ECE 595 HW5

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Exercise 1

ECE 595 HW5

1.

a) i) $g = h_1 = [\bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet]$

ii) Agree on 3 out-sample pts: f_8

2 : f_7, f_6, f_4

1 : f_2, f_3, f_5

0 : f_1

b)

i) $g = h_2 = [0, 0, 0, 0, 0, 0, 0, 0]$

ii) Agree on 3 out-samples pts : f_1

2 : f_2, f_3, f_5

1 : f_7, f_6, f_4

0 : f_8

c) i) $g = [0, \bullet, \bullet, 0, \bullet, 0, 0, \bullet]$

ii) Agree 3 out-sample pts: f_2

2 : f_1, f_6, f_4

1 : $f_3, \cancel{f_4}, f_5, f_8$

0 : f_7

d) i) $g = [0, \bullet, \bullet, 0, \bullet, \bullet, \bullet, 0]$

ii) Agree 3 pts: f_7

2 : f_3, f_5, f_8

1 : f_1, f_4, f_6

0 : f_2

Exercise 2

1. $\mu 1 = \mu rand = \mu min = 0.5$

2.

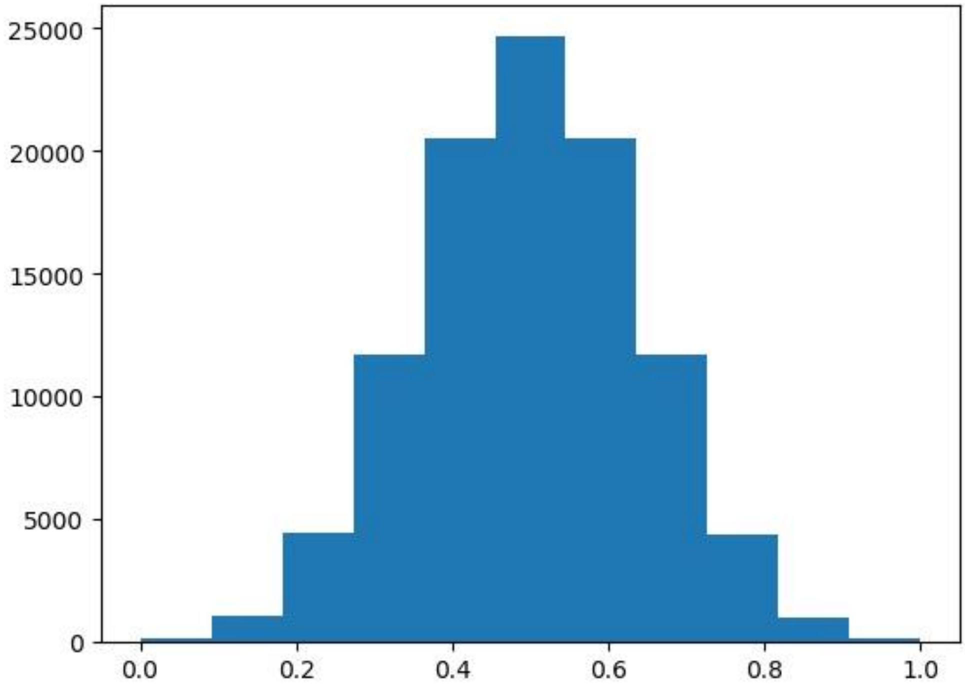


Figure 1. V1

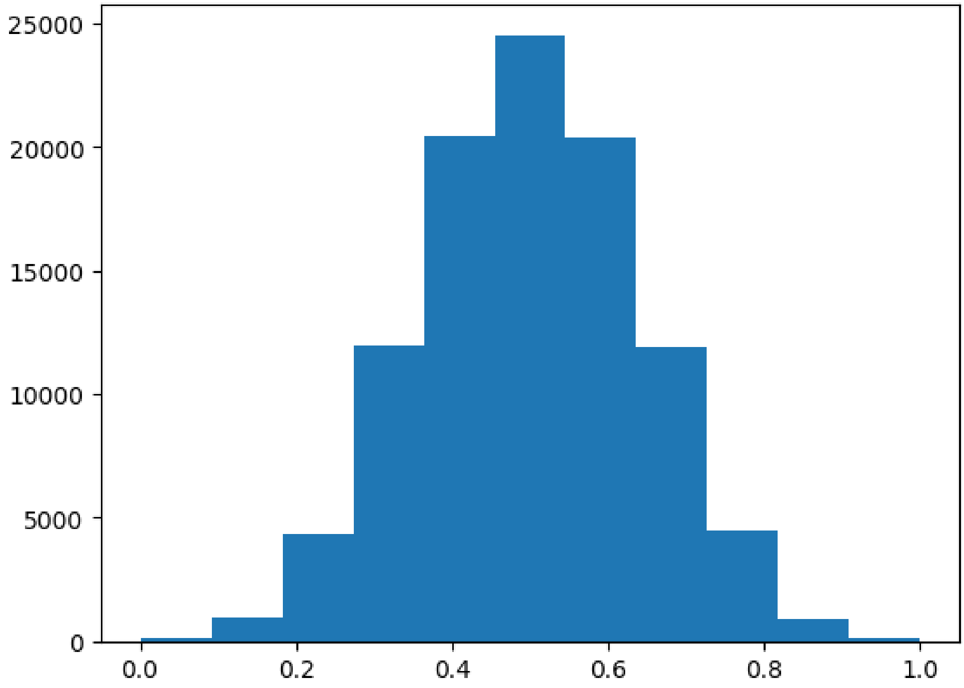


Figure 2. Vrand

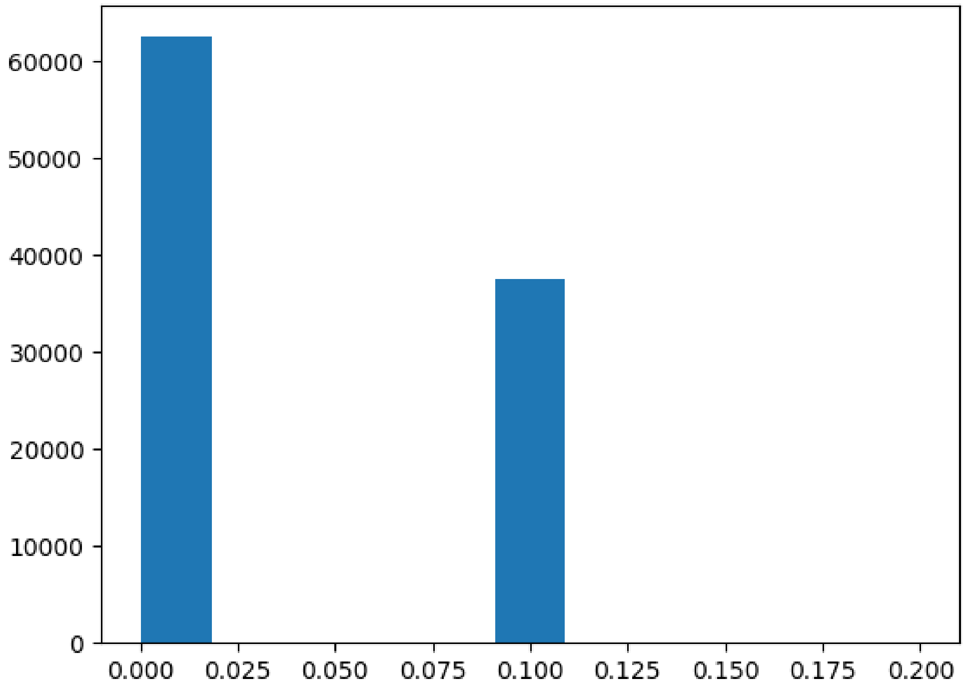
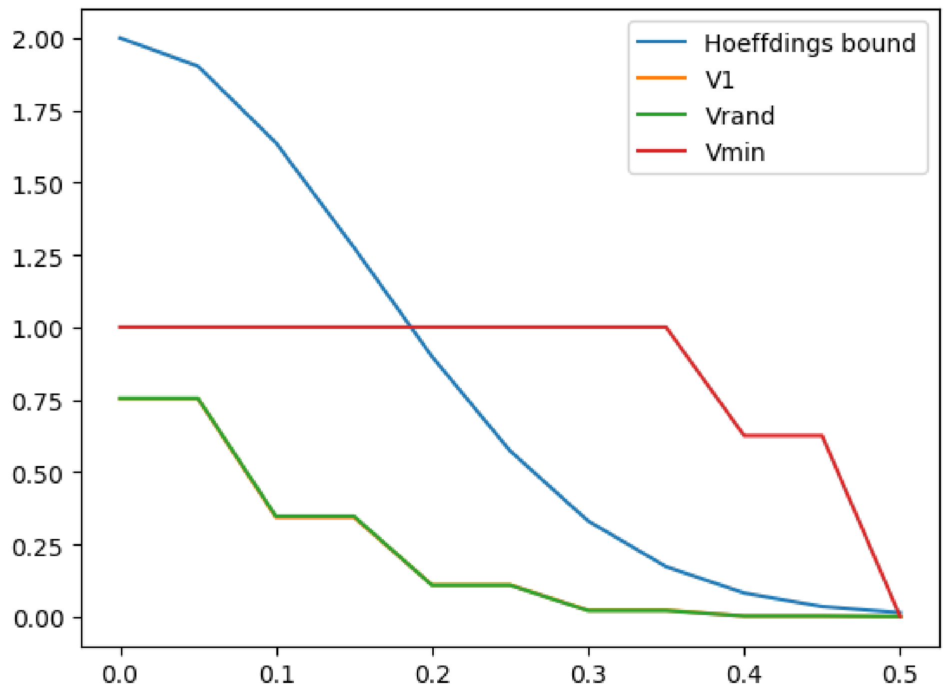


