HW1

Ruijie Song

Feb.4.2021

Exercise 1: Histogram and Cross-Validation

a)

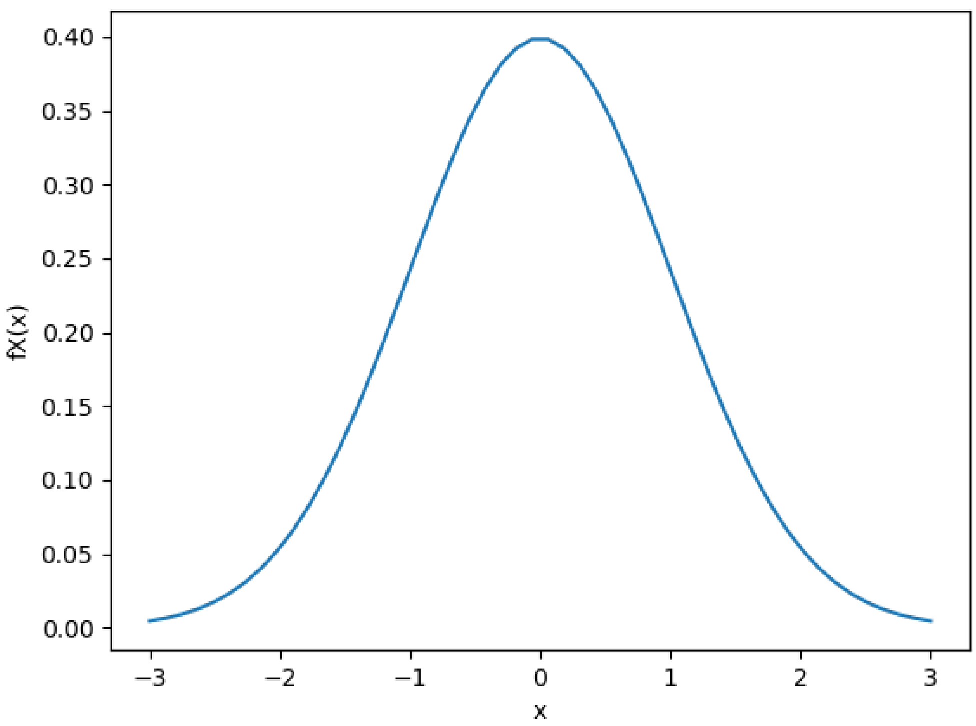


Figure 1. fX

b)
ii)

