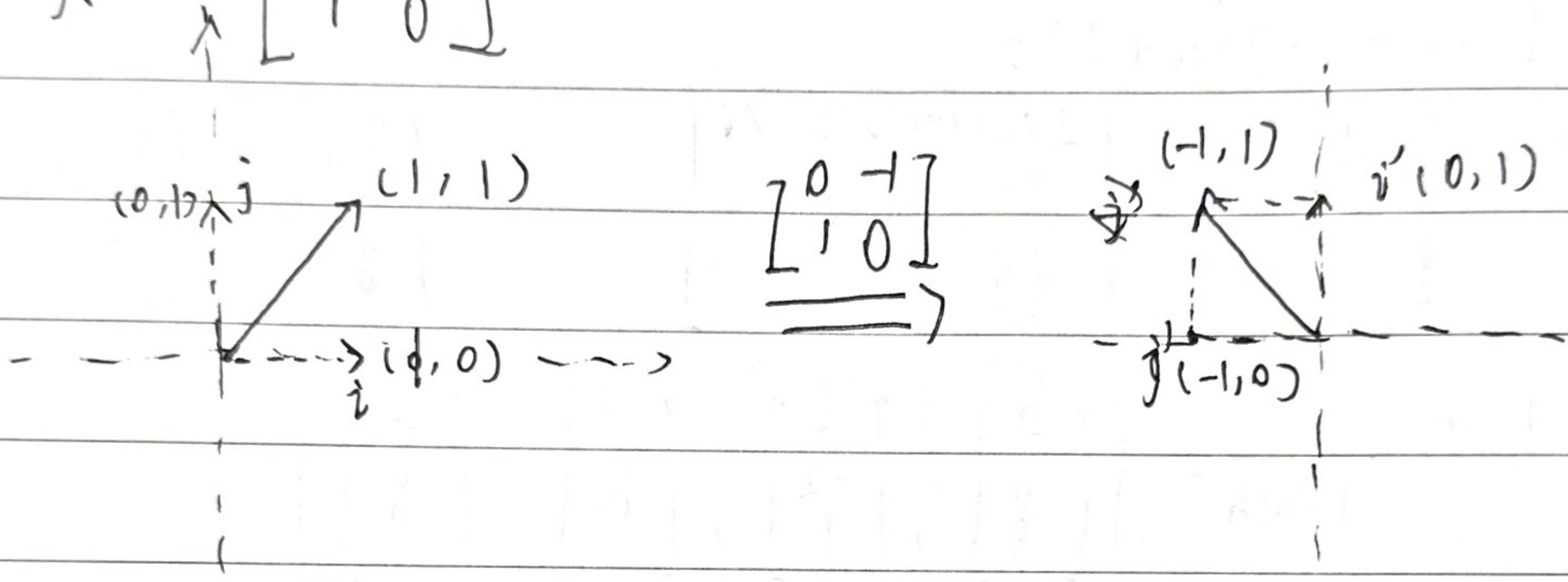
# Essense of Linear Algebra

1. $A\vec{x} = \vec{b}$

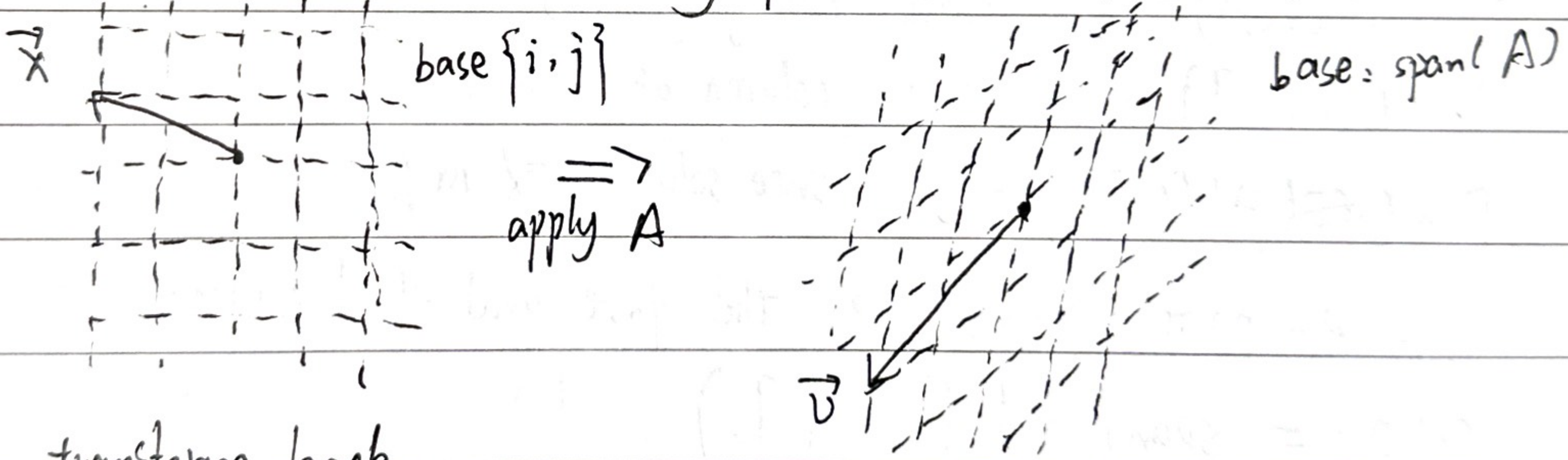
$$\begin{bmatrix} \vec{v_1}, \vec{v_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{v_1} x_1 + \vec{v_2} x_2$$

base change from $\overset{i}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}, \overset{j}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$ to $\vec{v_1}, \vec{v_2}$

eg. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

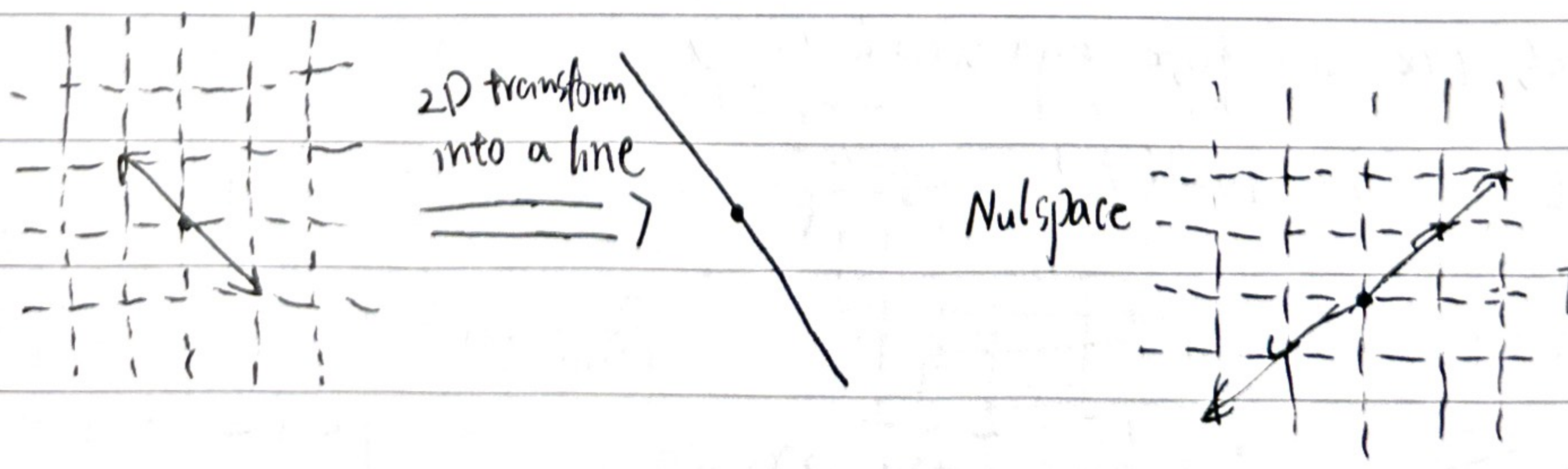



2. solve $A\vec{x} = \vec{b}$ means looking for $\vec{x}$ that after transformation is $\vec{b}$




