Cryptography: cryptographic hash functions and MACs

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Introduction

Encryption $\Rightarrow$ confidentiality against eavesdropping

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What about authenticity and integrity against an active attacker?
$\longrightarrow$ cryptographic hash functions and Message authentication codes
$\longrightarrow$ this lecture

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Multiplication of large primes IS a OWF:
integer factorisation is a hard problem - given $p \times q$ (where p and q are primes) it is hard to retrieve p and q

Collision-resistant functions (CRFs)

A function is a CRF if it is hard to find two messages that get mapped to the same value through this function

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Multiplication of large primes IS a CRF:
every positive integer has a unique prime factorisation

Quiz

Cryptographic hash functions

A cryptographic hash function takes messages of arbitrary length and returns a fixed-size bit string such that any change to the data will (with very high probability) change the corresponding hash value.

Definition (Cryptographic hash function)

A cryptographic hash function $H: \mathcal{M} \rightarrow \mathcal{T}$ is a function that satisfies the following 4 properties:

- $|\mathcal{M}| >> |\mathcal{T}|$
- it is easy to compute the hash value for any given message
- it is hard to retrieve a message from its hashed value (OWF)
- it is hard to find two different messages with the same hash value (CRF)

Examples: MD4, MD5, SHA-1, RIPEMD160, SHA-256, SHA-512, ...

$\rightsquigarrow$In new projects use SHA-256 or SHA-512

Cryptographic hash functions: applications

- **Commitments** - Allow a participant to commit to a value v by publishing the hash $H(v)$ of this value, but revealing v only later.
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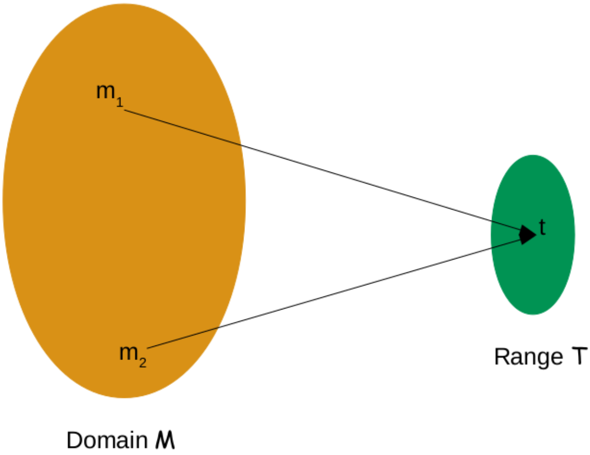
- **Building block of other crypto primitives** - Used to build MACs, block ciphers, PRG, ...

Collisions are unavoidable



The domain being much larger than the range, collisions necessarily exist

The birthday attack - attack on all schemes

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Theorem

Let $H : \mathcal{M} \rightarrow \{0,1\}^n$ be a cryptographic hash function ($|\mathcal{M}| >> 2^n$)
Generic algorithm to find a collision in time $O(\sqrt{2^n}) = O(2^{n/2})$ hashes:

1. *Choose $2^{n/2}$ random messages in $\mathcal{M}$: $m_1, \ldots, m_{2^{n/2}}$*
2. *For $i = 1, \ldots, 2^{n/2}$ compute $t_i = H(m_i)$*
3. *If there exists a collision ($\exists i, j.\ t_i = t_j$)*
   then return (m_i, m_j)
   else go back to 1

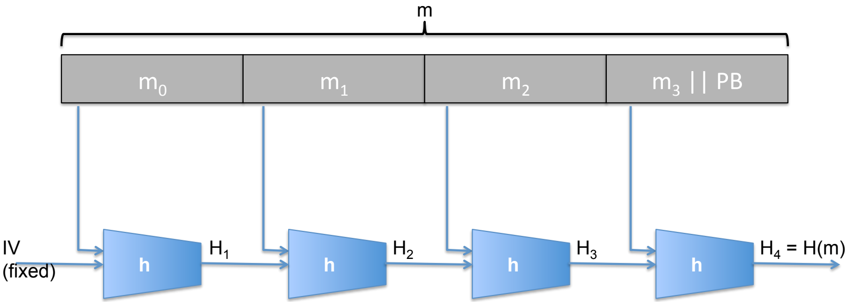
Birthday paradox Let $r_1, \ldots, r_\ell \in \{1, \ldots, N\}$ be independent variables.
For $\ell = 1.2 \times \sqrt{N}$, $Pr(\exists i \neq j.\ r_i = r_j) \geq \frac{1}{2}$

$\Rightarrow$ the expected number of iteration is 2

$\Rightarrow$ running time $O(2^{n/2})$

$\Rightarrow$ Cryptographic function used in new projects should have output length $n \geq 256$!

