Rank and solvability

rank : the number of pivots ⟹ r RZF to see redundant

for matrix A $m \times n$, there are $n-r$ free column

     the dimension of nullspace is $n-r$

Full column rank $r = n$

if all column of A are pivot column ⟹ all column linearly independent

     ⟸ no free columns (no free solution) ⟸ $Nul\ A = \{\vec{0}\}$

     ⟸ if $A\vec{x} = \vec{b}$ has a solution, it has exactly one solution

Full row rank $r = m$

if all rows of A have pivot position ⟹ all rows linearly independent

     ⟹ $n - r = n - m$ nullspace solutions

     ⟺ the column space of A spans all of R^m

     ⟸ $A\vec{x} = \vec{b}$ has a solution for every $\vec{b}$

eg.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{matrix} m = r = 3 \\ n = 4 \end{matrix}$$

Solvability

1. $r = m = n$ ⟹ $A\vec{x} = \vec{b}$ 1 solution,   square invertible

2. $r = m, r < n$ ⟹ $A\vec{x} = \vec{b}$ infinite solution

3. $r < m, r = n$ ⟹ $A\vec{x} = \vec{b}$ 0 or 1 solution

4. $r < m, r < n$ ⟹ $A\vec{x} = \vec{b}$ infinite or no solution