

Solution For Ass1

Chang Li

July 27, 2016

1 Solutions

1.1 Prob1.

If $\det(A) = 0$, then $\det(A^T) = 0$, therefore $\det(A) = \det(A^T)$.

If $\det(A) \neq 0$, then matrix has inverse. So A is a full rank matrix. Since every full rank matrix can be written as a product of elementary matrices:

$$R_1 \dots R_t I_n C_1 \dots C_s$$

where R_i are row elementary matrices and C_i are column elementary matrices. We know that for all elementary matrices, $\det(E) = \det(E^T)$ (because column expansion or row expansion of \det doesn't change the value) and $(A_1 \dots A_n)^T = A_n^T \dots A_1^T$. $\det(I_n) = \det(I_n^T) = 1$ Therefore,

$$\begin{aligned} \det(A^T) &= \det((R_1 \dots R_t I_n C_1 \dots C_s)^T) \\ &= \det(C_s^T \dots C_1^T I_n^T R_t^T \dots R_1^T) \\ &= \det(C_s^T) \dots \det(C_1^T) \det(I_n^T) \det(R_t^T) \dots \det(R_1^T) \\ &= \det(C_s) \dots \det(C_1) \det(I_n) \det(R_t) \dots \det(R_1) \\ &= \det(R_1) \dots \det(R_t) \det(I_n) \det(C_1) \dots \det(C_s) \\ &= \det(R_1 \dots R_t I_n C_1 \dots C_s) \\ &= \det(A) \end{aligned}$$

1.2 Prob 2.

For any n , an identity matrix $A \in R^{n \times n}$ is a matrix only have element equal to 1 on diagonal and all other places are 0s

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

We assume that when A is an identity matrix of $A \in R^{n \times n}$, $\det(A) = \prod_{i=1}^n a_{ii} = 1$. We prove this using induction.

When $n = 1$, A is a scalar and $\det(A) = a_{11} = 1$.

When $n > 1$, $\det(A_{n-1}) = \prod_{i=1}^{n-1} a_{ii} = 1$. When we consider the square matrix of size n , we expand the matrix along the first row. Since only the first element is 1 and all the other are 0s, we have:

$$\begin{aligned} & 1 \times \det(A_{n-1}) + 0 + \cdots + 0 \\ &= a_{11} \times \prod_{i=2}^n a_{ii} \\ &= \prod_{i=1}^n a_{ii} = 1 \end{aligned}$$

Therefore, $\det(A) = 1$, when A is an identity matrix and $A \in R^{n \times n}$.