$$f(y;\theta,\phi) = \exp\left\{ \frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi) \right\}$$

mean $b'(\theta)$, variance $b''(\theta)\,a(\phi)$

<u>Ex 1 Normal</u> $N(\mu,\sigma^2)$ $\quad f(y|\mu,\sigma^2) = \exp\left\{ \frac{y\mu}{\sigma^2} - \frac{\mu^2}{2\sigma^2} + \cdots \right\}$

$$\frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(y-\mu)^2}{2\sigma^2} \right\} \rightarrow$$

$\theta = \mu \quad a(\phi) = \sigma^2 \quad b(\theta) = \frac{\theta^2}{2} \quad b'(\theta) = \theta = \mu$

<u>Ex 2 Poisson</u> $f(y;\mu) = \frac{\mu^y e^{-\mu}}{y!} = \exp\{ y\log\mu - \mu - \log(y!) \}$

$\theta = \log\mu \quad b(\theta) = \mu = e^\theta \quad b'(\theta) = e^\theta = \mu$ | $y = 0, 1, 2, \ldots$

<u>Ex 3</u> ~~Bin~~ <u>Binomial</u> Let y be the <u>proportion</u> of successes in n trials with prob. of success $= p = \mu$

$$f(y;n,p) = \binom{n}{ny} p^{ny} (1-p)^{n-ny} \qquad y = 0, \frac{1}{n}, \frac{2}{n}, \ldots, 1$$

$$= \exp\left\{ ny\log p + n(1-y)\log(1-p) + \log\binom{n}{ny} \right\}$$

$$= \exp\left\{ ny\log\frac{p}{1-p} + n\log(1-p) + \cdots \right\}$$

$$= \exp\left\{ \frac{y\theta - b(\theta)}{\phi} + \cdots \right\}$$

$n = \frac{1}{\phi}$

$\theta = \log\frac{p}{1-p}$

$p = \frac{e^\theta}{1+e^\theta}$

$1-p = \frac{1}{1+e^\theta}$

$b(\theta) = +\log(1+e^\theta) \qquad b'(\theta) = \frac{e^\theta}{1+e^\theta} = 1 - \frac{1}{1+e^\theta}$ $(= p)$

$b''(\theta) = \frac{e^\theta}{(1+e^\theta)^2} = p(1-p)$

Mean $= b'(\theta) = p \qquad \mathrm{Var} = \phi b''(\theta) = \frac{p(1-p)}{n}$

$$f(y;\alpha,\beta) = \frac{\beta^{\alpha} y^{\alpha-1} e^{-\beta y}}{\Gamma(\alpha)} \qquad 0<y<\infty$$

mean $\frac{\alpha}{\beta}$ var. $\frac{\alpha}{\beta^2}$

Instead write $\nu = \alpha$, $\mu = \frac{\alpha}{\beta}$ mean μ, var. $\frac{\mu^2}{\nu}$

$$f(y;\mu,\nu) = \left(\frac{\nu}{\mu}\right)^{\nu} y^{\nu-1} e^{-\nu y/\mu} \cdot \frac{1}{\Gamma(\nu)}$$

$$= \exp\left\{ -\frac{\nu y}{\mu} - \nu \log\mu + \nu\log\nu + (\nu-1)\log y - \log\Gamma(\nu) \right\}$$

$\phi = \frac{1}{\nu}$ $\theta = \frac{1}{\mu}$ $a(\phi) = -\phi$ $-\frac{\nu y}{\mu} = \frac{y\theta}{a(\phi)}$ $-\nu\log\mu = \frac{-\log\theta}{a(\phi)}$

$$= \exp\left\{ + \frac{y-\theta}{a(\phi)} + c(y,\phi) \right\}$$

$b(\theta) = \log\theta$ $b'(\theta) = \frac{1}{\theta} = \mu$ $b''(\theta) = -\frac{1}{\theta^2} = -\mu^2$

variance of y: $b''(\theta)\cdot a(\phi) = (-\mu^2)\left(-\frac{1}{\nu}\right) = \frac{\mu^2}{\nu}$

Inverse Gaussian (Faraway Ch. 9)

$$f(y;\mu,\lambda) = \left(\frac{\lambda}{2\pi y^3}\right)^{\frac{1}{2}} \exp\left\{ -\frac{\lambda(y-\mu)^2}{2\mu^2 y} \right\} \qquad \begin{array}{l} y>0 \\ \lambda>0 \\ \mu>0 \end{array}$$

$$= \exp\left\{ -\frac{\lambda y}{2\mu^2} + \frac{\lambda}{\mu} \underbrace{- \frac{\lambda}{2y} + \frac{1}{2}\log\left(\frac{\lambda}{2\pi y^3}\right)}_{c(y,\phi)} \right\}$$

$\phi = \frac{1}{\lambda}$ $a(\phi) = -\phi$ $\theta = \frac{1}{2\mu^2}$

$\mu = (2\theta)^{-\frac{1}{2}}$ $b(\theta) = \frac{1}{\mu} = (2\theta)^{\frac{1}{2}}$ $b'(\theta) = 2^{\frac{1}{2}}\cdot\frac{1}{2}\theta^{-\frac{1}{2}} = (2\theta)^{-\frac{1}{2}}$

$b''(\theta) = -(2\theta)^{-3/2}$

mean $b'(\theta) = (2\theta)^{-\frac{1}{2}} = \mu$ $\cancel{b''(\theta)}$ = Mean

Variance is $b''(\theta)\,a(\phi) = \dfrac{(2\theta)^{-\frac{3}{2}}}{\lambda} = \dfrac{\mu^3}{\lambda}$

Louis Bachelier (1900)