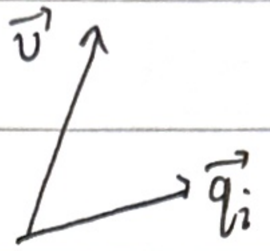
Check Orthogonal Bases for $Q = \{\vec{q_1}, \vec{q_2} \cdots \vec{q_n}\}$

$\Rightarrow$ whether q_i is orthonormal $= \begin{cases} q_i^T q_j = 0 \ (i \neq j) \ \& \ q_i^T q_i = 1 \\ \|\vec{q_i}\| = 1 \end{cases}$

If Q is the orthogonal basis for subspace $S \subset R^m$, for $\vec{v} \in S$

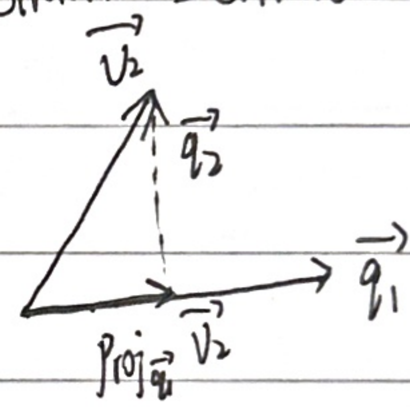
$$\vec{v} = c_1\vec{q_1} + c_2\vec{q_2} + \cdots + c_n\vec{q_n}$$

with $c_i = \dfrac{\vec{q_i}^T \vec{v}}{\vec{q_i}^T \vec{q_i}}$



$c_i\vec{q_i}$ means the projection of $\vec{v}$ on $\vec{q_i}$

Find Orthogonal basis by using Gram-Schmidt (G-S)



$$\vec{q_1} = \vec{v_1}$$

$$\vec{q_2} = \vec{v_2} - \frac{\vec{q_1}^T \cdot \vec{v_2}}{\vec{q_1}^T \cdot \vec{q_1}} \vec{q_1}$$

$$\vec{q_3} = \vec{v_3} - \vec{P_{31}} - \vec{P_{32}}$$

$$= \vec{v_3} - \frac{\vec{q_1}^T \vec{v_3}}{\vec{q_1}^T \vec{q_1}} \vec{q_1} - \frac{\vec{q_2}^T \vec{v_3}}{\vec{q_2}^T \vec{q_2}} \vec{q_2}$$

$\vdots$

$$\vec{q_n} = \vec{v_n} - \vec{P_{n1}} - \vec{P_{n2}} - \cdots - \vec{P_{n(n-1)}}$$

Find Orthonormal basis : normalize $\vec{q_1} \cdots \vec{q_n}$