Figure 3. Vmin

3.



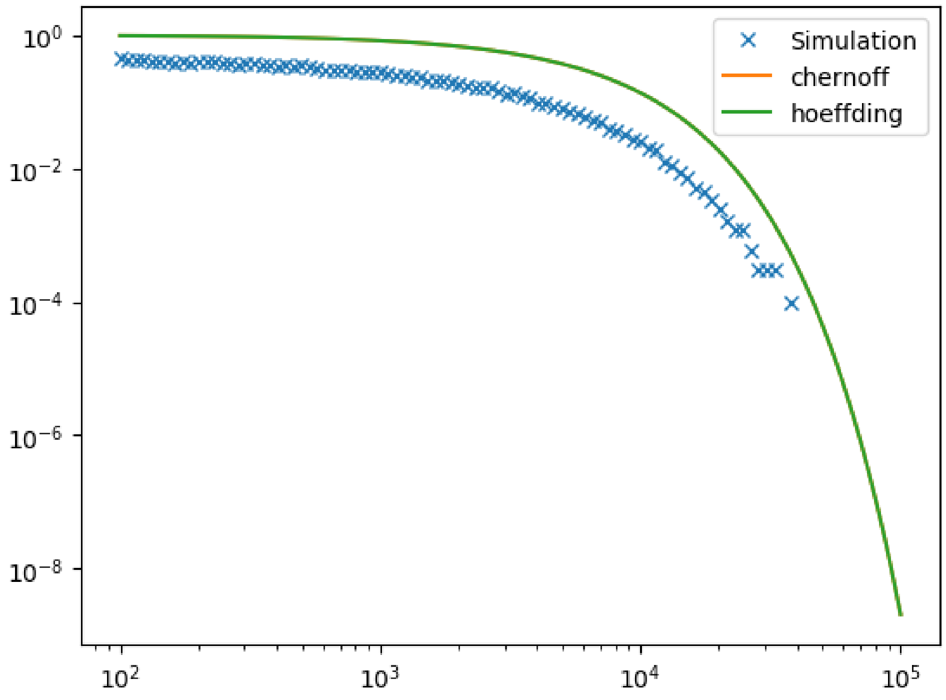
d.

The coin1 and coin_rand obey the Hoeffding's bound, and coin_min does not. That is because the coin_min is not independent of the samples. E[V1] = E[Vrand] = 0.5.

However, E[Vmin] = 0.04, which is far away from μ.

Exercise 3.

b.



a.

3. a) $\mathbb{P}[\bar{X}_N - \mu \geq \varepsilon] = \mathbb{P}[\bar{X}_N \geq \varepsilon + \mu] \leq \frac{\mathbb{E}[e^{s\bar{X}_N}]}{e^{s(\varepsilon+\mu)}}$

$= e^{-s(\varepsilon+\mu) + \ln(M_X(s))}$

$M_X(s) = (\frac{1}{2} + \frac{1}{2}e^s)^N$

$\Downarrow$

$\therefore \exp[-s(\varepsilon+\mu) + \ln[(\frac{1}{2} + \frac{1}{2}e^s)^N]]$

$s^* = \arg\min_{s>0}\{\ \} \downarrow = \arg\min_{s>0}\{-s(\varepsilon+\mu) + \ln[(\frac{1}{2}+\frac{1}{2}e^s)^N]\}$

