Determinants

columns in square matrix A determin a parallelepiped P

$\det(A)$ = volumn of P

Find Determinants (property)

- If a row is k · another row $\det A = 0$
- If a row is $\vec{0}$, $\det A = 0$
- If A is triangular, $\det A$ = product of element in diagnal

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & a_{nn} \end{pmatrix} \qquad \det A = a_{11}a_{22}\cdots a_{nn}\det(Id_n) = a_{11}a_{22}\cdots a_{nn}$$

- $\det(A \cdot B) = \det(A)\det(B)$
- If A is invertible, then $\det(A) \neq 0$ vise versa
- $\det(A^T) = \det(A)$

prove: case 1 ($\det(A) = 0$): reduction of A^T has column of $0 \Rightarrow \det(A^T) = 0$

case 2 ($\det(A) \neq 0$):

A is invertible, $PA = LU$

$$A^T P^T = U^T L^T$$

$$|A^T||P^T| = |U^T||L^T|$$

$$|A^T|(\pm 1) = |L||U|$$

$$|A^T| = |LU|$$

$$|A^T| = |A|$$

- If two row or column exchanges, determinant $\cdot(-1)$