Vector Field

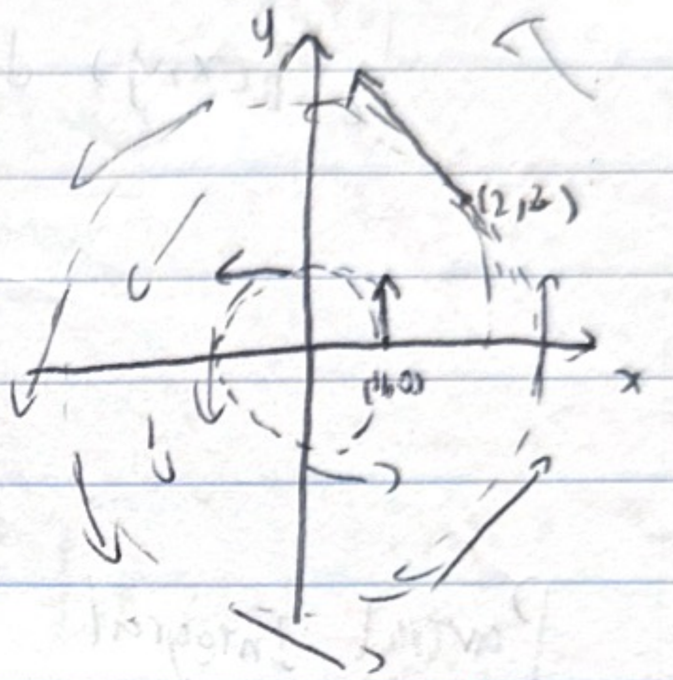
def: F that assign each point (x, y) in D a two-D vector $F(x, y)$

$\overrightarrow{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$

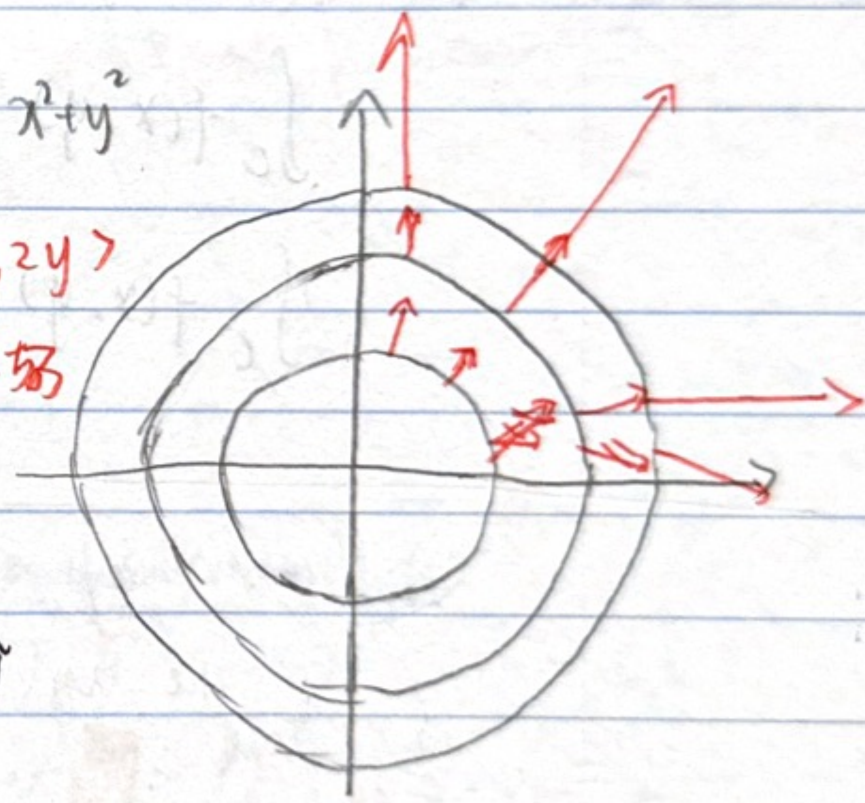
eg. $F(x,y) = -y\,i + x\,j$

~~(1, 0)~~

(x, y)	$F(x,y)$
$(1, 0)$	$(0, 1)$
$(2, 2)$	$(-2, 2)$
$(3, 0)$	$(0, 3)$
...	...



Scatch:



potential function: $f(x,y) = x^2 + y^2$

conservative vector field $\overrightarrow{F} = \nabla f = \langle 2x, 2y \rangle$

f 最快变化向量由 梯度向量场给出

Gradient Field: - a special vector field

$\nabla f(x,y) = f_x(x,y)\,i + f_y(x,y)\,j$

$\overrightarrow{F} = \nabla f = \langle f_x, f_y \rangle$

if $F = \nabla f$, F (vector field) is called conservative vector field 保守向量场
f is the potential function 势函数

eg. $F = i$

$\nabla f(x,y) = \langle 1, 0 \rangle \qquad f(x,y) = x + C$

eg. Find potential function for $\overrightarrow{F} = \langle y^2 + yz,\ 2xy + xz + 2yz,\ xy + y^2 \rangle$

$f_x(x,y,z) = xy^2 + xyz + g(y,z)$

~~$f(x,y,z) = y^2 z$~~ $\quad g(y,z) = zy^2 + h(z)$

$h(z) = C$

$f(x,y,z) = xy^2 + xyz + y^2 z + C$