The Merkle-Damgard construction



- Compression function: $h: \mathcal{T} \times \mathcal{X} \to \mathcal{T}$
- PB: $1000\ldots0||$mes-len (add extra block if needed)

Theorem

Let H be built using the MD construction to the compression function h. If H admits a collision, so does h.

Example of MD constructions: MD5, SHA-1, SHA-2, ...

Compression functions from block ciphers

Let $E: \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ be a block cipher

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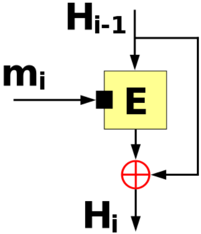
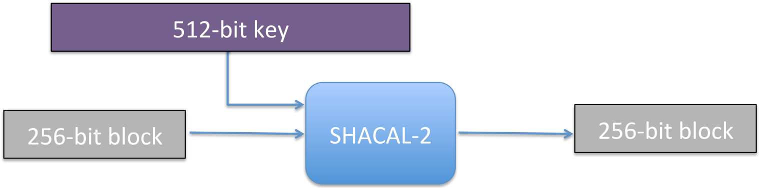


Davies-Meyer

Source: `https://en.wikipedia.org/wiki/One-way_compression_function`

Example of cryptographic hash function: SHA-256

- Structure: Merkle-Damgard
- Compression function: Davies-Meyer
- Bloc cipher: SHACAL-2



Message Authentication Codes (MACs)

$\xrightarrow{e=E(K_E,\text{Transfer 100 € on Bob's account})}$ Royal Bank of Scotland

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What if the encryption scheme E is the OTP - $e = K_E \oplus \text{Transfer 100 € on Bob's account}$?

Encryption is not always enough

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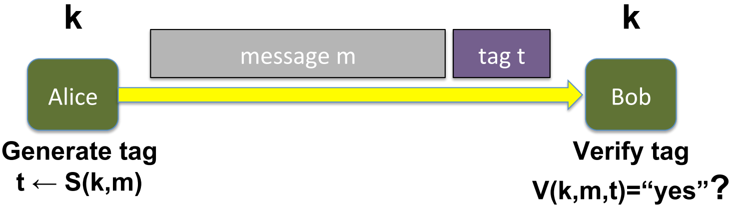
What if the encryption scheme E is the OTP -
$e = K_E \oplus$ Transfer 100 € on Bob's account?

$\xrightarrow{e}$

$\xrightarrow[=E(K_E,\text{Transfer 100 € on Eve's account})]{e\oplus 0\ldots 0\text{Bob}0\ldots 0\oplus 0\ldots 0\text{Eve}0\ldots 0}$ Royal Bank of Scotland

Goal: message integrity



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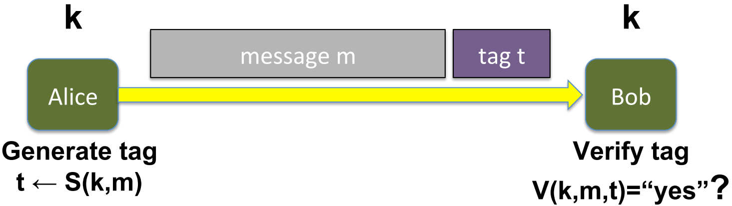


A MAC is a pair of algorithms (S, V) defined over $(\mathcal{K}, \mathcal{M}, \mathcal{T})$:

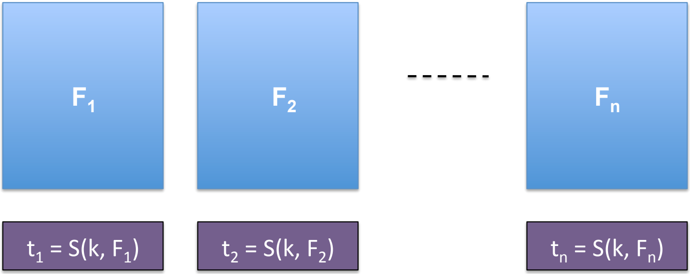
- $S : \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{T}$
- $V : \mathcal{K} \times \mathcal{M} \times \mathcal{T} \rightarrow \{\top, \bot\}$
- Consistency: $V(k, m, S(k, m)) = T$

Unforgeability

It is hard to computer a valid pair $(m, S(k, m))$ without knowing k

File system protection

- At installation time



k derived from user password

- To check for virus file tampering/alteration:
  - reboot to clean OS
  - supply password
  - any file modification will be detected

Block ciphers and message integrity

Block ciphers and message integrity

Let (E, D) be a block cipher. We build a MAC (S, V) using (E, D) as follows:

- $S(k, m) = E(k, m)$
- $V(k, m, t) =$ if $m = D(k, t)$
  then return $\top$
  else return $\perp$

