Solution For Ass1

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1 Solutions

1.1 Prob1.

If det(A) = 0, then $det(A^T) = 0$, therefore $det(A) = det(A^T)$. If $det(A) \neq 0$, then matrix has inverse. So A is a full rank matrix. Since every full rank matrix can be written as a product of elementary matrices:

$$R_1 \dots R_t I_n C_1 \dots C_s$$

where R_i are row elementary matrices and C_i are column elementary matrices. We know that for all elementary matrices, $det(E) = det(E^T)$ (because column expansion or row expansion of det doesn't change the value) and $(A_1
ldots A_n^T
ldots det(I_n) = det(I_n^T) = 1$ Therefore,

$$det(A^{T}) = det((R_{1} \dots R_{t}I_{n}C_{1} \dots C_{s})^{T})$$

$$= det(C_{s}^{T} \dots C_{1}^{T}I_{n}^{T}R_{t}^{T} \dots R_{1}^{T})$$

$$= det(C_{s}^{T}) \dots det(C_{1}^{T})det(I_{n}^{T})det(R_{t}^{T}) \dots det(R_{1}^{T})$$

$$= det(C_{s}) \dots det(C_{1})det(I_{n})det(R_{t}) \dots det(R_{1})$$

$$= det(R_{1}) \dots det(R_{t})det(I_{n})det(C_{1}) \dots det(C_{s})$$

$$= det(R_{1} \dots R_{t}I_{n}C_{1} \dots C_{s})$$

$$= det(A)$$

1.2 Prob 2.

For any n, an identity matrix $A \in \mathbb{R}^{n \times n}$ is a matrix only have element equal to 1 on diagonal and all other places are 0s

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

We assume that when A is an indentity matrix of $A \in \mathbb{R}^{n \times n}$, $det(A) = \prod_{i=1}^{n} a_{ii} = 1$. We prove this using induction.

When n = 1, A is a scalar and $det(A) = a_{11} = 1$.

When n > 1, $det(A_{n-1}) = \prod_{i=1}^{n-1} a_{ii} = 1$. When we consider the square matrix of size n, we expand the matrix along the first row. Since only the first element is 1 and all the other are 0s, we have:

$$1 \times det(A_{n-1}) + 0 + \dots + 0$$
$$= a \cdot 11 \times \prod_{i=2}^{n} a_{ii}$$
$$= \prod_{i=1}^{n} a_{ii} = 1$$

Therefore, det(A)=1, when A is an identity matrix and $\mathbf{A} \in \mathbb{R}^{n \times n}$.