

# Solution For Ass1

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## 1 Solutions

### 1.1 Prob1.

If  $\det(A) = 0$ , then  $\det(A^T) = 0$ , therefore  $\det(A) = \det(A^T)$ .

If  $\det(A) \neq 0$ , then matrix has inverse. So  $A$  is a full rank matrix. Since every full rank matrix can be written as a product of elementary matrices:

$$R_1 \dots R_t I_n C_1 \dots C_s$$

where  $R_i$  are row elementary matrices and  $C_i$  are column elementary matrices. We know that for all elementary matrices,  $\det(E) = \det(E^T)$  (because column expansion or row expansion of  $\det$  doesn't change the value) and  $(A_1 \dots A_n)^T = A_n^T \dots A_1^T$ .  $\det(I_n) = \det(I_n^T) = 1$  Therefore,

$$\begin{aligned} \det(A^T) &= \det((R_1 \dots R_t I_n C_1 \dots C_s)^T) \\ &= \det(C_s^T \dots C_1^T I_n^T R_t^T \dots R_1^T) \\ &= \det(C_s^T) \dots \det(C_1^T) \det(I_n^T) \det(R_t^T) \dots \det(R_1^T) \\ &= \det(C_s) \dots \det(C_1) \det(I_n) \det(R_t) \dots \det(R_1) \\ &= \det(R_1) \dots \det(R_t) \det(I_n) \det(C_1) \dots \det(C_s) \\ &= \det(R_1 \dots R_t I_n C_1 \dots C_s) \\ &= \det(A) \end{aligned}$$

### 1.2 Prob 2.

For any  $n$ , an identity matrix  $A \in R^{n \times n}$  is a matrix only have element equal to 1 on diagonal and all other places are 0s

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

We assume that when  $A$  is an identity matrix of  $A \in R^{n \times n}$ ,  $\det(A) = \prod_{i=1}^n a_{ii} = 1$ . We prove this using induction.

When  $n = 1$ ,  $A$  is a scalar and  $\det(A) = a_{11} = 1$ .

When  $n > 1$ ,  $\det(A_{n-1}) = \prod_{i=1}^{n-1} a_{ii} = 1$ . When we consider the square matrix of size  $n$ , we expand the matrix along the first row. Since only the first element is 1 and all the other are 0s, we have:

$$\begin{aligned} & 1 \times \det(A_{n-1}) + 0 + \cdots + 0 \\ &= a_{11} \times \prod_{i=2}^n a_{ii} \\ &= \prod_{i=1}^n a_{ii} = 1 \end{aligned}$$

Therefore,  $\det(A) = 1, A \in R^{n \times n}$ .