1. **How many ways are there to arrange the letters in the word "probabilistic"?**

$$\frac{13!}{3! \times 2!}$$

2. **With reference to the previous problem, how many ways if all the b's have to precede all the i's?**

$$\binom{8+5}{5} * 8! = \binom{13}{5} * 8!$$

3. **How many ways are there to put 100 (unlabelled) balls into 50 labelled boxes?**

$$\binom{50-1+100}{100} = \binom{149}{100}$$

4. **We computed the "mean time to failure" to be 1/p if the probability of failure is p. What is the mean time to the second failure? In a sense, the answer is obvious, but prove it from first principles. That is, write down an expression for q_k, the probability that the second failure occurs on the k-th trial, and then compute and simplify the weighted sum $\sum kq_k$.**

   HINT: Recall that in class we computed the mean time to failure by determining p_k, the probability that the first failure occurs on the k-th trial, to be the quantity in Definition 19.4.6 of the MIT notes. We then simplified $\sum kp_k$, the expected number of trials, using the formula $(1-x)^{-2}=1+2x+3x^2+\cdots$, which we obtained by differentiating $(1-x)^{-1}=1+x+x^2+\ldots$

   $q_k = (1-p)^{k-1}p$, then $\sum_{k=1}(p)_k = \sum_{k\geq 1}(1-p)^{k-1}p = p\sum_{k\geq 0}(1-p)^k$

   $\sum_{k\geq 1} k(1-p)^{k-1}p = p\sum_{k\geq 1} k(1-p)^{k-1} = p \bullet \frac{1}{(1-(1-p))^2} = \frac{p}{p^2} = \frac{1}{p}$

5. **What number do you get if you subtract the binomial coefficients $\binom{n}{k}$ with an even k from those with an odd k, where n is fixed?**

   If n is even, then $\binom{n}{k}$ equal to $\binom{n-1}{k-1} + \binom{n-1}{k}$, and $\binom{n}{k+1}$ equal to $\binom{n-1}{k} + \binom{n-1}{k+1}$, thus the sum is 0.

And for n is odd, then k is odd, and n-k is even, thus $\binom{n}{n-k}$ equal to $\binom{n}{k}$, and $\binom{n}{n-k-1}$ = $\binom{n}{k+1}$, thus the sum is 0.
In conclusion, the sum is 0

6. **What difference of binomial coefficients is equal to the sum**

$$\binom{12}{5} + \binom{11}{5} + \binom{10}{5}+\binom{9}{5}+\binom{8}{5} ?$$

$$\binom{12}{5} + \binom{11}{5} + \binom{10}{5}+\binom{9}{5}+\binom{8}{5} = \binom{13}{6} - \binom{8}{6}$$

7. **Imagine a maze created in a m by n grid. Assume that there is a unique path from any cell to any other cell. What is the total length of the walls in the maze as a function of m and n? For example, below is a 2 by 2 grid with the required path property and the total length of walls is 9. Explain your answer.**

```
+----+----+
|         |
+----+    +
|         |
+----+----+
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The formula is (m+1)*(n+1). To be precise, assume it is a square, then if it is a 3 by 3 grid, then the total length of walls is 16, which is (3+1)*(3+1); if it is a 4 by 4 grid, then the total length of walls is 25, which is (4+1)*(4+1). Thus, the formula should be (m+1)*(n+1).

Assume it is a rectangle, then if it is a 3 by 2 grid, then the total length of walls is 12, which is (3+1)*(2+1); if it is a 4 by 3 grid, then the total length of walls is 20, which is (4+1)*(3+1). Thus, the formula should be m+1)*(n+1).