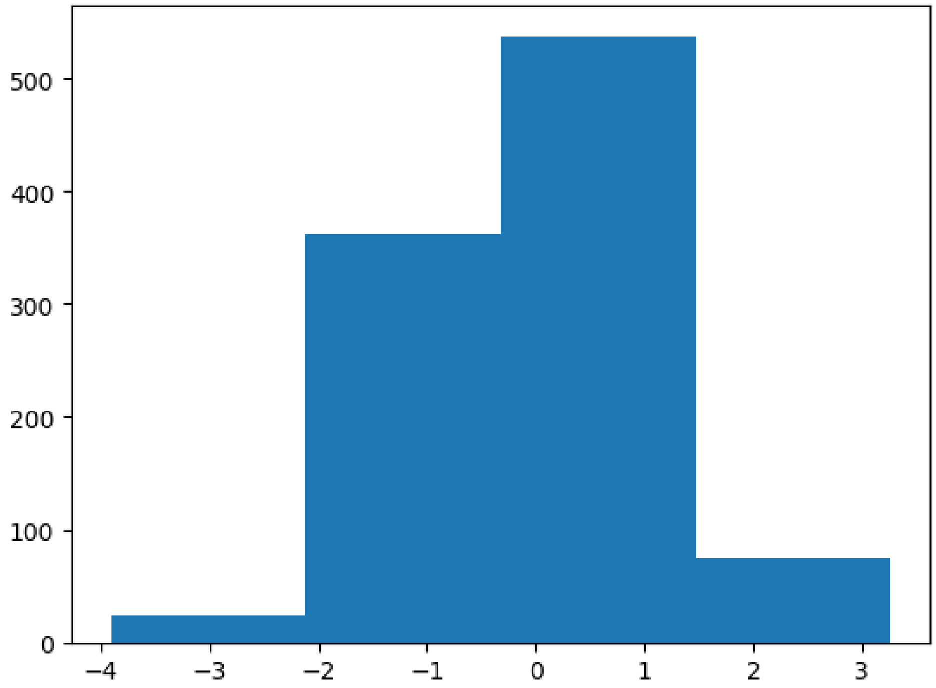


Figure 2. m=4

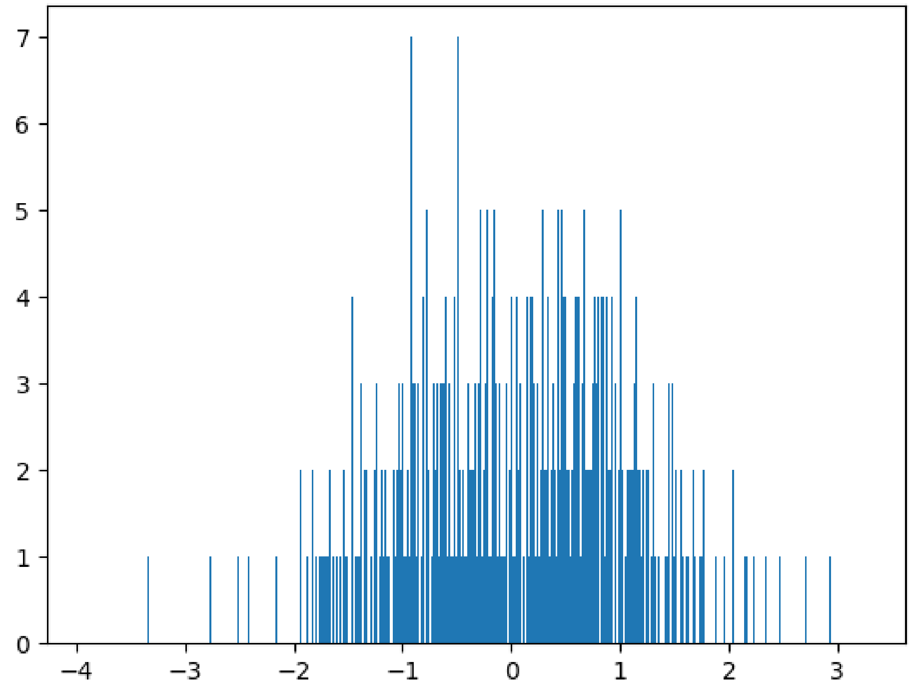


Figure 3. m=1000

iii)
mean = -0.021858244152l9112
standard deviation = 1.0469359084378456

iv)

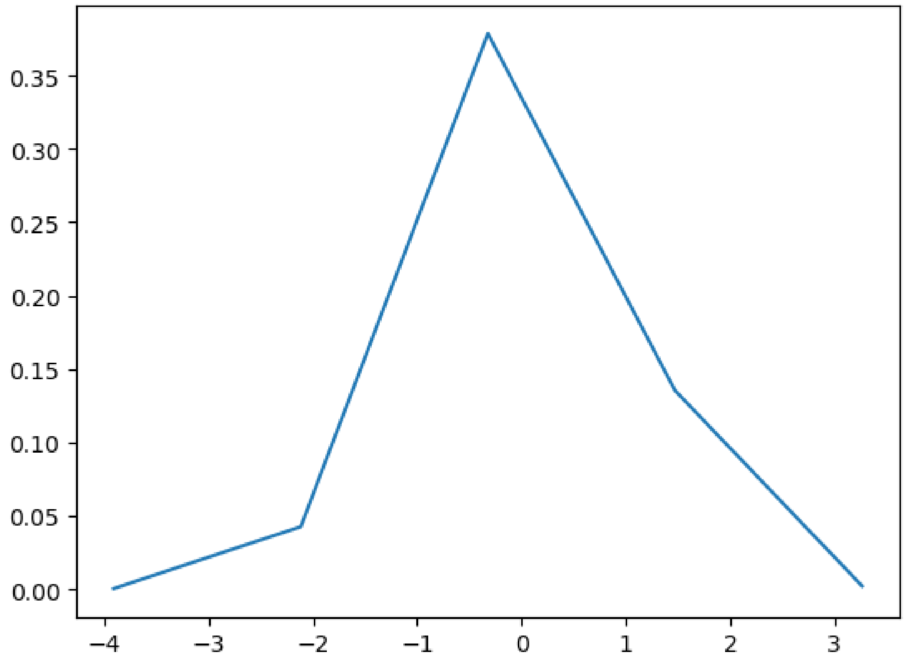


Figure 4. m=4

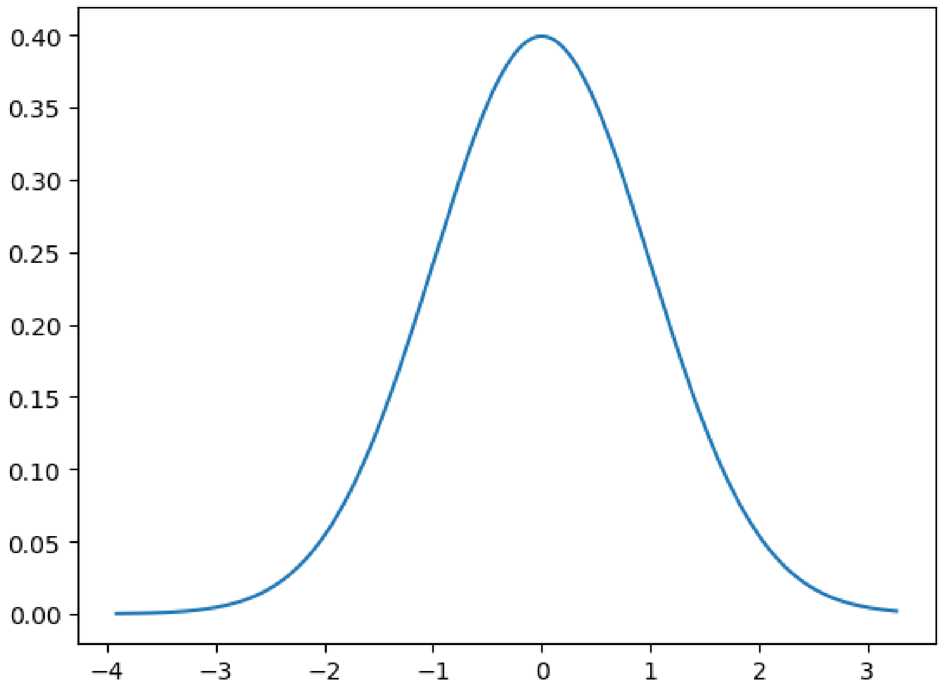


Figure 5. m=1000

c)
i)

