Find best-fit line by using least Square approximation

eg. find best-fit line through the point $(0,6)$, $(1,0)$ and $(2,0)$

1. put the equation into the line equation to get matrix equation $A\vec{x}=\vec{b}$

$$\begin{cases}(0)D+C=(6)\\(1)D+C=(0)\\(2)D+C=(0)\end{cases} \Rightarrow \begin{bmatrix}0&1\\1&1\\2&1\end{bmatrix}\begin{bmatrix}D\\C\end{bmatrix}=\begin{bmatrix}6\\0\\0\end{bmatrix}$$

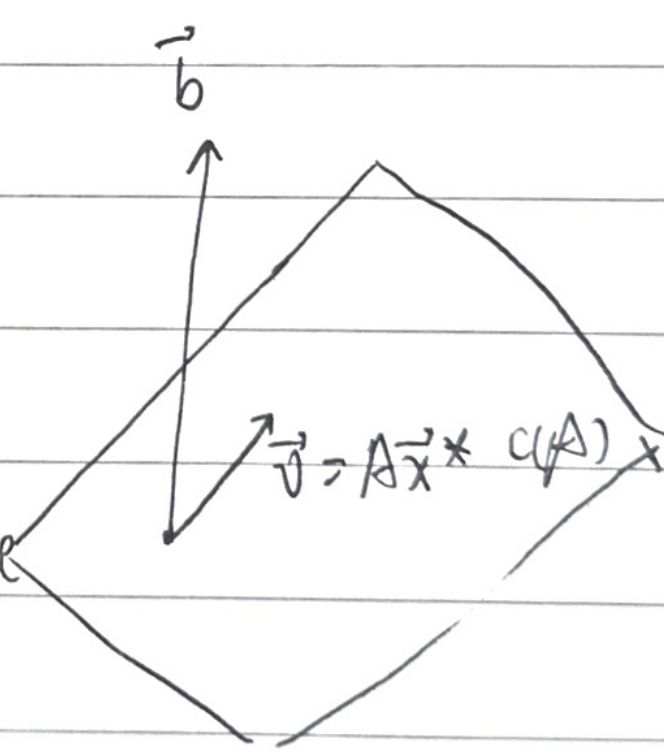
2. Use Least Square Approximation to solve the matrix equation

$$A\vec{x}=\vec{b} \Rightarrow (A^TA)\vec{x}=A^T\vec{b}$$

$$\begin{bmatrix}0&1&2\\1&1&1\end{bmatrix}\begin{bmatrix}0&1\\1&1\\2&1\end{bmatrix}\begin{bmatrix}\hat{D}\\\hat{C}\end{bmatrix}=\begin{bmatrix}0&1&2\\1&1&1\end{bmatrix}\begin{bmatrix}6\\0\\0\end{bmatrix}$$

$$\Rightarrow \hat{C}=5 \quad \hat{D}=-3$$

Principle behind Least Square Approximation



$A\vec{x}$ must be in the space $C(A)$

No solution to $A\vec{x}=\vec{b} \Rightarrow$

$\vec{x}^*$ (least square solution)

where $A\vec{x}^*$ is as close to $\vec{b}$ as possible

$$A\vec{x}^*-\vec{b}\in C(A)^\perp$$

$$C(A)^\perp=N(A^T)$$

$$A\vec{x}^*-\vec{b}\in N(A^T)$$

$$A^T(A\vec{x}^*-\vec{b})=0$$

$$A^TA\vec{x}^*=A^T\vec{b}$$