

Rank

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rank: the number of pivots $= r$ RZF to see redundant

for matrix $A_{m \times n}$, there are $n - r$ free column
the dimension of nullspace is $n - r$

Full column rank $r = n$

if all column of A are pivot column \Leftrightarrow all column linearly independent
 \Leftrightarrow no free columns (no free solution) $\Leftrightarrow \text{Nul } A = \{ \vec{0} \}$
 \Leftrightarrow if $A\vec{x} = \vec{b}$ has a solution, it has exactly one solution

Full row rank $r = m$

if all rows of A have pivot position \Leftrightarrow all rows linearly independent
 $\Leftrightarrow n - r = n - m$ nullspace solutions
 \Leftrightarrow the column space of A spans all of \mathbb{R}^m
 $\Leftrightarrow A\vec{x} = \vec{b}$ has a solution for every \vec{b}

eg.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} m = r = 3 \\ n = 4 \end{matrix}$$