CS 171, Fall 2017 ps1

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Problem 1. (5 points)

$p(x|y=A) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu_A)^T(x-\mu_A)}$

$p(x|y=B) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu_B)^T(x-\mu_B)}$

Answer:

$p(y=A|x) = \frac{\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu_A)^T(x-\mu_A)}p_A}{p(x)}$

$p(y=B|x) = \frac{\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu_B)^T(x-\mu_B)}p_B}{p(x)}$

$p(y=A|x) > p(y=B|x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu_A)^T(x-\mu_A)}p_A > \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}(x-\mu_B)^T(x-\mu_B)}p_B$

$= e^{-\frac{1}{2\sigma^2}(x-\mu_A)^T(x-\mu_A)}p_A > e^{-\frac{1}{2\sigma^2}(x-\mu_B)^T(x-\mu_B)}p_B$

$= -\frac{1}{2\sigma^2}(x-\mu_A)^T(x-\mu_A) + \ln(p_A) > -\frac{1}{2\sigma^2}(x-\mu_B)^T(x-\mu_B) + \ln(p_B)$

$= -\frac{1}{2\sigma^2}(x-\mu_A)^T(x-\mu_A) + \ln(p_A) > -\frac{1}{2\sigma^2}(x-\mu_B)^T(x-\mu_B) + \ln(p_B)$

$= -\frac{(x^T-\mu_A^T)(x-\mu_A)}{2\sigma^2} + \ln(p_A) > -\frac{(x^T-\mu_B^T)(x-\mu_B)}{2\sigma^2} + \ln(p_B)$

$= (x^T-\mu_A^T)(x-\mu_A) + \ln(p_A) > (x^T-\mu_B^T)(x-\mu_B) + \ln(p_B)$

$= x^Tx - x^T\mu_A - \mu_A^Tx + \mu_A^T\mu_A + \ln(p_A) > x^Tx - x^T\mu_B - \mu_B^Tx + \mu_B^T\mu_B + \ln(p_B)$

$= \ln(p_A) - x^T\mu_A - \mu_A^Tx + \mu_A^T\mu_A > \ln(p_B) - x^T\mu_B - \mu_B^Tx + \mu_B^T\mu_B$

When $\ln(p_A) - x^T\mu_A - \mu_A^Tx + \mu_A^T\mu_A = \ln(p_B) - x^T\mu_B - \mu_B^Tx + \mu_B^T\mu_B$ there is a linear decision boundary