

minor M_{ij} the determinant of submatrix does not include the row and column of a_{ij}

$$\text{cofactor } C_{ij} = (-1)^{i+j} M_{ij}$$

Use cofactor Expansion to find determinants

for matrix $A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$

along row 1, $\det(A) = a_{11}C_{11} + a_{12}C_{12} + \dots + a_{1n}C_{1n}$

along column 1: $\det(A) = a_{11}C_{11} + a_{21}C_{21} + \dots + a_{n1}C_{n1}$

$$= a_{11}M_{11} - a_{21}M_{21} + \dots + (-1)^{1+n} a_{n1}M_{n1}$$

Use cofactor to find the inverse of matrix

$$A^{-1} = \frac{1}{\det A} C^T$$

A is $n \times n$ matrix

C is the matrix of cofactors of element in A

eg. find the inverse of $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$C = \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$$

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$$