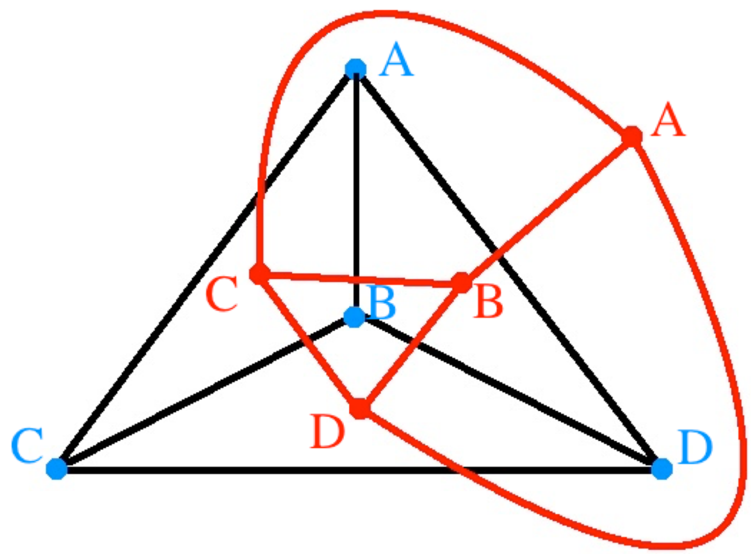
- Find a plane graph that is self-dual (you can do this by trial-and-error or by looking online; e.g., at Mathworld or Wikipedia, but let us know what method you use). A graph G is *self-dual* if it is isomorphic to its dual. Give the bijection between the vertices of the two graphs that demonstrates that they are isomorphic.



Based on the graph, the black edges with blue vertices are the original graph, and the red edge with red vertices are the self-dual expression. The corresponding relations are: **A->A, B->B, C->C, D->D**

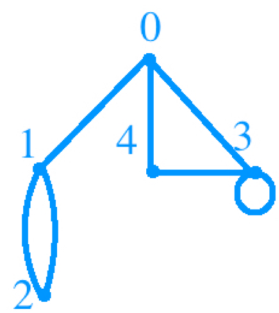
According to g = (2 – n + m – f) / 2, (n is the # of vertices, m is the # of edges, f is the # of faces)

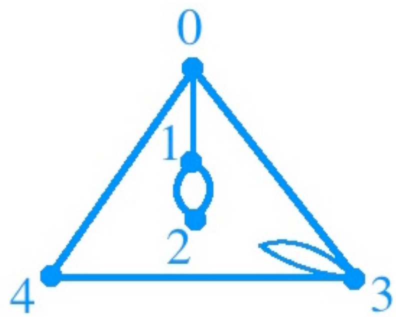
Original Graph(black, blue)	Self-dual(Red)
g = (2 – 4 + 6 – 4) / 2 = 0	g = (2 – 4 + 6 – 4) / 2 = 0

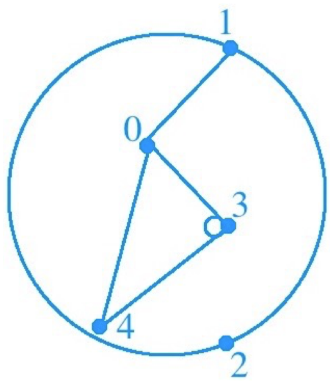
The two results are the same, which can demonstrate they are isomorphic. As both results of g are 0, which demonstrate they are both plane graph.

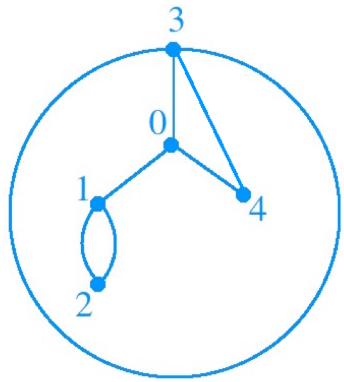
- Below is the rotation system of a plane graph. Draw all possible embeddings in the plane (i.e., each face should be an outer face in exactly one of the embeddings).

0: 4,1,3
1: 0,2,2
2: 1,1
3:0,4,3,3
4: 3,0



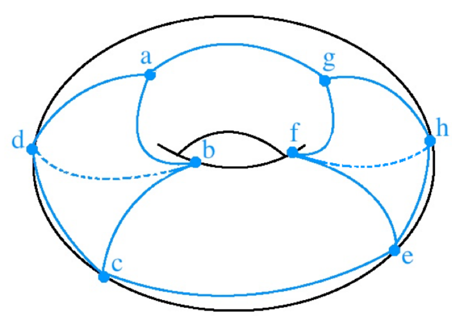






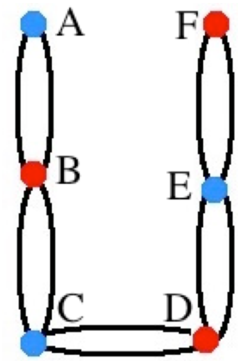
- Below is the rotation system for a graph embedded on a surface. What surface is it embedded on? Draw a nice diagram of the graph as embedded on this surface.

a: b,d,g
b: d,c,a
c: b,e,d
d: a,c,b
e: h,c,f
f: h,g,e
g: a,h,f
h: f,e,g



F0: (a, b) (b, d) (d, a) (a, g) (g, h) (h, f) (f, g) (g, a) (a, b)
F1: (a, d) (d, c) (c, b) (b, a) (a, d)
F2: (b, c) (c, e) (e, f) (f, h) (h, e) (e, c) (c, d) (d, b) (b, c)
F3: (e, h) (h, g) (g, f) (f, e) (e, h)
According to surface expression, there are 4 surfaces, 8 vertices, and 12 edges, that is to say, n = 8, m = 12, f = 4. And for g = (2 – n + m – f) / 2, g = (2 – 8 + 12 - 4) / 2 = 1. As the result is 1, thus, the graph should be a torus.

- Find an example of a biconnected bipartite non-Hamiltonian graph. Noting that any Hamiltonian bipartite graph must be *balanced* in the sense that the cardinalities of the partite sets must be equal, it is not hard to find an example graph that is unbalanced. Can you find one that is balanced? Explain why your graph has the required properties.



According to this graph, every red spot connects to a blue spot, thus, it is a bipartite graph. And if I cut one of the edge, for example, edge F to E, the whole graph is still connected, because there are two edges connected F and E, thus, it is a biconnected graph. And, as every vertices have two edges connect with other vertices, so I need to go trough vertices twice. Thus, it is a non-Hamiltonian graph.

- What happens to Bellman-Ford if there is a negative cycle on the path from s to v and then you call pathTo(v)?

  A negative cycle will effect shortest path when using Bellman-Ford algorithm. For example, the correct shortest path of the following graph is AB(weight is 3). However, if choosing negative cycle in this graph, the shortest path will be -4, which is against the rule that the shortest path in a graph cannot be a negative number. And if I call pathTo(v), there will be an error, which will throw exceptions.