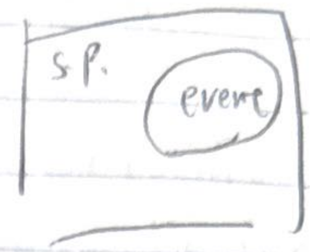
Sample space : the set of all possible outcome of a given experiment
Event : Any subject of the sample space



For counting :

① Does order Matter ?

② Is repetition allowed ?

Permutation (order matter)

① The number of premutations of n distinct objects taken r at a time

$$A_n^r = P(n,r) = \frac{n!}{(n-r)!}$$

② if not all distinct

eg. Permutation in letters in MISSISSIPPI

$$\frac{11!}{4!\,4!\,2!}$$

permutation of 3 letters in TEEXAS

① 3 distinct : $\frac{5!}{2!} = 60$

② 2 same : $4 \times 3 = 12$

$\}$ 72

③ If distinct and ~~d~~repetition allowed

TEXAS ⇒ 5^3 $\left(\begin{smallmatrix} n=5 \\ r=3 \end{smallmatrix}\right)$

Combination (order doesn't matter)

take ~~2~~3 students out of 10 students for permutation : A_{10}^3

~~tot~~ Permutation inside of 3-student party : A_3^3

With no order, A_3^3 is only 1 case, so combination is $\frac{A_{10}^3}{A_3^3}$

① repetition not allowed

TEXAS ⇒ $\frac{5!}{(5-3)!\,3!}$ $\left(\begin{smallmatrix} n=5 \\ r=3 \end{smallmatrix}\right)$

② The number of combinations of n objects taken r at a time

$$C(n,r) = nCr = \binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \binom{n}{n-r}$$

③ Repetition is allowed (stars and bars) number of ways = $\binom{n+r-1}{r}$

eg. different ways of ordering ~~10~~16 bagels which have 5 types $\left(\begin{smallmatrix} n=5 \\ r=16 \end{smallmatrix}\right)$

⇒ ****|**|*****|***|** → Find 4 positions among 20 positions

Suppose all *, | as one position, choose 4 position among 20 position for split sigh |

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!} \quad \text{binomial Coefficient}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Partition: A set of n distinct objects divided into r distinct groups with respective sizes $n_1, n_2, \ldots n_r$ (no overlap) is called a partition of n objects into ~~non-empty~~ groups

\# of partitions of n distinct objects into r groups with distinct sizes

$$\binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3} \cdots \binom{n-n_1-\cdots n_{r-1}}{n_{r-1}}\binom{n_r}{n_r}$$

$$= \frac{n!}{n_1!\cancel{(n-n_1)!}} \times \frac{\cancel{(n-n_1)!}}{\cancel{(n-n_1-n_2)!}\,n_2!} \times \frac{\cancel{(n-n_1-n_2)!}}{\cancel{(n-n_1-n_2-n_3)!}} \times \cdots \frac{\cancel{(n_{r-1}+n_r)!}}{(n_{r-1})!\,\cancel{(n_r)!}} \times \frac{\cancel{n_r!}}{n_r!\,1}$$

$$= \frac{n!}{n_1! \times n_2! \times \cdots n_r!} = \binom{n}{n_1, n_2, \ldots n_r} \quad \text{Multinomial coefficient}$$

If some groups have same size:

Divide the # of groups having same size (exclude the permutation of same-size group)

eg. 15 kid divided into 3 groups of size 5 each

$$\frac{15!}{5!\,5!\,5!} \Big/ 3!$$

Use Multinomial Theorem to expand polynomial $(x_1+x_2+\cdots x_k)^n$

$$(x_1+x_2+\ldots x_n)^n = \sum_{i_1+i_2+\cdots i_k=n} \binom{n}{i_1, i_2, \cdots i_k} x_1^{i_1} x_2^{i_2} \cdots x_k^{i_k}$$