Minimization Word Problems

1. Manufacturing Problem

Low sulfur and high sulfur fuels are burned in a generator to provide power. Each gallon of low sulfur costs P160 and produces 4 kilowatts of power while emitting 3 units of sulfur dioxide over the course of an hour; while each gallon of high sulfur costs P150, produces 4 kilowatts of power, and releases 5 units of sulfur dioxide.

The maximum quantity of sulfur dioxide that can be released per hour, according to an environmental agency, is 15 units. How many gallons of high sulfur and low sulfur must be used per hour to reduce fuel cost if at least 16 kilowatts must be produced per hour?

Solution:

Let:

x = amount of low sulfur fuel in gallon

y = amount of high sulfur fuel in gallon

z = minimum fuel cost

Objective function:

$$z = 160x + 150y$$

Constraints:

	x	y		
POWER PRODUCED:	4	4	≥ 16	$\rightarrow 4x + 4y \geq 16$
SULFUR DIOXIDE EMISSION:	3	5	≤ 15	$3x + 5y \leq 15$

$\underline{4x + 4y = 16}$

Let $x = 0$

$4(0) + \frac{4y}{4} = \frac{16}{4}$

$y = 4$

$(0, 4)$

Let $y = 0$

$4x + 4(0) = 16$

$\frac{4x}{4} = \frac{16}{4}$

$x = 4 \quad (4, 0)$

Let $x, y = 0$

$0 \geq 16$

False

$3x + 5y = 15$

Let $x = 0$

$3(0) + 5y = 15$

$\frac{5y}{5} = \frac{15}{5}$

$y = 3 \quad (0, 3)$

Let $x, y = 0$

$0 \leq 15$

TRUE

Let $y = 0$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5 \quad (5, 0)$

Intersection:

$(3)(4x + 4y) = (16)(3) \rightarrow 12x + 12y = 48$

$(-4)(3x + 5y) = (15)(-4) \quad + \; -12x - 20y = -60$

$\frac{-8y}{-8} = \frac{-12}{-8}$

$y = \frac{3}{2} = 1.5$

$4x + 4y = 16$

$4x + 4\left(\frac{3}{2}\right) = 16$

$4x + \frac{12}{2} = 16$

$4x = 16 - 6$

$\frac{4x}{4} = \frac{10}{4}$

$x = \frac{5}{2} = 2.5$

Intersection $\left(\frac{5}{2}, \frac{3}{2}\right)$

Corner Points

(4, 0) z= 160 (4) + 150 (3) = 640

$\mathbf{(\frac{5}{2}, \frac{3}{2})}$ z= 160 $(\frac{5}{2})$ + 150 $(\frac{3}{2})$ = 400 + 225 = 625 (minimum cost)

(5, 0) z= 160 (5) + 150 (0) = 800

Final Answer:

5/2 (or 2.5) gallons of low sulfur fuel and 3/2 (or 1.5) gallons of high sulfur fuel should be burned to minimize the cost to P650.

2. *Diet Problem*

A person wants to design a daily diet plan using two food items, Meal A and Meal B. Each meal has a cost in pesos and provides carbohydrates, protein, and fat. The person has minimum daily requirements for each nutrient and aims to minimize the cost of this diet.

<u>Meal A:</u>

Cost per serving: 50 pesos
Carbohydrates: 400 grams per serving
Protein: 10 grams per serving
Fat: 20 grams per serving

<u>Meal B:</u>

Cost per serving: 30 pesos
Carbohydrates: 300 kcal per serving
Protein: 15 grams per serving
Fat: 10 grams per serving

<u>Nutritional Requirements:</u>

The person needs at least:
Carbohydrates: 1200 grams
Protein: 30 grams
Fat: 30 grams

Solution:

Let x = number of servings of meal A
y = number of servings of meal B
z = minimum cost

Objective Function

$z = 50x + 30y$

Constraints

	x	y		
Carbohydrates :	400	300	≥ 1200	$\rightarrow 400x + 300y \geq 1200$
Protein :	10	15	≥ 30	$\rightarrow 10x + 15y \geq 30$
Fat :	20	10	≥ 30	$\rightarrow 20x + 10y \geq 30$

