

Complete Solution

Complete Solution:

A is $m \times n$, $m < n$, will have free variables with free columns
infinite solution or no solution

Find Complete Solution

eg.
$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 4 & 8 \\ 4 & 8 & 6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}$$

1. Get RREF form

$$[A|B] \Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 0 & -4 & 7 \\ 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

2. Get System of equations

$$\begin{cases} x_1 + 2x_2 - 4x_4 = 7 \\ x_3 + x_4 = -3 \end{cases}$$

3. Get x in form of free variable

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 + 4x_4 - 2x_2 \\ x_2 \\ -3 - x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ -3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$\left\{ \begin{array}{l} \text{particular solution } A\vec{x}_p = \vec{b} \\ \text{null space solution } A\vec{x}_n = \vec{0} \end{array} \right.$

the complete solution $A\vec{x} = \vec{b}$ where $\vec{x} = \vec{x}_p + \vec{x}_n$