Complete Solution

Complete Solution:

A is $m \times n$, $m < n$, will have free ids with free columns

infinite solution or no solution

Find Complete Solution

eg. $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 4 & 8 \\ 4 & 8 & 6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}$

1. Get RREF form of

$[A|B] \Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

2. Get System of equation

$\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ x_3 + x_4 = -3 \end{cases}$

3. Get x

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 - 2x_2 - x_3 \\ x_2 \\ x_3 \\ -3 - x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

particular solution | null space solution

particular solution $A\vec{x}_p = \vec{b}$

null space solution $A\vec{x}_n = \vec{0}$

the complete solution $A\vec{x} = \vec{b}$ where $\vec{x} = \vec{x}_p + \vec{x}_n$