italic"))
```




```
#Omitting Bladen and Robeson counties with weights
wts = dataset$PNR<0.1
#Extract PNR and other predicted variables
pnr.df = dataset[,2:11]
#Build up full model
full.lm = lm(formula = PNR ~ ., data = pnr.df, weights=as.numeric(wts))
print(summary(full.lm))
```

```
##
## Call:
## lm(formula = PNR ~ ., data = pnr.df, weights = as.numeric(wts))
##
## Weighted Residuals:
##       Min        1Q    Median        3Q       Max
## -0.011413 -0.003979 -0.001140  0.003621  0.018394
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.031e-02  2.230e-02  -1.359 0.177628
## Pop          9.990e-09  5.701e-09   1.752 0.083198 .
## Rural        4.073e-05  4.252e-05   0.958 0.340743
## MedAge      -1.727e-04  1.796e-04  -0.962 0.338798
## Travel       2.161e-04  2.954e-04   0.732 0.466414
## Hsgrad       5.177e-04  2.585e-04   2.003 0.048266 *
## Collgrad    -1.196e-04  1.753e-04  -0.682 0.496791
## MedInc      -1.397e-08  1.692e-07  -0.083 0.934397
## Black        1.825e-04  4.662e-05   3.913 0.000179 ***
```

```
## Hisp         1.969e-04  2.025e-04   0.973 0.333428
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.006275 on 88 degrees of freedom
## Multiple R-squared:  0.2739, Adjusted R-squared:  0.1996
## F-statistic: 3.688 on 9 and 88 DF,  p-value: 0.0005898
```

```
#Apply backward selection model
full.backward = step(full.lm, direction = "backward")
```

```
## Start:  AIC=-1003.02
## PNR ~ Pop + Rural + MedAge + Travel + Hsgrad + Collgrad + MedInc +
##     Black + Hisp
##
##            Df  Sum of Sq       RSS      AIC
## - MedInc    1 0.00000027 0.0034653 -1009.01
## - Collgrad  1 0.00001833 0.0034834 -1008.49
## - Travel    1 0.00002107 0.0034861 -1008.41
## - Rural     1 0.00003613 0.0035012 -1007.98
## - MedAge    1 0.00003642 0.0035014 -1007.97
## - Hisp      1 0.00003725 0.0035023 -1007.95
## <none>                   0.0034650 -1007.02
## - Pop       1 0.00012091 0.0035859 -1005.59
## - Hsgrad    1 0.00015796 0.0036230 -1004.56
## - Black     1 0.00060297 0.0040680  -992.98
##
## Step:  AIC=-1005.01
## PNR ~ Pop + Rural + MedAge + Travel + Hsgrad + Collgrad + Black +
##     Hisp
##
##            Df  Sum of Sq       RSS      AIC
## - Travel    1 0.00003364 0.0034989 -1010.05
## - MedAge    1 0.00003622 0.0035015 -1009.97
## - Hisp      1 0.00003713 0.0035024 -1009.95
## - Collgrad  1 0.00003877 0.0035041 -1009.90
## - Rural     1 0.00004073 0.0035060 -1009.84
## <none>                   0.0034653 -1009.01
## - Pop       1 0.00012087 0.0035862 -1007.58
## - Hsgrad    1 0.00016061 0.0036259 -1006.48
## - Black     1 0.00063826 0.0041036  -994.11
##
## Step:  AIC=-1006.05
## PNR ~ Pop + Rural + MedAge + Hsgrad + Collgrad + Black + Hisp
##
##            Df  Sum of Sq       RSS      AIC
## - MedAge    1 0.00003743 0.0035364 -1010.98
## - Hisp      1 0.00004405 0.0035430 -1010.80
## - Collgrad  1 0.00004682 0.0035458 -1010.72
## <none>                   0.0034989 -1010.05
```