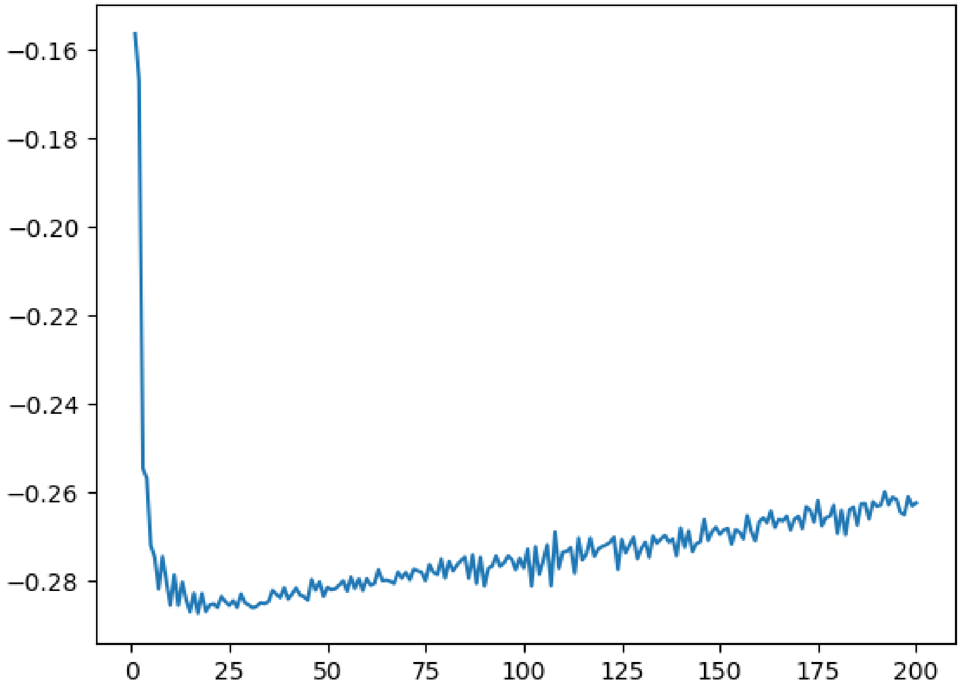


Figure 6. J(h)

ii)

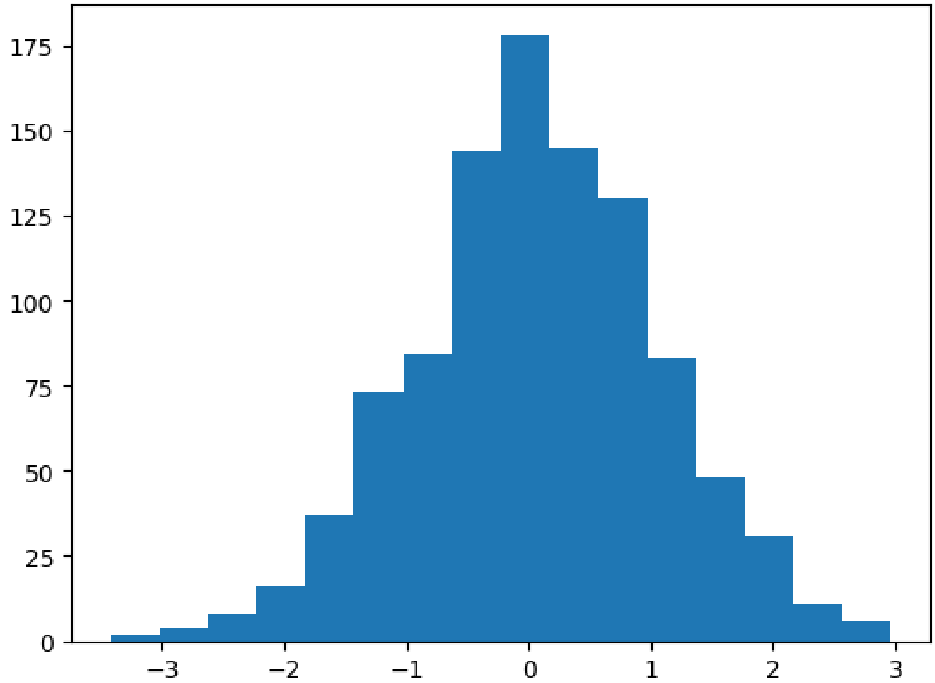


Figure 7. histogram of your data with that m∗

m* = 16
iii)

