Set Theory

Def: A set is an unordered collection of elements

$x \in A$: element x belongs to A; ~~the~~ x is an element of set A

$B \subset A$: set B is a subset of set A

If $B \not\subset A$, then there exists an element in B that is not in A

empty set $\{\} = \phi \neq \{\phi\}$ empty set is subset of everyset

Union of set $A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$

u 

Intersection of sets $A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$

Complement of a set A: $A' = A^c = \bar{A} = \{x \in U \mid x \notin A\}$

$A \cup A^c = U \quad A \cap A^c = \phi$

For 2 events A, B, they are $\boxed{\text{mutually exclusive}}$ if $A \cap B = \phi$

$\Downarrow$

if A happens, B must not happen, $\neq$ independent

Def: The probability of an event A is the sum of all the "weights" of elements in A

Axiom

① $P(S) = 1$ and $P(\phi) = 0$

② $0 \le P(A) \le 1$

③ If A B are mutually exclusive ($A \cap B = \phi$), then $P(A \cup B) = P(A) + P(B)$

exercise: Show $P(A \cup B)$

$A \cup B = A \cup (A' \cap B)$

$P(A \cup B) = P(A \cup (A' \cap B)) = P(A) + P(A' \cup B)$ axiom ③

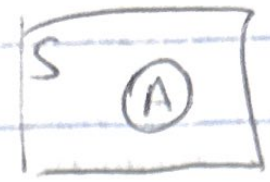
$= P(A) + P(B) - P(A \cap B)$ axiom ③

De Morgan's Law

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

Experiment $\longrightarrow S$ ← universe set



$A \cup A^c = S$
$A \cap A^c = \emptyset$

If $A \subset S$, A is an event

$|A|$ = cardinality (number of elements) of set A

exercises:

7 balls are randomly withdrawn from urn containing 12 red, 16 blue, 18 green

find the P(exactly 3 red balls or exactly 3 blue balls)

$$= \frac{\binom{12}{3}\binom{34}{4} + \binom{16}{3}\binom{30}{4} - \binom{12}{3}\binom{16}{3}\binom{18}{1}}{\binom{46}{7}}$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

probability of A given B

$$P(A \cap B) = P(A|B) \cdot P(B)$$

methods:

① Given intersection → ask to find condition

eg. 14% have A, ... how much percent have A and B or given B

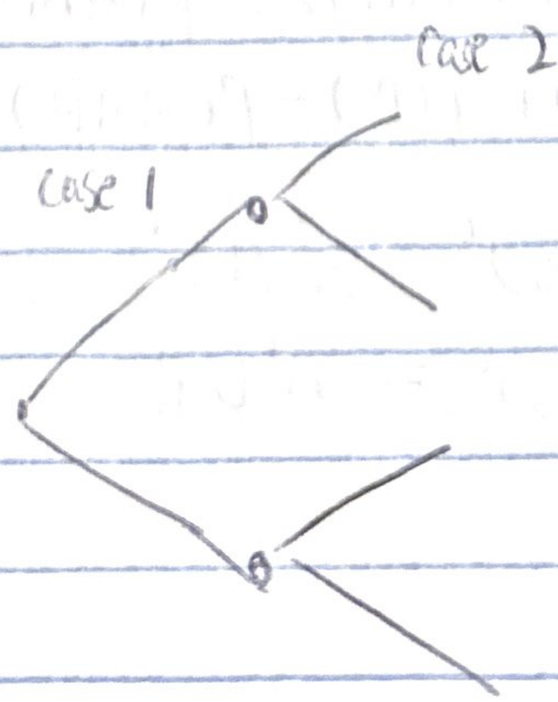
Use charts:

	x	y	z
A			
B			

② Given conditions → ask for find intersection

eg.

Use Tree:



Independent

def. 2 events are independent if $P(A|B) = P(A)$ and $P(B|A) = P(B)$

$\Leftrightarrow P(A \cap B) = P(A)P(B)$

properties: If A and B are independent, so too are:

① A and B^c, ② A^c and B, ③ A^c and B^c

exercise: show A and B^c are independent if A and B are independent

$$P(A \cap B) = P(A)P(B) = P(A)(1 - P(B^c))$$
$$= P(A) - P(A)P(B^c)$$

$$P(A)P(B^c) = P(A) - P(A \cap B) = P(A \cap B^c)$$

Bernoulli trials

experiment with binary outcomes: $P(A) = p$ $P(\bar{A}) = 1-p$

$$P_n(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where n = experiment number, p = success rate

k = # of success

Law of Total Probability

$$P(A) = P((A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3)) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)\overbrace{P(A|B_3)}$$

$P(A|B_3)$

Bayes' Theorem

$$P(B_1|A) = \frac{P(A \cap B_1)}{P(A)} = \frac{P(B_1)P(A|B_1)}{\text{Law of Total Probability}}$$

If experiment are done by 2 steps

use either or both of them