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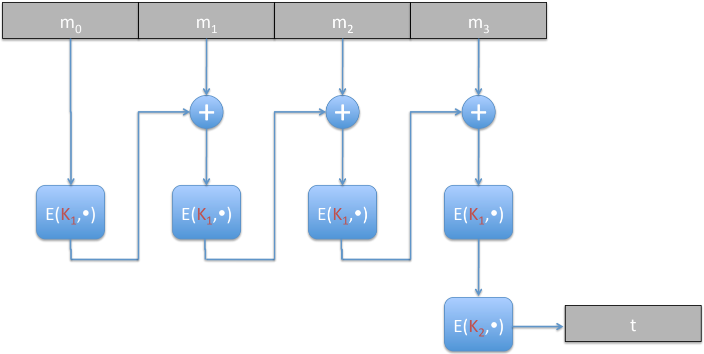
Our goal now: construct MACs for long messages

ECBC-MAC

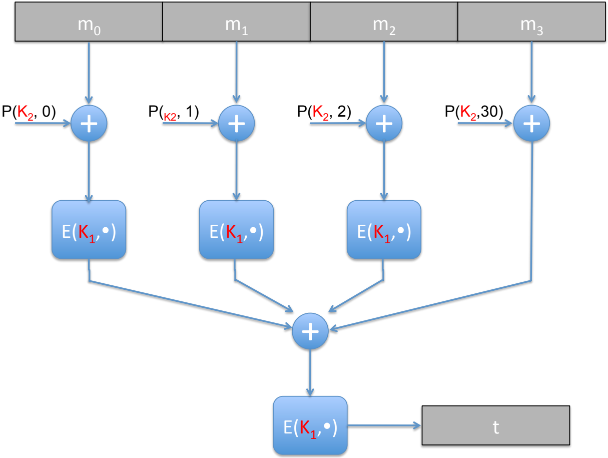


- $E : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ a block cipher
- $ECBC\text{-}MAC : \mathcal{K}^2 \times \{0,1\}^* \to \{0,1\}^n$

$\to$ the last encryption is crucial to avoid forgeries!!

Ex: 802.11i (WPA for Wi-Fi) uses AES based ECBC-MAC

PMAC



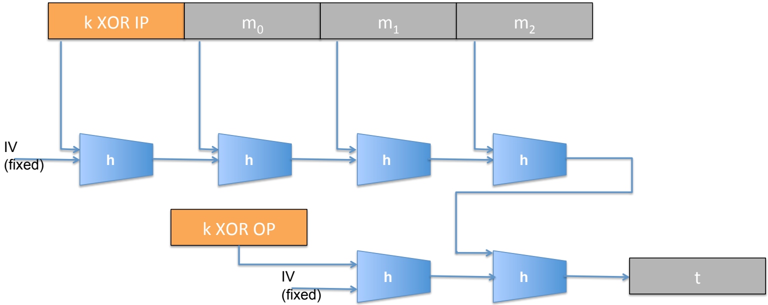
- $E: \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ a block cipher
- $P: \mathcal{K} \times \mathbb{N} \to \{0,1\}^n$ any easy to compute function
- $PMAC: \mathcal{K}^2 \times \{0,1\}^* \to \{0,1\}^n$

HMAC

MAC built from cryptographic hash functions

$$HMAC(k, m) = H(k \oplus OP || H(k \oplus IP || m))$$

IP, OP: publicly known padding constants



Ex: SSL, IPsec, SSH, . . .

Authenticated encryption

Plain encryption is malleable

- The decryption algorithm never fails
- Changing one bit of the i^{th} block of the ciphertext
  - CBC decryption: will affect i^{th} and $i+1^{th}$ block of the plaintext
  - ECB decryption: will only affect the i^{th} block of the plaintext
  - CTR decryption: will only affect one bit of the i^{th} block of the plaintext

Decryption should fail if a ciphertext was not computed using the key

Goal

Simultaneously provide data confidentiality, integrity and authenticity
$\rightsquigarrow$ decryption combined with integrity verification in one step

Encrypt-then-MAC

1. Always compute the MACs on the ciphertext, never on the plaintext
2. Use two different keys, one for encryption (K_E) and one for the MAC (K_M)

Encryption

1. $C \leftarrow E_{AES}(K_E, M)$
2. $T \leftarrow HMAC\text{-}SHA(K_M, C)$
3. return $C||T$

Decryption

1. if $T = HMAC\text{-}SHA(K_M, C)$
2. then return $D_{AES}(K_E, C)$
3. else return $\perp$

Do not:

- Encrypt-and-MAC: $E_{AES}(K_E, M)||HMAC\text{-}SHA(K_M, M)$
- MAC-then-Encrypt: $E_{AES}(K_E, M||HMAC\text{-}SHA(K_M, M))$

AES-GCM

Galois Counter Mode

Combines

1. Galois field based One-time MAC for authentication
2. AES based Counter Mode for encryption.

- Trick: One-time MAC is encrypted too
  $\Rightarrow$ secure for many messages
- Widely adopted for its performance
- Many good implementations of this mode