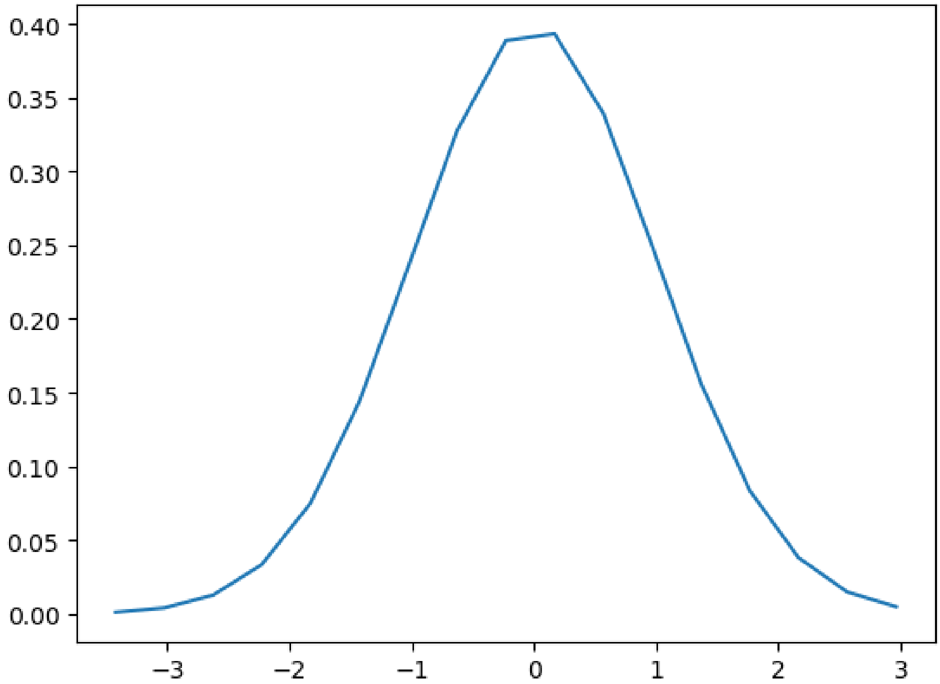


Figure 8. Gaussian curve

Exercise 2: Gaussian Whitening

a)

i)

2.

a) $X \sim N(\mu, \Sigma)$ $\quad f_X(x) = \frac{1}{\sqrt{(2\pi)^2 |\Sigma|}} \exp\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\}$

$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mu = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

i) $|\Sigma| = 4 - 1 = 3 \quad x - \mu = \begin{bmatrix} x_1 - 2 \\ x_2 - 6 \end{bmatrix}$

$\Sigma^{-1} = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$

$\therefore f_X(x) = \frac{1}{\sqrt{(2\pi)^2 3}} \exp\{-\frac{1}{2} [x_1 - 2, x_2 - 6] \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} x_1 - 2 \\ x_2 - 6 \end{bmatrix}\}$

$= \frac{1}{\sqrt{(2\pi)^2 3}} \exp\{\frac{2x_1^2 + 4x_1 - 2x_1 x_2 + 2x_2^2 + 56 - 20x_2}{-6}\}$

ii)

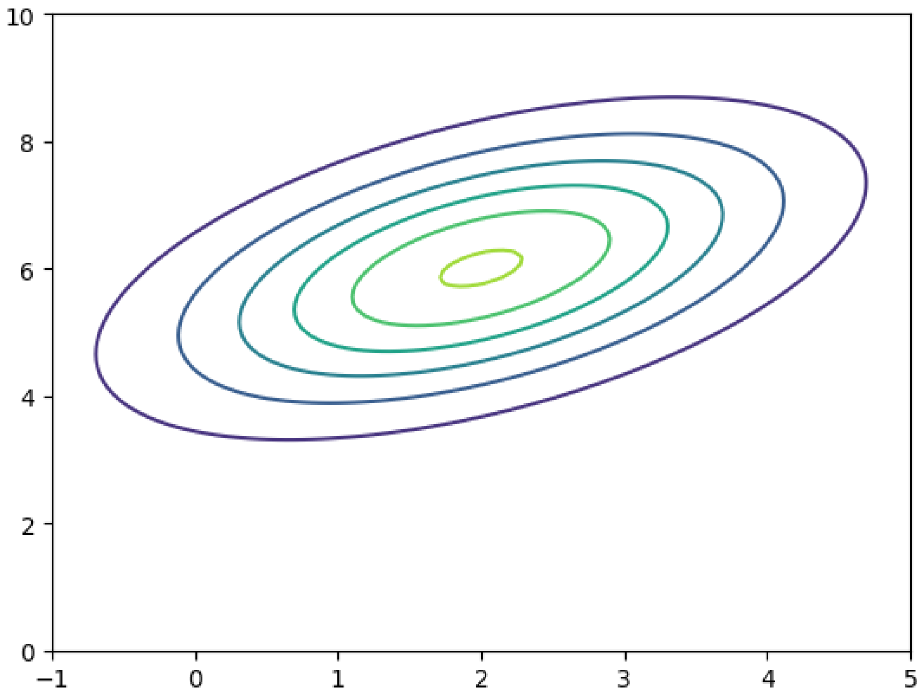


Figure 9. The contour of fX

b)

b) $A \in \mathbb{R}^{d\times d}$ $\quad b \in \mathbb{R}^{d}$ $\quad Y = AX + b$ $\quad X$: r.v $\sim N(0, I)$

