

Find Column Space and Row Space, Basis, and Dimension

Find column space and row space

eg. $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \\ 3 & 5 \end{bmatrix}$

1. Find REF

$$A \rightarrow \begin{bmatrix} 1 & 4 \\ 0 & -1 \\ 0 & -7 \end{bmatrix}$$

2. Column space is the span of pivot columns (original)

$$\text{col } A = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix} \right\}, \text{ a plane space in } \mathbb{R}^3$$

$$\beta_{\text{col } A} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix} \right\} \quad \dim(\text{col } A) = 2$$

3. row space is the span of pivot rows (original)

$$\text{row } A = \text{span} \left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \end{pmatrix} \right\}, \mathbb{R}^2$$

$$\beta_{\text{row } A} = \left\{ \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \end{pmatrix} \right\}$$

$$\dim(\text{row } A) = 2$$

$$\text{col } A = \text{row } A$$

$$\dim(\text{col } A) = \dim(\text{row } A) = \text{rank}(A)$$

Basis = set of vector span V , minimal generating set V

eg. $\text{col } A = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix} \right\}$

$$\beta = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix} \right\}$$

all $\vec{v} \in V$ can be expressed as linear combination of basis uniquely

$$\beta_{M_{2 \times 2}} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Dimension = number of vectors in basis β

$$\dim(\text{col } A) = r$$

Dimension : number of vectors in basis $\{^3$

$$\dim(\text{Col } A) = r$$

$$\dim(\text{Row } A) = r = \dim(\text{Col } A^T)$$

$$\dim(\text{Nul } A) = n - r$$

$$\dim(\text{Nul } A^T) = m - r$$