$\frac{d}{ds}(\cdot) = 0$

$\therefore -(\varepsilon+\mu) + N\frac{e^s}{1+e^s} = 0 \qquad \cancel{N}\frac{e^s}{1+e^s} = \frac{\varepsilon+\mu}{N}$

$\frac{1+e^s}{e^s} = \frac{\cancel{N}\,1}{\varepsilon+\mu} \qquad e^{-s} + 1 = \frac{N}{\varepsilon+\mu} \qquad e^{-s} = \frac{N}{\varepsilon+\mu} - 1 = \frac{\cancel{N}\,1 - \varepsilon - \mu}{\varepsilon+\mu}$

$+s^* = -\ln(\frac{\cancel{N}\,1 - \varepsilon - \mu}{\varepsilon+\mu})$

$\therefore e^{[\ln(\frac{\cancel{N}\,1-\varepsilon-\mu}{\varepsilon+\mu})](\varepsilon+\mu) + \ln[(\frac{1}{2}+\frac{1}{2}(\frac{\varepsilon+\mu}{\cancel{N}\,1-\varepsilon-\mu}))^N]}$

$= (\frac{N-\varepsilon-\mu}{\varepsilon+\mu})^{\varepsilon+\mu} \cdot (\frac{1}{2}+\frac{1}{2}(\frac{\varepsilon+\mu}{\cancel{N}\,1-\varepsilon-\mu}))^N$

$= (\frac{\cancel{N}\,1-\varepsilon-\frac{1}{2}}{\varepsilon+\frac{1}{2}})^{\varepsilon+\frac{1}{2}} \cdot (\frac{1}{2}+\frac{1}{2}(\frac{\varepsilon+\frac{1}{2}}{\cancel{N}\,1-\varepsilon-\frac{1}{2}}))^{N}$

$= (\frac{\cancel{N}\,1-\varepsilon-\frac{1}{2}}{\varepsilon+\frac{1}{2}})^{\varepsilon+\frac{1}{2}} \cdot (\frac{N-\cancel{\varepsilon}-\cancel{\frac{1}{2}}+\cancel{\varepsilon}+\cancel{\frac{1}{2}}}{2(N-\varepsilon-\frac{1}{2})})^N$

$= (\frac{\cancel{N}\,1-\varepsilon-\frac{1}{2}}{\varepsilon+\frac{1}{2}})^{\varepsilon+\frac{1}{2}} \cdot (\frac{\cancel{N}\,1}{2(N-\varepsilon-\frac{1}{2})})^N$

$= 2 \wedge \log_2[(\frac{N-\varepsilon-\frac{1}{2}}{\varepsilon+\frac{1}{2}})^{\varepsilon+\frac{1}{2}} \cdot (\frac{N}{2(N-\varepsilon-\frac{1}{2})})^N]$

$2 \wedge ((\varepsilon+\frac{1}{2})\log_2(\frac{N-\varepsilon-\frac{1}{2}}{\varepsilon+\frac{1}{2}}) + N\log_2\frac{N}{2(N-\varepsilon-\frac{1}{2})})$

$= 2 \wedge ((\varepsilon+\frac{1}{2})[\log_2(N-\varepsilon-\frac{1}{2}) - \log_2(\varepsilon+\frac{1}{2})] + N[\log_2 N - 1 - \log_2(N-\varepsilon-\frac{1}{2})])$

$N = 1$

$= 2 \wedge ((\varepsilon+\frac{1}{2})[\log_2(\frac{1}{2}-\varepsilon) - \log_2(\varepsilon+\frac{1}{2})] + [-1 - \log_2(\frac{1}{2}-\varepsilon)])$

$= 2 \wedge ((\varepsilon+\frac{1}{2})\log_2(\frac{1}{2}-\varepsilon) - (\varepsilon+\frac{1}{2})\log_2(\varepsilon+\frac{1}{2}) - 1 - \log_2(\frac{1}{2}-\varepsilon))$

$= 2 \wedge (-1 - (\varepsilon+\frac{1}{2})\log_2(\varepsilon+\frac{1}{2}) - (\frac{1}{2}-\varepsilon)\log_2(\frac{1}{2}-\varepsilon))N$

$\therefore \beta = \sim\sim \qquad \mu = 0.5$