$\mu_Y \overset{def}{=} E[Y]$ $\quad \Sigma_Y \overset{def}{=} E[(Y-\mu_Y)(Y-\mu_Y)^T]$

i) $E[Y] = E[AX + b] = A\cdot \underbrace{E[X]}_{0} + b = b = \mu_Y$

$\Sigma_Y = E[(AX + \cancel{b} - \cancel{b})(AX + \cancel{b} - \cancel{b})^T] = E[(AX)(AX)^T]$

$= E(AXX^TA^T) = A\,E[XX^T]\,A^T = AA^T$

ii) symmetric if $\Sigma = \Sigma^T$, positive semi-definite if $x^T\Sigma x \geq 0$ for any $x \in \mathbb{R}^n$

$\cancel{AA^T}$ $\Sigma_Y = AA^T$ must be symmetric since $A \in \mathbb{R}^{d\times d}$

Let x be a non-zero vector

$x^T\Sigma x = x^TA^TAx = (Ax)^T(Ax) = \|Ax\|^2 \geq 0.$

$\therefore$ Σ is symmetric positive semi-definite when $A \in \mathbb{R}^{d\times d}$

iii) $x^T\Sigma x > 0$ for $x \in \mathbb{R}^n$ when $\|Ax\| \neq 0$

iv) $Y \sim N(\mu_Y, \Sigma_Y)$ $\quad \mu_Y = \begin{bmatrix}2\\6\end{bmatrix}$, $\Sigma_Y = \begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$

$b = \mu_Y = \begin{bmatrix}2\\6\end{bmatrix}$ $\quad AA^T = \begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$

$A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$ $\quad \begin{bmatrix}a & b\\c & d\end{bmatrix}^{-1}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}^T = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

$\begin{bmatrix}2a+c & 2b+d\\a+2c & b+2d\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}$ $\quad \begin{cases}\cancel{2}a+c = \cancel{a}\ 0\\ \cancel{2}b+d = \cancel{b}\\ a+\cancel{2}c = \cancel{c}\\ b+\cancel{2}d = \cancel{d}\ 0\end{cases}$

$a+c=0$ $\quad a=-c$

$b+d=0$ $\quad b=-d$ $\quad \therefore A = \begin{bmatrix}a & b\\-a & -b\end{bmatrix}$

$U = \begin{bmatrix}-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$ $\quad \Sigma = \begin{bmatrix}1 & 0\\0 & 3\end{bmatrix}$

$\Sigma^{\frac{1}{2}} = A = U\Lambda^{\frac{1}{2}}U^T = \begin{bmatrix}1 & \frac{1}{2}\\ \frac{1}{2} & 1\end{bmatrix}$

c)
i)

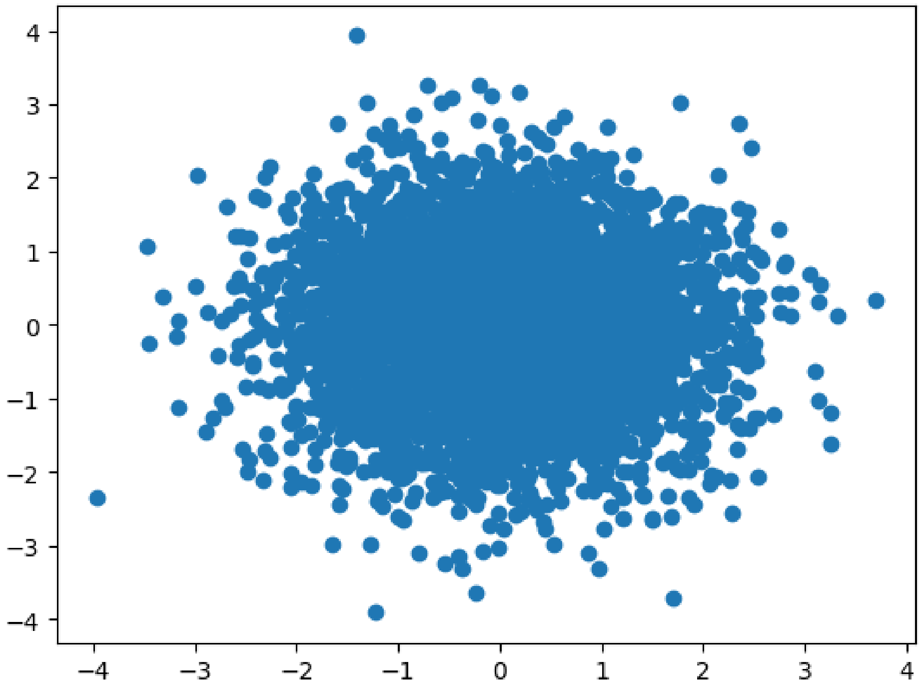


Figure 10. scatter plot

ii)