$400x + 300y = 1{,}200$

You may reduce the equation above by 100

$4x + 3y = 12$

let $x = 0$

$4(0) + 3y = 12$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{12}{3}$

$y = 4$

$(0,4)$

let $y = 0$

$4x + 3(0) = 12$

$\frac{4x}{4} = \frac{12}{4}$

$x = 3$

$(3,0)$

let $x, y = 0$

$400(0) + 300(0) \geq 1200$

$0 \geq 1200$

FALSE

$10x + 15y = 30$

Reduce by 5

$2x + 3y = 6$

let $x = 0$

$2(0) + 3y = 6$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{6}{3}$

$y = 2$

$(0,2)$

Let $y = 0$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$

$x = 3$

$(3,0)$

Let $x, y = 0$

$10(0) + 15(0) \geq 30$

$0 \geq 30$

FALSE

$20x + 10y = 30$

Reduce by 10

$2x + y = 3$

let $x = 0$

$2(0) + y = 3$

$y = 3$

$(0,3)$

let $y = 0$

$2x + 0 = 3$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{3}{2}$

$x = \frac{3}{2} = 1.5$

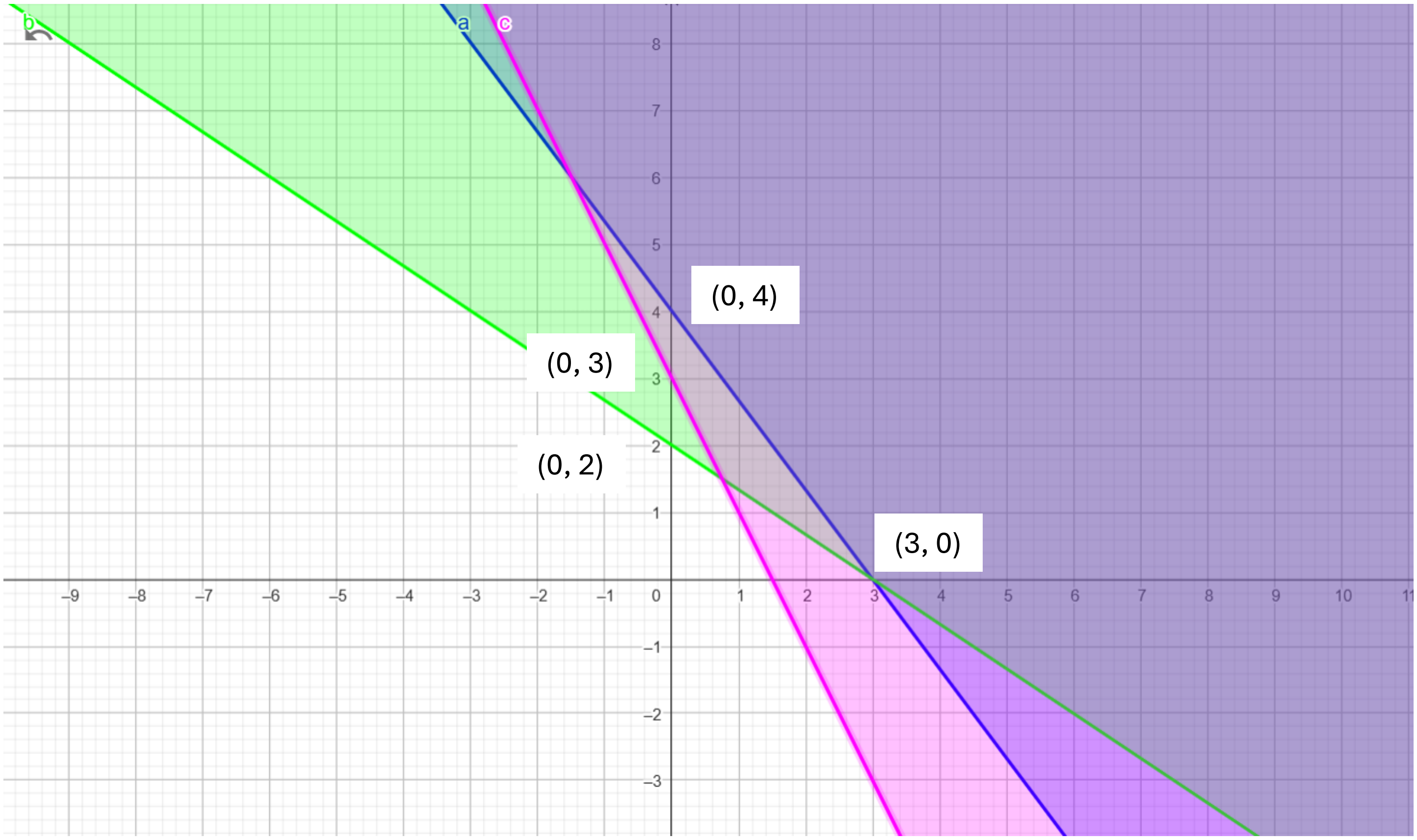
$(3/2, 0)$

let $x, y = 0$

$20(0) + 10(0) \geq 30$

$0 \geq 30$

FALSE



Note: If an intersection is not in the optimal region, there's no need to compute/determine the coordinates)

Corner Points:

(0, 4) z = 50 (0) + 30 (4) = 120 (minimum cost)

(3, 0) z = 50 (3) + 30 (0) = 150

Final Answer:

The person should have 4 servings of meal B to minimize the cost to P120.