```
## - Rural     1 0.00010637 0.0036053 -1009.05
## - Pop       1 0.00015082 0.0036498 -1007.83
## - Hsgrad    1 0.00019829 0.0036972 -1006.53
## - Black     1 0.00068003 0.0041790  -994.29
##
## Step:  AIC=-1006.98
## PNR ~ Pop + Rural + Hsgrad + Collgrad + Black + Hisp
##
##            Df  Sum of Sq       RSS      AIC
## - Collgrad  1 0.00005003 0.0035864 -1011.58
## <none>                   0.0035364 -1010.98
## - Rural     1 0.00007514 0.0036115 -1010.88
## - Hisp      1 0.00008153 0.0036179 -1010.70
## - Pop       1 0.00015908 0.0036954 -1008.58
## - Hsgrad    1 0.00020594 0.0037423 -1007.32
## - Black     1 0.00078865 0.0043250  -992.85
##
## Step:  AIC=-1007.58
## PNR ~ Pop + Rural + Hsgrad + Black + Hisp
##
##          Df  Sum of Sq       RSS      AIC
## - Hisp    1 0.00006453 0.0036509 -1011.79
## <none>                 0.0035864 -1011.58
## - Rural   1 0.00008404 0.0036704 -1011.26
## - Pop     1 0.00011727 0.0037037 -1010.36
## - Hsgrad  1 0.00016430 0.0037507 -1009.10
## - Black   1 0.00085991 0.0044463  -992.09
##
## Step:  AIC=-1007.79
## PNR ~ Pop + Rural + Hsgrad + Black
##
##          Df  Sum of Sq       RSS      AIC
## - Rural   1 0.00005063 0.0037015 -1012.42
## <none>                 0.0036509 -1011.79
## - Hsgrad  1 0.00011645 0.0037674 -1010.65
## - Pop     1 0.00016668 0.0038176 -1009.33
## - Black   1 0.00080052 0.0044514  -993.97
##
## Step:  AIC=-1008.42
## PNR ~ Pop + Hsgrad + Black
##
##          Df  Sum of Sq       RSS      AIC
## <none>                 0.0037015 -1012.42
## - Hsgrad  1 0.00007514 0.0037767 -1012.41
## - Pop     1 0.00011641 0.0038180 -1011.32
## - Black   1 0.00075157 0.0044531  -995.93
```

```
print(summary(full.backward))
```

```
##
## Call:
## lm(formula = PNR ~ Pop + Hsgrad + Black, data = pnr.df, weights = as.numer
ic(wts))
##
## Weighted Residuals:
##       Min        1Q    Median        3Q       Max
## -0.013330 -0.004575 -0.001610  0.003681  0.019052
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -7.239e-03  1.391e-02  -0.520   0.6041
## Pop          7.561e-09  4.398e-09   1.719   0.0888 .
## Hsgrad       2.277e-04  1.648e-04   1.381   0.1704
## Black        1.833e-04  4.195e-05   4.369 3.22e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.006275 on 94 degrees of freedom
## Multiple R-squared:  0.2243, Adjusted R-squared:  0.1996
## F-statistic: 9.061 on 3 and 94 DF,  p-value: 2.498e-05
```

```
#Apply forward selection model
full.forward <- step(lm(PNR ~ 1, data=pnr.df), list(upper=full.lm), direction
='forward')
```

```
## Start:  AIC=-837.59
## PNR ~ 1
##
##            Df  Sum of Sq      RSS     AIC
## + Black     1 0.00138120 0.021198 -841.90
## + MedAge    1 0.00072221 0.021857 -838.84
## <none>                   0.022580 -837.59
## + MedInc    1 0.00038528 0.022194 -837.31
## + Hsgrad    1 0.00034363 0.022236 -837.12
## + Pop       1 0.00023380 0.022346 -836.63
## + Hisp      1 0.00017338 0.022406 -836.36
## + Collgrad  1 0.00011878 0.022461 -836.12
## + Travel    1 0.00003466 0.022545 -835.74
## + Rural     1 0.00000129 0.022578 -835.59
##
## Step:  AIC=-841.9
## PNR ~ Black
##
##            Df  Sum of Sq      RSS     AIC
## <none>                   0.021198 -841.90
## + MedAge    1 0.00039665 0.020802 -841.79
## + Hisp      1 0.00020613 0.020992 -840.88
## + Pop       1 0.00020608 0.020992 -840.88
## + MedInc    1 0.00007419 0.021124 -840.25
```

