Find λ-eigenspace

$$Z_\lambda = \{ \vec{v} \mid A\vec{v} = \lambda \vec{v} \}$$

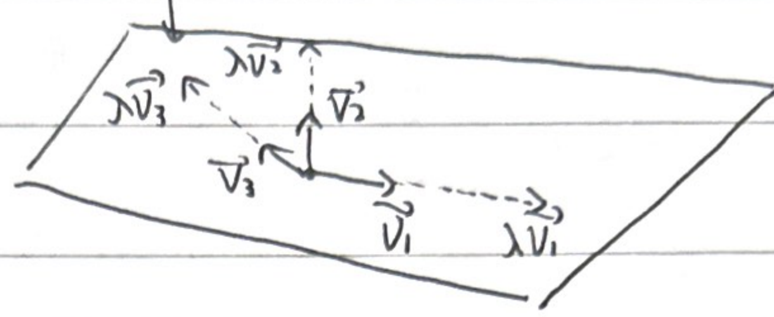
$$= \{ \vec{v} \mid (A - \lambda I)\vec{v} = 0 \}$$

$$= Nul(A - \lambda I)$$

λ: eigenvalue - extend multiplier

$\vec{v}$: eigenvector - vector can be extended

Z_λ: λ-eigenspace



Generate eigenvalue by Finding Characteristic Polynomial

eigenvector $\vec{x}$ in $Nul(A-\lambda I)$

eigenvalue λ satisfies $\det(A - \lambda I) = 0$

reason: ① if $\vec{x} = \vec{0}$ is the only solution, $(A-\lambda I)$ is linearly independent, no eigenvalue

② if $\vec{x} \neq \vec{0}$, then $(A - \lambda I)$ has free columns, so $\det(A - \lambda I) = 0$

$p(\lambda) = \det(A - \lambda I)$ Characteristic Polynomial $p(\lambda)$

$p(\lambda) = 0 \Rightarrow$ get λ

eg. given $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Find the eigenvalues and eigenvectors

step 1. eigenvalues: $p(\lambda) = |A - \lambda I|$

$$= \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix}$$

step 2. $p(\lambda) = 0$

$\lambda_1 = 2$ and $\lambda_2 = 4$ ← eigenvalues

step 3. Find eigenvectors: $Nul(A - \lambda I)$ by case

$\lambda = 2$: $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $x_1 = -x_2$, x_2 free column $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} x_2$

$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is eigenvector, $span\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$ eigenspace