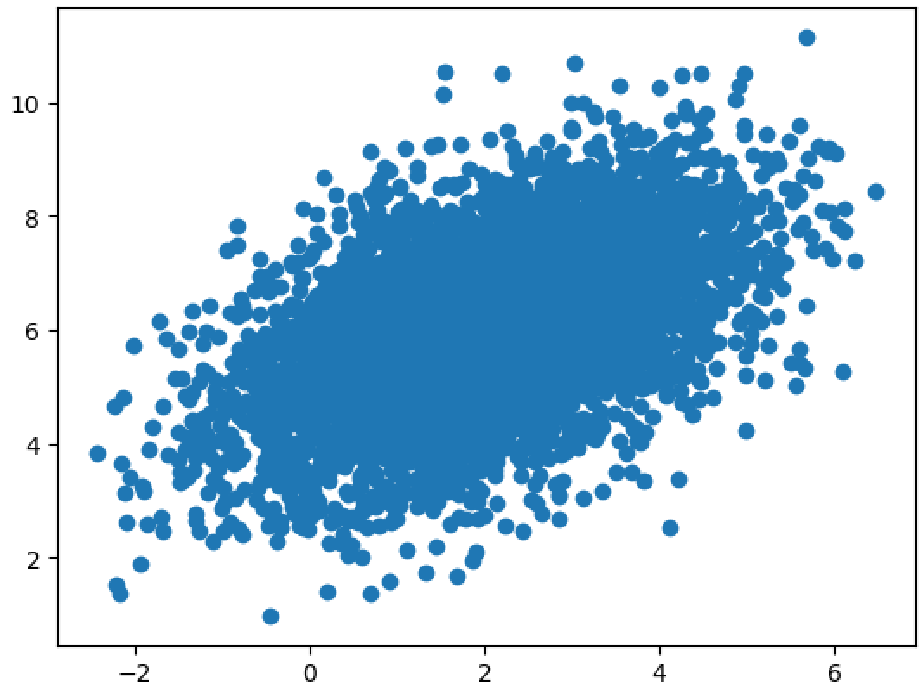


Figure 11. data calculated by numpy.random.multivariate_normal

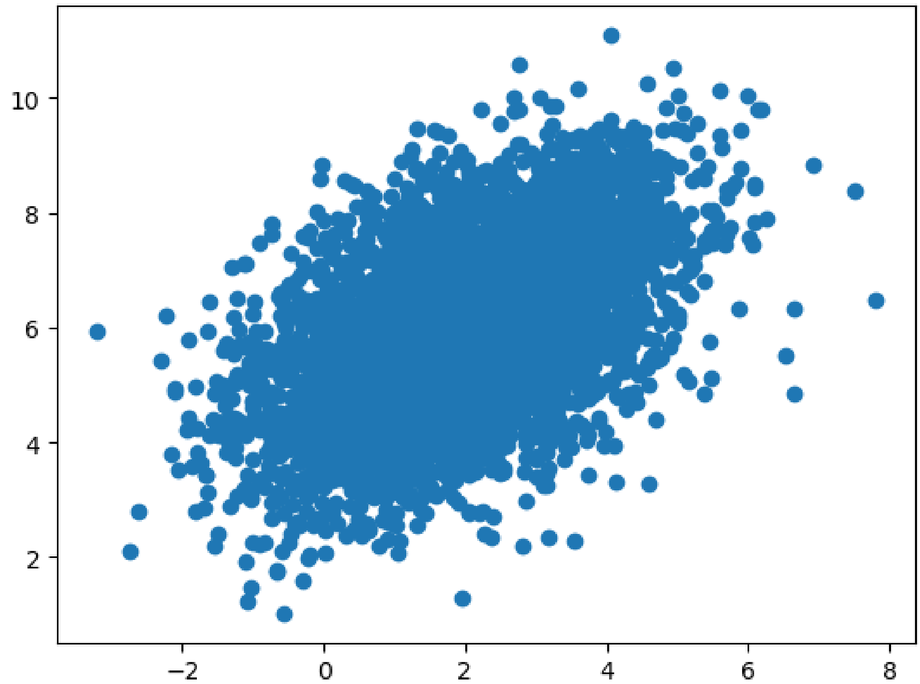
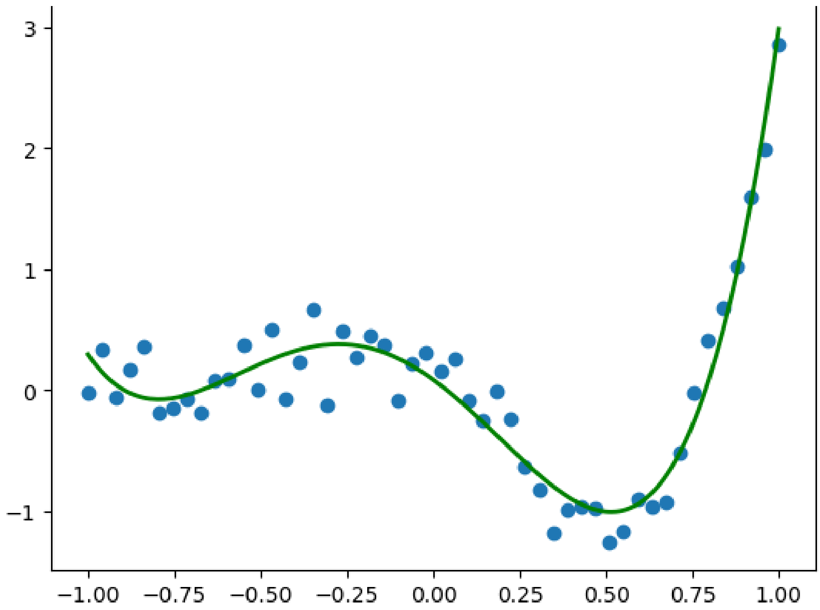


