1. Symmetric Matrix $A = A^T$

* If matrix has 2 (or more) distinct eigenvalues, those eigenvectors are orthogonal

eg. $\begin{cases} A\vec{x} = \lambda_1 \vec{x} \\ A\vec{y} = \lambda_2 \vec{y} \end{cases} \Rightarrow \begin{cases} \vec{y}^T A\vec{x} = \lambda_1 \vec{y}^T \vec{x} \text{ ①} \\ \vec{x}^T A\vec{y} = \lambda_2 \vec{x}^T \vec{y} \text{ ②} \end{cases} \xrightarrow[\text{transpose}]{\text{① take}} \begin{cases} \vec{x}^T A\vec{y} = \lambda_1 \vec{x}^T \vec{y} \\ \vec{x}^T A\vec{y} = \lambda_2 \vec{x}^T \vec{y} \end{cases}$

$\Rightarrow 0 = (\lambda_1 - \lambda_2)\vec{x}^T \vec{y}$ since $\lambda_1 \neq \lambda_2$ $\vec{x}^T \vec{y} = 0$ $\vec{x} \perp \vec{y}$

different eigenspaces are orthogonal

If eigenvalues are orthonormal

$A = X \Lambda X^{-1} = X \Lambda X^T$

2. Positive definite matrix ($\vec{x}$ is stretch positively $(0, +\infty)$)

① $A = A^T$ (A is symmetric)

② all eigenvalues are positive

$\Leftrightarrow$ all pivot are positive after RZF

$\Leftrightarrow$ all upper left determinants are positive

$\Leftrightarrow$ $x^T A \vec{x}$ is positive for all $\vec{x} \neq 0$

eg. $\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$ $\det([a]) > 0$, $\det\left(\begin{bmatrix} a & d \\ b & e \end{bmatrix}\right) > 0$, $\det\left(\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}\right) > 0$

if A is positive definite, so is A^{-1}

prove: $A\vec{x} = \lambda \vec{x} \Rightarrow A^{-1}A\vec{x} = A^{-1}\lambda \vec{x} \Rightarrow A^{-1}\vec{x} = \frac{1}{\lambda}\vec{x}$