1) (5 points)

Module 3 Homework

Suppose a random variable X has pdf as $f(x) = 2e^{-2(x-1)}$, $x > 1$. Which of the following represents $P(0 < X < 4)$? (Note: you do not need to solve for exact number).

(a) $\int_0^4 2e^{-2(x-1)}dx$;

(b) $\int^4 2e^{-2(x-1)}dx$;

(c) $\int_0^4 x2e^{-2(x-1)}dx$;

(d) $\sum^4 2e^{-2(x-1)}$; $x=0$

(e) $\int^{\infty} x2e^{-2(x-1)}dx$.

b. x range from 1 and it is calculating the possibility not the expected value.

2) (10 points)

A random variable X has pdf

$f(x) = \frac{}{x!}e$, $x=0,1,2,....$

Find $P(X = 1)$.

Thenfind $P(-2<X<4)$.
Give your answers to at least four decimal places.

1) P(x=1) = 0.2706706

2) P(−2<X<4) = 0.8571235

3) (5 points)

If two carriers of the gene for albinism marry and have children, then each of their children has a probability of 1/4 of being albino. Let the random variable Y denote the number of their albino children out of all 3 of their children. Then Y follows a binomial(n, p) distribution. Find the values for n and p.

n =__3___ p = __0.25____

4) (10 points)

For Y following a binomial (n = 3, p = 0.25) distribution, compute the following:

P(Y ≤ 2) = 0.984375

E(Y) = 0.7500

Var(Y) = 0.5625

Give your answers to at least four decimal places.

5) (20 points)

For X following a Chi-square distribution with degree of freedom m = 3, compute the following:

P(1 < X < 4) = 0.5397878

E(X) = 3.000

Var(X) = 6.000

Give your answers to at least four decimal places.

Also, use a Monte Carlo simulation with sample size n=100,000 to estimate $P(1 < X < 4)$. What is your Monte Carlo estimate? Does it agrees with the answer above?

Monte Carlo estimate $P(1<x<4) = 0.54038$

It is almost the same with the true value with a little difference.

6) (10 points)

Suppose X follows a Chi-square distribution with degree of freedom m = 5 so that E(X) = 5 and Var(X) = 10. Also, let Y = 4X - 10. Find E(Y) and Var (Y). Does Y follow a Chi-square distribution with degree of freedom m=10?

E(Y) = 10

Var(Y) = 80

Does Y follow a Chi-square distribution with degree of freedom m = 10?

No.

7) (20 points)

The Zyxin gene expression values are distributed according to $N(\mu = 1.6, \sigma = 0.4)$.

(a) What is the probability that a randomly chosen patient have the Zyxin gene expression values between 1 and 1.6?

A:0.4331928 with absolute error < 4.8e-15

(b) Use a Monte Carlo simulation of sample size n=500,000 to estimate the probability in part (a). Give your R code, and show the value of your estimate.

Just use the sample function to generate 500,000 samples then choose the value between 1 and 1.6

Monte Carlo simulation gives the answer of 0.43307 with little difference each time running. However, it is almost the same with the true value.

(c) What is the probability that exactly 2 out of 5 patients have the Zyxin gene expression values between 1 and 1.6?

Please show your work on how to arrive at the answer. Give your answer to at least four decimal places.

0.3417468. This is consider to be a binomal distribution. Choose 2 out of 5 with a P of 0.43307

8)

(20 points)

(a) Hand in a R script that calculates the mean and variance of two random variables X~F(m=2,n=5) and Y~F(m=10,n=5) from their density functions.

(b) Use the formula in Table 3.4.1 to calculate the means and variances directly.

(c) Run your script in (a), and check that your answers agree with those from part (b).

a)

X_mean =1.6667

X_var= 13.8889

Y_mean = 1.6667

Y_var= 7.2222

b) X_mean = n /n–2 =1.6667

X_var = $2n^2(m+n–2)/m(n – 2)^2 (n – 4)$ = 13.8889

Y_mean = n/n-2 = 1.667

Y_var = $2n^2(m+n–2)/m(n – 2)^2 (n – 4)$ = 7.2222

c) Yes they agree. They come to the same value. Because the table is the human calculation summary of the function.