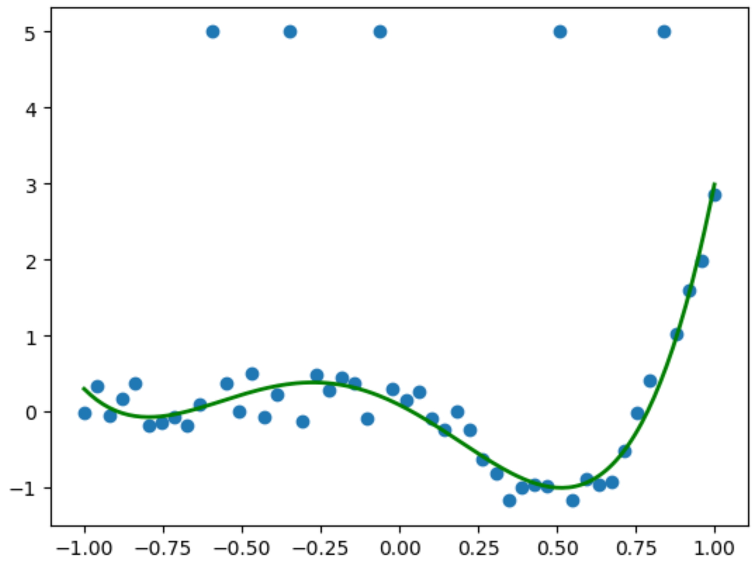
Figure 12. data calculated by affine transformation

Exercise 3: Linear Regression

a & c.



d.



There are a few differences between those 2 plot. However, the outliers do not affect much.

b & e.

3. b) $\hat{\beta} = \underset{\beta}{\arg\min} \, \|y - X\beta\|^2$

$$\beta = (X^T X)^{-1} X^T \underline{y}$$

X: data

$$y = g_{\beta}(X) = \beta^T X$$

e) $\hat{\beta} = \underset{\beta}{\arg\min} \, \|y - X\beta\|_1$

$$\Downarrow$$

$$\min_{\beta, \cancel{\{u_n\}}} \sum_{n=1}^{N} |y - X\beta|$$

~~s.t. $u_n \geq -(y - X\beta)$~~

~~$u_n \geq y - X\beta$~~

$$\Downarrow$$

$$\min_{\beta, \{u_n\}} \sum_{n=1}^{N} u_n \quad \text{s.t.} \quad u_n = |y - X\beta|$$

$$\Downarrow$$

$$\min_{\beta, \{u_n\}} \sum_{n=1}^{N} u_n \quad \text{s.t.} \quad \begin{aligned} u_n &\geq -(y - X\beta) \\ u_n &\geq (y - X\beta) \end{aligned}$$

$$\Downarrow$$

$$\min_{\beta, u} \left(\underbrace{[0 \cdots 0, 1 \cdots 1]}_{c} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \\ u_1 \\ \vdots \\ u_N \end{bmatrix} \right)$$

$$\underset{x}{\text{minimize}} \quad c^T x$$

$$\text{s.t.} \quad Ax \leq b$$

$$\underbrace{\begin{bmatrix} X_1 & -1 & & \\ \vdots & & \ddots & \\ X_N & & & -1 \\ -X_1 & -1 & & \\ \vdots & & \ddots & \\ -X_N & & & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_p \\ u_1 \\ \vdots \\ u_N \end{bmatrix}}_{x} \leq \underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_N \\ -y_1 \\ \vdots \\ -y_N \end{bmatrix}}_{b}$$

f.

