

# Vector Space and REF

★ Given Vector Space  $V$ , then  $W \subset V$  is a subspace if

1.  $\vec{0} \in W$
2.  $\vec{w}_1 + \vec{w}_2 \in W$  if  $\vec{w}_1, \vec{w}_2 \in W$
3.  $c\vec{w} \in W$  if  $\vec{w} \in W$  and  $c \in \mathbb{R}$

check subspace

$$A = \langle \vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n \rangle$$

$$\text{col } A = \langle A\vec{c} \mid \vec{c} \in \mathbb{R}^n \rangle$$

★ Null Space

consist of all solutions  $\vec{x}$  to  $A\vec{x} = \vec{0}$

$$\text{Nul } A = \{ \vec{x} \mid A\vec{x} = \vec{0} \}$$

REF (row-echelon form)  $\left\{ \begin{array}{l} \text{all zero rows beneath other rows} \\ \text{All pivot must lie to right of any pivot above it} \end{array} \right.$

pivot column: a column exactly 1 pivot  
free column: a column with no pivot

eg.  $\begin{bmatrix} \textcircled{1} & 0 & -1 & 1 \\ 0 & 0 & \textcircled{1} & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  pivot

Reduced row-echelon form (RREF)

1. get REF

2. make pivot 1

3. make element above pivot 0