3. $A^{-1}$, transform back

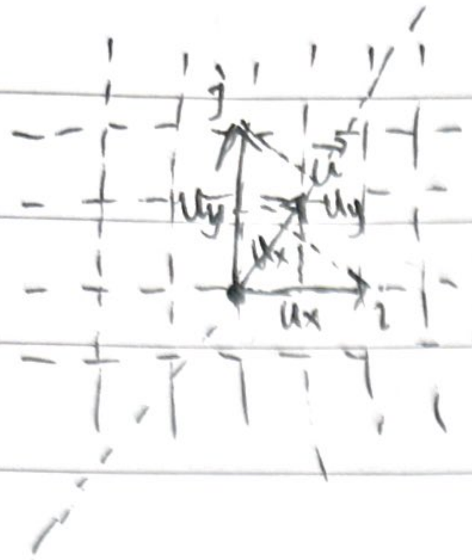
4. Nulspace: space of all vectors that squished onto the $\vec{0}$ vector after ~~the~~ transformation




Since line in difference directions full of vectors squished onto the origin

5. dot produce

to find dot produce of $\vec{u}$ and other vector



see $\vec{u}$ as a linear transformation resule

$A = [u_x, u_y]$

(by symetric, $\hat{i}$ transform to $u_x$, $\hat{j}$ transform to $u_y$)

$A\vec{x}$ means transform all vectors onto the basis of $\vec{u}$

performing the linear transformation is the same as taking a dot produce with $\vec{u}$

6. basis transformation

normal: $\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} \xrightarrow[\text{to}]{\text{transform}} \begin{bmatrix}a_1\\a_2\end{bmatrix}\begin{bmatrix}b_1\\b_2\end{bmatrix}$ $\quad \vec{b} = \begin{bmatrix}a_1 & b_1\\a_2 & b_2\end{bmatrix}\vec{x}$

$\xleftarrow[\text{back}]{\text{transform}}$ $\quad \vec{x} = \begin{bmatrix}a_1 & b_1\\a_2 & b_2\end{bmatrix}^{-1}\vec{b}$

Show changes in normal bases reflected on the transformed basis

change: $\begin{bmatrix}c_1 & c_3\\c_2 & c_4\end{bmatrix} \xrightarrow{\text{reflect}} \begin{bmatrix}a_1 & b_1\\a_2 & b_2\end{bmatrix}^{-1}\begin{bmatrix}c_1 & c_3\\c_2 & c_4\end{bmatrix}\begin{bmatrix}a_1 & b_1\\a_2 & b_2\end{bmatrix}$

7. eigen vector: $\vec{v}$ ~~such that~~ that just expand after linear transformation

If transformation have lots of eigenvectors: eg. $\begin{bmatrix}3 & 1\\0 & 2\end{bmatrix}$ has eigenvectors $\begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}$

Use eigenvectors to change basis matrix: $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \rightarrow \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$

change reflect on the changed basis: $\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}^{-1}\begin{bmatrix}3 & 1\\0 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$

resule ~~diagona~~ must be diagonal matrix with eigen value on diagonal since basis vector get scaled during transformation: $\begin{bmatrix}3 & 0\\0 & 2\end{bmatrix}$

diagonalize: $A = X \Lambda X^{-1}$

($X$ eigenvector basis, $\Lambda$ extend rate (eigenvalue))

transform from eigen basis back to normal base, apply change on eigenbasis, transform back to eigenbasis

$A$: change on normal basis, $\Lambda$: change reflect on eigen basis