# Diagonalisation

1. $A$ is similar to $B$ if there's invertible matrix such that $A = PBP^{-1}$

$$A = PBP^{-1} \Leftrightarrow P^{-1}AP = (P^{-1}P)B(P^{-1}P) \Leftrightarrow B = P^{-1}AP$$

2. If $A$, $B$ are similar, they have same characteristic polynomial and eigenvalue

prove: $B = P^{-1}AP$

$$B - \lambda I = P^{-1}AP - \lambda P^{-1}P = P^{-1}(AP - \lambda P) = P^{-1}(A - \lambda I)P$$

$$\det(B - \lambda I) = \det(P^{-1})\det(A - \lambda I)\det(P) = \det(A - \lambda I)$$

3. Why diagonalised? To find $A^K$

$$A^K = X\Lambda(X^{-1}X)\Lambda(X^{-1}X)\Lambda(X^{-1} \cdots \cdots X)\Lambda X^{-1} = X\Lambda^K X^{-1}$$

4. Diagonalize the matrix $(n \times n)$

step 1. Find eigenvalues $c_1, c_2 \cdots c_m$

step 2: Find eigenvectors $\vec{v_1}, \vec{v_2}, \cdots \vec{v_n}$ $\Rightarrow \begin{cases} \text{if } n \text{ linearly independent} \Rightarrow \text{diagonalizable} \\ \text{if linearly dependent} \Rightarrow \text{not diagonalizable} \end{cases}$

step 3: $X = [\vec{v_1}, \vec{v_2}, \cdots \vec{v_n}]$ $\quad \Lambda = \begin{bmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_m \end{bmatrix}$

$$A = X\Lambda X^{-1}$$