

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_i \end{pmatrix} \begin{pmatrix} a_1 & a_2 & \dots & a_j \end{pmatrix} = \begin{pmatrix} p_1 a_1 & p_1 a_2 & \dots & p_1 a_j \\ p_2 a_1 & p_2 a_2 & \dots & p_2 a_j \\ \vdots & \vdots & \dots & \vdots \\ p_i a_1 & p_i a_2 & \dots & p_i a_j \end{pmatrix}$$

$$Cov(a, b) = \frac{1}{m-1} \sum_{n=1}^m (a_i - \mu_a)(b_i - \mu_b) \approx \frac{1}{m} \sum_{i=1}^m a_i b_i$$

$$X = \begin{pmatrix} a_1 & a_2 & \dots & a_m \\ b_1 & b_2 & \dots & b_m \end{pmatrix}$$

$$\frac{1}{m}XX^T = \begin{pmatrix} \frac{1}{m}\sum_{i=1}^m a_i^2 & \frac{1}{m}\sum_{i=1}^m a_i b_i \\ \frac{1}{m}\sum_{i=1}^m a_i b_i & \frac{1}{m}\sum_{i=1}^m b_i^2 \end{pmatrix} = \begin{pmatrix} Cov(a, a) & Cov(a, b) \\ Cov(a, b) & Cov(b, b) \end{pmatrix}$$

$$D = \frac{1}{m}YY^T = \frac{1}{m}(PX)(PX)^T = P(\frac{1}{m}XX^T)P^T = PCP^T$$

$$E^TCE = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix}$$

$$X_{approx} = P^TY$$