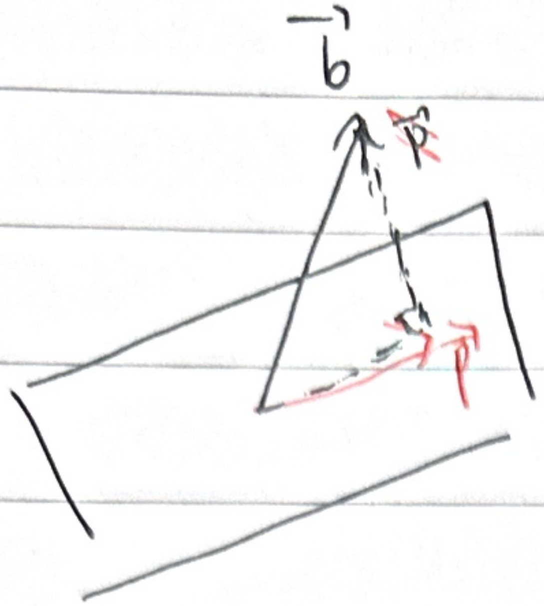
Find projection $\vec{b}$ on ~~$\vec{x}$~~ A (span(col(A))



There exist $\vec{x}$ such that

$$A\vec{x} = \vec{p}$$

$$A^T(\vec{b} - A\vec{x}) = \vec{0}$$

$$A^TA\vec{x} = A^T\vec{b}$$

If A^TA invertible (all linear independent case of A)

$$\vec{x} = (A^TA)^{-1}A^T\vec{b}$$

$$\vec{p} = A\vec{x} = A(A^TA)^{-1}A^T\vec{b}$$

$$= P\vec{b}$$

projection matrix $P = A(A^TA)^{-1}A^T\cancel{\vec{b}}$

Prove $P_A\vec{b} = P_{\vec{a_1}}\vec{b} + P_{\vec{a_2}}\vec{b} + P_{\vec{a_3}}\vec{b} + \cdots P_{\vec{a_n}}\vec{b}$