Transportation Problem

Transportation problem is a special kind of **Linear Programming Problem (LPP)** in which *goods are transported from a set of sources* to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is **minimized.** It is also sometimes called the Hitchcock problem.

1. *Northwest Corner Cell Method*

	D1	D2	D3	D4	**Supply**
S1	12	18	9	11	**105**
S2	19	7	30	15	**145**
S3	8	10	14	16	**50**
Demand	**80**	**60**	**70**	**90**	**300**

S1, S2, and S3 are the three sources of certain products, while D1, D2, D3, and D4 are the four different destinations of delivery. The numbers with black font colors are the prices of each delivery with the specific source and destination. For example, 12 in row 2 column 2 is the price when S1 delivers in D1. To solve this problem, we need to allocate number of units for a specific destination from a certain source. Since this is Northwest Corner Cell method, you will start with the allocation in the northwest corner cell which is 12 in this example. When you allocate, you choose between the demand and supply of the cell, and you allocate whichever is lower. In 12, the demand is 80 and the supply is 125. Since 80 is lower, you allocate 80 in destination 1, from source 1.

	D1	D2	D3	D4	**Supply**
S1	12 **(80)**	18	9	11	**105 - 80**
S2	~~19~~	7	30	15	**145**
S3	~~8~~	10	14	16	**50**
Demand	~~**80 – 80 = 0**~~	**60**	**70**	**90**	**300**

In the table above, once 80 is allocated, you cannot allocate to the other cells aligned to 80. On the other hand, 105 in the demand will be reduced by 80 since that number is already allocated to 12. Once you continue the process, you must find the next northwest corner cell until such time that all cells are allocated or removed once the demand or supply becomes zero.

	D1	D2	D3	D4	**Supply**
S1	12 **(80)**	18 **(25)**	~~9~~	~~11~~	**25 – 25 = 0**
S2	~~19~~	7	30	15	**145**
S3	~~8~~	10	14	16	**50**
Demand	**0**	**60 – 25 = 35**	**70**	**90**	**300**

The next northwest corner cell is 18, so you allocate in 18.

	D1	D2	D3	D4	Supply
S1	12 **(80)**	18 **(25)**	~~9~~	~~11~~	**0**
S2	~~19~~	7 **(35)**	30	15	**145 -35 = 110**
S3	~~8~~	~~10~~	14	16	**50**
Demand	**0**	**35 – 35 = 0**	**70**	**90**	**300**

The northwest corner cell is 7.

	D1	D2	D3	D4	Supply
S1	12 **(80)**	18 **(25)**	~~9~~	~~11~~	**0**
S2	~~19~~	7 **(35)**	30 **(70)**	15	**110 – 70 = 40**
S3	~~8~~	~~10~~	~~14~~	16	**50**
Demand	**0**	**0**	**70 – 70 = 0**	**90**	**300**

The northwest corner cell is 30.

	D1	D2	D3	D4	Supply
S1	12 **(80)**	18 **(25)**	~~9~~	~~11~~	**0**
S2	~~19~~	7 **(35)**	30 **(70)**	15 **(40)**	**40 – 40 = 0**
S3	~~8~~	~~10~~	~~14~~	16	**50**
Demand	**0**	**0**	**0**	**90 – 40 = 50**	**300**

The northwest corner cell is 15.

	D1	D2	D3	D4	Supply
S1	12 **(80)**	18 **(25)**	~~9~~	~~11~~	**0**
S2	~~19~~	7 **(35)**	30 **(70)**	15 **(40)**	**0**
S3	~~8~~	~~10~~	~~14~~	16 **(50)**	**50 – 50 = 0**
Demand	**0**	**0**	**0**	**50 – 50 = 0**	**300**

The last northwest corner cell is 16.

Total Cost = 12 (80) + 18 (25) + 7 (35) + 30 (70) + 15 (40) + 16 (50) = 5, 155

The total cost of this transportation problem using the Northwest Corner Cell Method is 5, 155.