```
## + Hsgrad    1 0.00003431 0.021164 -840.06
## + Travel    1 0.00002421 0.021174 -840.01
## + Rural     1 0.00000629 0.021192 -839.93
## + Collgrad  1 0.00000001 0.021198 -839.90
```

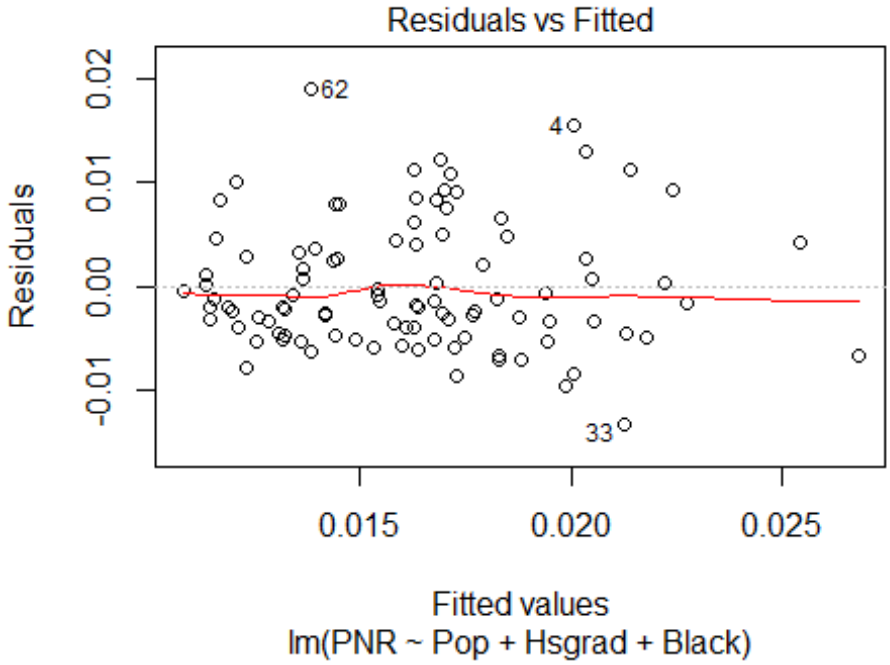
```
print(summary(full.forward))
```

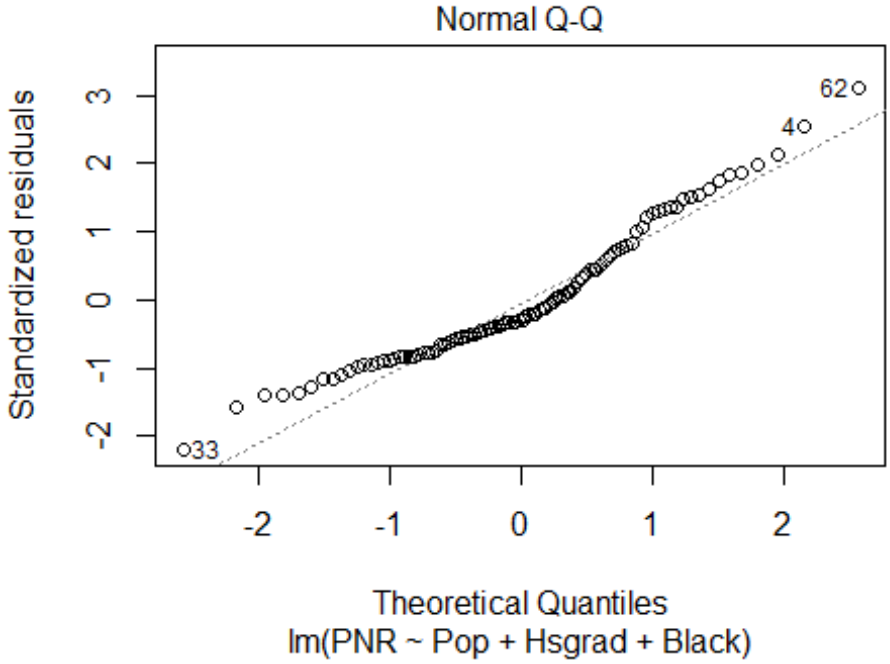
```
##
## Call:
## lm(formula = PNR ~ Black, data = pnr.df)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.018546 -0.005792 -0.003218  0.003136  0.091644
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.352e-02  2.351e-03   5.750 1.02e-07 ***
