Solution For Ass1

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1 Solutions

1.1 Prob1.

If $det(A) = 0$, then $det(A^T) = 0$, therefore $det(A) = det(A^T)$.
If $det(A) \neq 0$, then matrix has inverse. So A is a full rank matrix. Since every full rank matrix can be written as a product of elementary matrices:

$$R_1 \dots R_t I_n C_1 \dots C_s$$

where R_i are row elementary matrices and C_i are column elementary matrices. We know that for all elementary matrices, $det(E) = det(E^T)$ (because column expansion or row expansion of det doesn't change the value) and $(A_1 \dots A_n)^T = A_n^T \dots A_1^T$. $det(I_n) = det(I_n^T) = 1$ Therefore,

$$\begin{aligned}
det(A^T) &= det((R_1 \dots R_t I_n C_1 \dots C_s)^T) \\
&= det(C_s^T \dots C_1^T I_n^T R_t^T \dots R_1^T) \\
&= det(C_s^T) \dots det(C_1^T) det(I_n^T) det(R_t^T) \dots det(R_1^T) \\
&= det(C_s) \dots det(C_1) det(I_n) det(R_t) \dots det(R_1) \\
&= det(R_1) \dots det(R_t) det(I_n) det(C_1) \dots det(C_s) \\
&= det(R_1 \dots R_t I_n C_1 \dots C_s) \\
&= det(A)
\end{aligned}$$

1.2 Prob 2.

For any n, an identity matrix $A \in R^{n \times n}$ is a matrix only have element equal to 1 on diagonal and all other places are 0s

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

We assume that when A is an indentity matrix of $A \in R^{n \times n}$, $det(A) = \prod_{i=1}^{n} a_{ii} = 1$. We prove this using induction.

When $n = 1$, A is a scalar and $det(A) = a_{11} = 1$.

When $n > 1$, $det(A_{n-1}) = \prod_{i=1}^{n-1} a_{ii} = 1$. When we consider the square matrix of size n, we expand the matrix along the first row. Since only the first element is 1 and all the other are 0s, we have:

$$\begin{aligned}
&1 \times det(A_{n-1}) + 0 + \cdots + 0 \\
&= a11 \times \prod_{i=2}^{n} a_{ii} \\
&= \prod_{i=1}^{n} a_{ii} = 1
\end{aligned}$$

Therefore, $det(A) = 1, when \mathrm{A} is an identity matrix and A \in R^{n \times n}$.