Linear Transformation

$$\begin{cases} T(\vec{v_1}+\vec{v_2}) = T(\vec{v_1}) + T(\vec{v_2}) \\ T(a\vec{v_1}) = aT(\vec{v_1}) \end{cases}$$

$$T(\vec{v}) = T(c_1\vec{v_1}) + T(c_2\vec{v_2}) + T(c_3\vec{v_3}) + T(\vec{v_4}) + T(c_5\vec{v_5}) \cdots T(c_n\vec{v_n})$$

$$= c_1T(\vec{v_1}) + c_2T(\vec{v_2}) + c_3T(\vec{v_3}) + c_4T(\vec{v_4}) \cdots + cT(\vec{v_5})$$

$$= [T(\vec{v_1}), T(\vec{v_2}), \ldots T(\vec{v_n})] \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$$

$$= A\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$$

explain: $[\vec{v_1}, \vec{v_2}, \vec{v_3} \cdots \vec{v_n}] \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \vec{v}$

$A\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = T(\vec{v})$ after transform

Translate: Linear Transformation from $R^n \to R^n$ change of basis from β_1 to β_2

eg. $\beta_1 = \left\{\begin{pmatrix} -9 \\ 1 \end{pmatrix}, \begin{pmatrix} -5 \\ -1 \end{pmatrix}\right\}$, $\beta_2 = \left\{\begin{pmatrix} 1 \\ -4 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \end{pmatrix}\right\}$

$$[\vec{v_1}, \vec{v_2}, \ldots \vec{v_n}] \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = [\vec{w_1}, \vec{w_2} \ldots \vec{w_m}] \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$$

$$V\vec{c} = W\vec{d} \Rightarrow \vec{c} = V^{-1}W\vec{d}$$

eg. $\begin{bmatrix} -9 & -5 \\ 1 & -1 \end{bmatrix} \vec{c} = \begin{bmatrix} 1 & 3 \\ -4 & -5 \end{bmatrix} \vec{d}$

change basis. $A\begin{bmatrix} -9 & -5 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -4 & -5 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 3 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} -9 & -5 \\ 1 & -1 \end{bmatrix}^{-1}$$