## Black       2.268e-04  8.977e-05   2.527   0.0131 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01471 on 98 degrees of freedom
## Multiple R-squared:  0.06117,    Adjusted R-squared:  0.05159
## F-statistic: 6.385 on 1 and 98 DF,  p-value: 0.01311
```

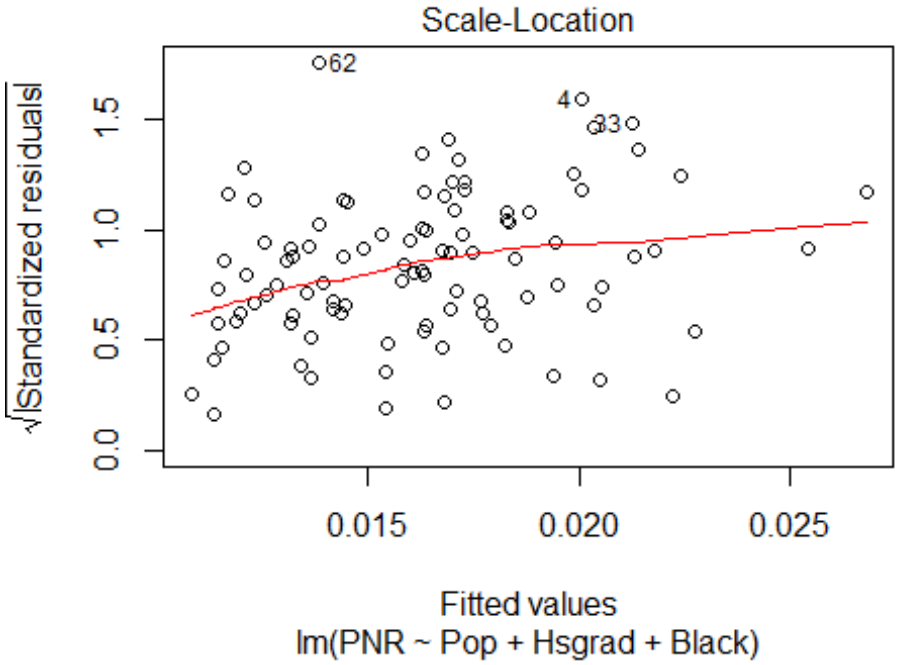
I apply both forward selection and backward selection to the full model, and get two different optimal models. The optimal model of backward selection is *PNR ~ Pop + Hsgrad + Black*, while the one of forward selection is *PNR ~ Black*.

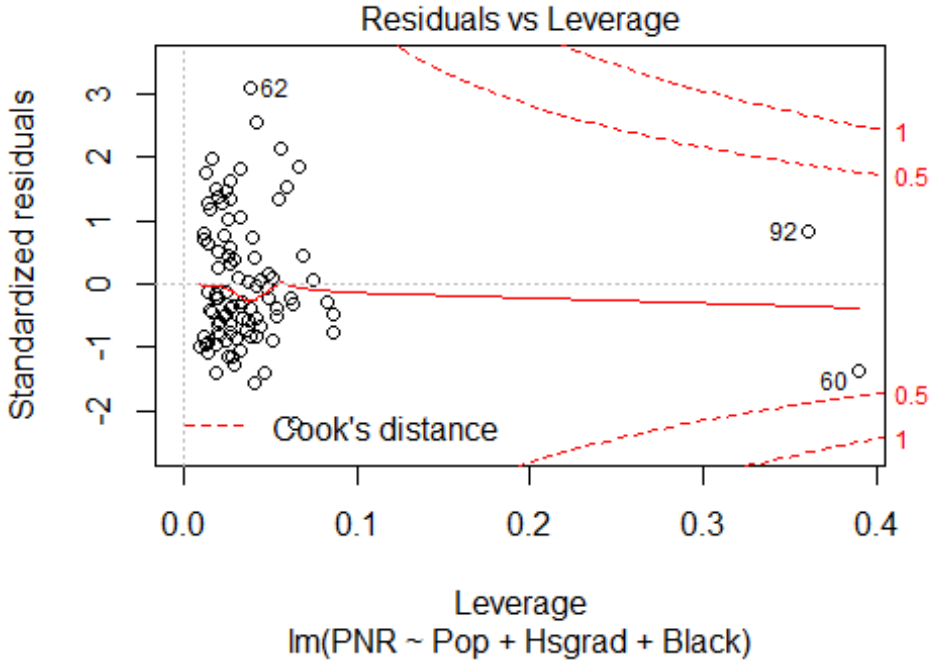
In the following step, I will use diagnostics to assess various measures of fit and choose the better model.

