CS 171, Fall 2017 PS4

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Problem 1. (5 points)

Say we have m items and we have k items for left hand side of the rule. This means there are $\binom{m}{n}$ combinations of items. With the remaining items we have $\binom{m-k}{n}$ combinations. With this, we can then make the rule R.

$$R = \sum_{k=1}^{m} \binom{m}{k} \sum_{n=1}^{m-k} \binom{m-k}{n}$$

$$= \sum_{k=1}^{m} \binom{m}{k} (2^{m-k} - 1)$$

$$= \sum_{k=1}^{m} \binom{m}{k} (2^{m-k}) - \sum_{k=1}^{m} \binom{m}{k}$$

$$= \sum_{k=1}^{m} \binom{m}{k} (2^{m-k}) - (2^m + 1)$$

$$(1+x)^m = \sum_{k=1}^{m} \binom{m}{k} x^{m-k} + x^m$$

if $x = 2$,

$$3^m = \sum_{k=1}^{m} \binom{m}{k} 2^{m-k} + 2^m$$

Substitute in R,

$$R = 3^m - 2^m - (2^m + 1)$$

$$R = 3^m - 2^{m_1} + 1$$