Ass5 Solutions

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1 Solutions

1.1 Linear Programming

1.2 Regularized Maximum Likelihood Estimation

1.2.1 a

Because dataset D is drawn from a Gaussian distribution with mean μ and covariance Σ . Suppose $x \in \mathbb{R}^n$, so the density is:

$$p(x) = (2\pi)^{-n/2} det(\Sigma)^{-1/2} exp(-(x-\mu)^T \Sigma^{-1}(x-\mu)/2)$$

The average log-likelihood function has the form:

$$l(D) = E(\log p(x_1, x_2, \dots, x_m))$$

= $E(-\log (2\pi) - \log det \Sigma - \sum_{i=1}^{M} (-(x_i - \mu)^T \Sigma^{-1} (x_i - \mu)))$

Since $(x - \mu)^T \Sigma^{-1}(x - \mu)$ is a number and $\operatorname{trace}(AB) = \operatorname{trace}(BA)$ therefore,

$$(x-\mu)^T \Sigma^{-1}(x-\mu) = \operatorname{tr}((x-\mu)^T \Sigma^{-1}(x-\mu)) = \operatorname{tr}(\Sigma^{-1}(x-\mu)^T (x-\mu))$$

Plugging into expectation we get:

$$E[(x - \mu)^{T} \Sigma^{-1} (x - \mu)] = E[tr((x - \mu)^{T} (x - \mu) \Sigma^{-1}])]$$

$$= tr(E[(x - \mu)^{T} (x - \mu) \Sigma^{-1}])$$

$$= tr(\hat{\Sigma} \Sigma^{-1})$$

Plugging into the third term of l(D) we can reformulate it as:

$$l(D) = -\log \det \Sigma - tr(\hat{\Sigma}\Sigma^{-1}) + const$$
$$= \log \det \Sigma^{-1} - tr(\hat{\Sigma}\Sigma^{-1}) + const$$

1.2.2 b

Because $K \succ 0$, we use the fact in section 3.1.5 [1] that the log of determinant is concave so its negation is convex and $\operatorname{tr}(\hat{\Sigma}K)$ is also convex. And the absolute value function and summation operation are also convex operations. Therefore the regularized maximum likelihood optimization problem is a summation of convex functions and henceTotal Variation Denoising convex.

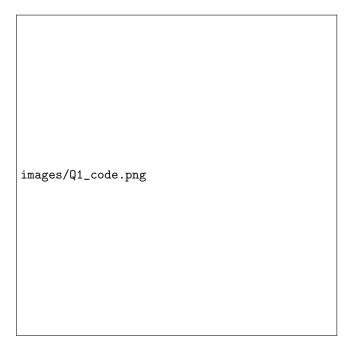


Figure 1: "fawe"

1.2.3 c

1.3 Total Variation Denoising

For this question we are looking for a convex optimization problem. You do not need to give a closed-form solution.

Bibliography

[1] S. Boyd and L. Vandenberghe, ${\it Convex~optimization}.$ Cambridge university press, 2004.