```
#Optimal model of backward selection
backward.lm = lm(formula = PNR ~ Pop + Hsgrad + Black, data = pnr.df, weights
 = as.numeric(wts))
#Diagnostics of backward optimal model
plot(backward.lm)
```


Residuals vs Fitted
Residuals
0.02
0.01
0.00
-0.01
62
4
33
0.015
0.020
0.025
Fitted values
lm(PNR ~ Pop + Hsgrad + Black)

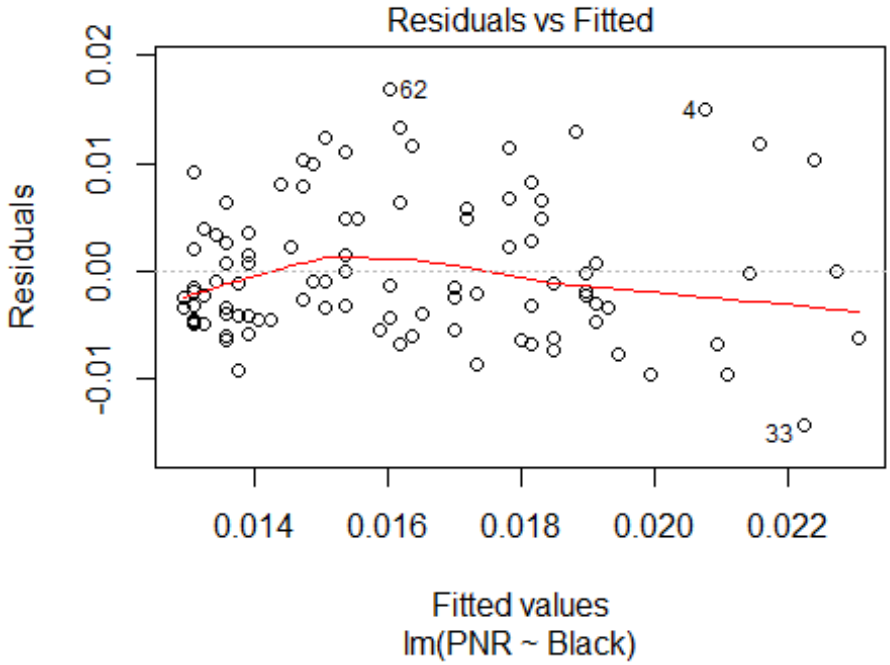

Normal Q-Q
Standardized residuals
3
2
1
0
-1
-2
62
4
33
-2
-1
0
1
2
Theoretical Quantiles
lm(PNR ~ Pop + Hsgrad + Black)

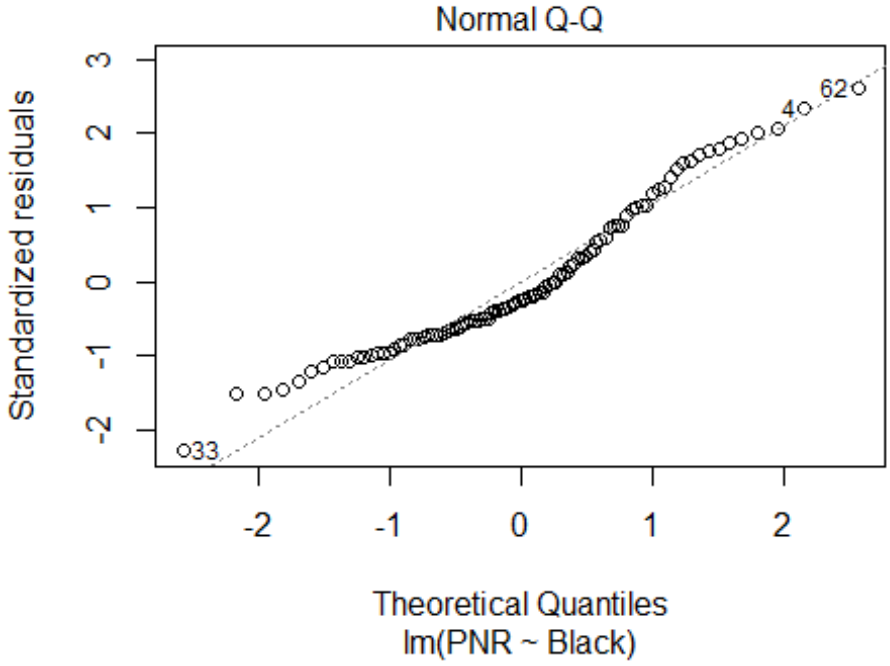


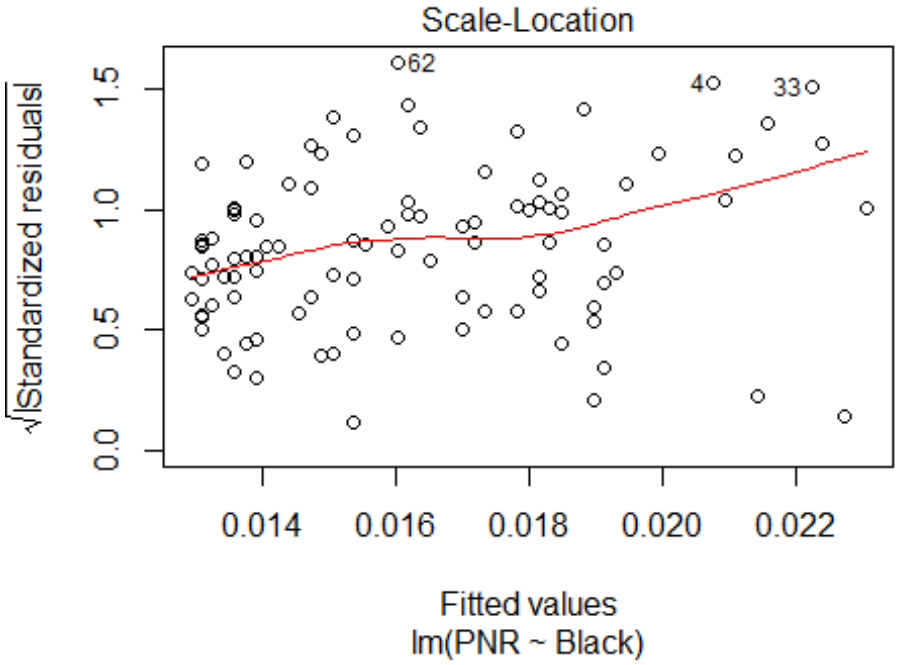


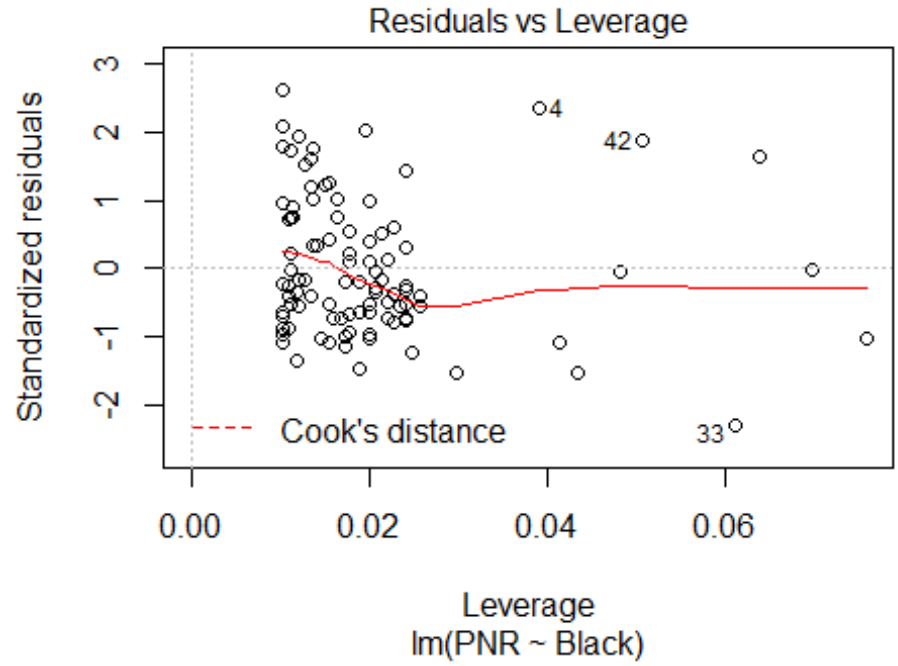

```r
#Optimal model of forward selection
forward.lm = lm(formula = PNR ~ Black, data = pnr.df, weights = as.numeric(wt
s))
```

```r
#Diagnostics of backward optimal model
plot(forward.lm)
```









Diagnosis Results: From the *Residuals vs. fitted values* plots of both models, we can see that both scatterplots distribute randomly, with some exception outliers in the bottom right corner, but the backward model's is more haphazard.

From the *Normal Probability plot* of both models, the errors of the backward model is more distributed normally, with a better straight line than the forward model.

From the *Scale Lotion* plots, both models show radom patterns.

From the *Cook's distance* plots, we can see there are some outliers affecting both models.

Thus, according to the results above, I will pick the backward model,*PNR ~ Pop + Hsgrad + Black*, as my optimal model for the future steps.

PNR prediction interval for Bladen and Robeson counties

```
#Choose backward model as the optimal model
opt.lm = backward.lm
#Predict based on 99% prediction interval
opt.pre.99=predict(opt.lm,se.fit=T,interval='prediction',level=0.99,weights=1)
```

```
## Warning in predict.lm(opt.lm, se.fit = T, interval = "prediction", level =
 0.99, : predictions on current data refer to _future_ responses
```

```
opt.pre.99$fit[c(9,78),]
```

```
##           fit           lwr        upr
## 9  0.01728530  0.0005795402 0.03399106
## 78 0.01555308 -0.0013119958 0.03241816
```

The 99% prediction interval of Bladen and Robeson are *(0.00058, 0.03399)*, and *(-0.00131, 0.03242)*.

Estimate excess PNR for Bladen and Robeson counties

```
excess.PNR=pnr.df[c(9,78),'PNR']-opt.pre.99$fit[c(9,78),'upr']
excess.PNR
```

```
##          PNR
## 1 0.07910894
## 2 0.07758184
```

The excee PNR of Bladen and Robeson are *0.079*, and *0.078* resepectively.

Estimate the total number of absentee ballots that are unaccounted for

```
total=sum(excess.PNR*dataset[c(9,78),'AbsBal'])
print(total)
```

```
## [1] 1888.236
```

The total number of absentee ballots that are unaccounted for is 1888.236.

Question The actual number of votes by which Mark Harris was leading at the time the count was stopped was 905. The Harris campaign responded to the allegations by asserting that the number of potentially missing votes was very small and certainly less than 905. Does your analysis support that conclusion - why or why not?

```
print(905/(905+total))
```

```
## [1] 0.323997
```

Answer Compared to the predicted missing votes of 1888, 905 only represents a minor group of people. According to my analysis, the valid votes only represent 32.40% of the whole. Thus, my analysis does not support Harris's conclusion.