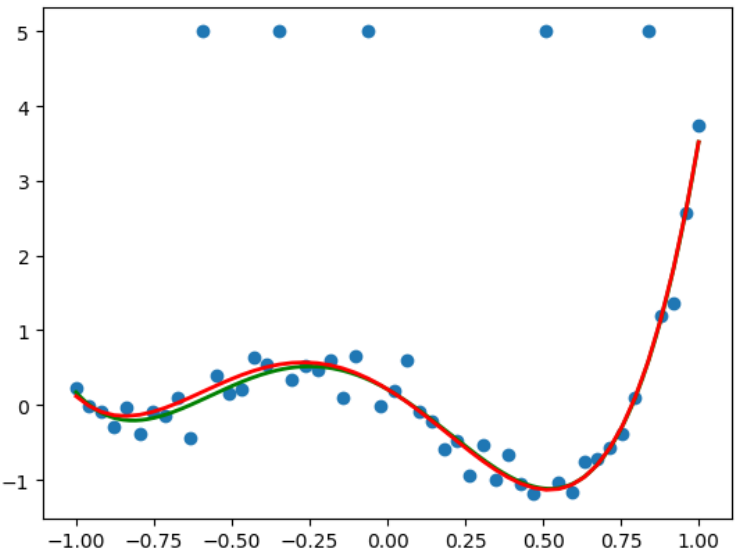


Figure 13. The green curve is for d, The red curve is for f

```python
# -*- coding: utf-8 -*-
"""
Created on Wed Feb  3 01:40:44 2021

@author: 11327
"""

import numpy as  np
import matplotlib.pyplot as plt
from scipy.stats import norm
from scipy.special import eval_legendre
from scipy.optimize import linprog


# 1a

x = np.linspace(-3,3)
fX = 1/np.sqrt(2*np.pi) * np.exp(-(x*x) / 2)

plt.plot(x,fX)
plt.xlabel('x')
plt.ylabel('fX(x)')
plt.savefig('D:/phd2/ece595/1a.png')
plt.show()

# 1b
x = np.random.normal(0, 1, 1000)

plt.figure(1)
hp1 = plt.hist(x,4)

plt.figure(2)
hp2 = plt.hist(x,1000)

m,sd = norm.fit(x)
plt.figure(3)
plt.plot(hp1[1],norm.pdf(hp1[1]))
plt.figure(4)
plt.plot(hp2[1],norm.pdf(hp2[1]))

# 1c
x = np.random.normal(0, 1, 1000)
n = 1000
maxx = np.max(x)
minx = np.min(x)
diff = maxx - minx
# pj = x[0:200:1]
# sumpj = np.sum(pj**2)
J = []
M = np.linspace(1,200,200)
for m in M:
    m = int(m)
    pj = (np.histogram(x,m))[0] / np.sum((np.histogram(x,m))[0])
    sumpj = np.sum(pj**2)
    Jh = 2/((n-1)*(diff/m)) - (n+1)/((diff/m)*(n-1))*sumpj
    J = np.append(J,Jh)

plt.figure(1)
plt.plot(M,J)

Jmin = np.min(J)
```

```
mmin = np.where(J==Jmin)

plt.figure(2)
hp1 = plt.hist(x,int(mmin[0]))
plt.figure(3)
plt.plot(hp1[1],norm.pdf(hp1[1]))

# 2a
x1 = np.linspace(-1,5,100)
x2 = np.linspace(0,10,100)
X1,X2 = np.meshgrid(x1,x2)
fX = (1/np.sqrt(3*(2*np.pi)**2))*np.exp((2*X1**2 + 4*X1 - 2*X1*X2 + 2*X2**2+56-20*X2)/(-6))

plt.figure(1)
plt.contour(X1,X2,fX)

# 2c
x = np.random.multivariate_normal([0, 0], [[1, 0], [0, 1]], 5000)
plt.figure(1)
plt.scatter(x[:, 0], x[:, 1])

X = np.random.multivariate_normal([2, 6], [[2, 1], [1, 2]], 5000)
plt.figure(2)
plt.scatter(X[:, 0], X[:, 1])

S = [[2, 1], [1, 2]]
b = [2,6]
L,U = np.linalg.eig(S)
A = np.dot(np.dot(U,np.diag(L**(0.5))),U.T)
y = np.dot(A,x.T)
y1 = y[0,:] + 2
y2 = y[1,:] + 6
plt.figure(3)
plt.scatter(y1, y2)
y = [y1,y2]

# 3a
b0 = -0.001
b1 = 0.01
b2 = 0.55
b3 = 1.5
b4 = 1.2
x = np.linspace(-1,1,50) # 50 points in the interval [-1,1]
L1 = eval_legendre(1,x)
L2 = eval_legendre(2,x)
L3 = eval_legendre(3,x)
L4 = eval_legendre(4,x)
error = np.random.normal(0,0.2,50)
y = b0 + b1*L1 + b2*L2 + b3*L3 + b4*L4 + error


X = np.column_stack((eval_legendre(0,x), eval_legendre(1,x), \
                     eval_legendre(2,x), eval_legendre(3,x), \
                     eval_legendre(4,x)))
theta = np.linalg.lstsq(X, y, rcond=None)[0]
t     = np.linspace(-1, 1, 200);
yhat  = theta[0]*eval_legendre(0,t) + theta[1]*eval_legendre(1,t) + \
        theta[2]*eval_legendre(2,t) + theta[3]*eval_legendre(3,t) + \
        theta[4]*eval_legendre(4,t)

plt.figure(1)
```

```
plt.scatter(x, y)
plt.plot(t,yhat,'g', linewidth=2)
plt.show()

idx = [10,16,23,37,45] # these are the locations of the outliers
y[idx] = 5 # set the outliers to have a value 5
plt.figure(2)
plt.scatter(x, y)
plt.plot(t,yhat,'g', linewidth=2)
plt.show()


N = 50
for p in range(5):
  X[:,p] = eval_legendre(p,x)

Au = np.hstack((X,-np.eye(N)))
Al = np.hstack((-X,-np.eye(N)))
A = np.vstack((Au,Al))
b = np.hstack((y,-y))
c = np.hstack((np.zeros(5),np.ones(N)))

sol = linprog(c,A,b,bounds=(None,None))
beta = sol.x[0:5]

t2     = np.linspace(-1, 1, 50);
yhat2  = beta[0]*eval_legendre(0,t2) + beta[1]*eval_legendre(1,t2) + \
        beta[2]*eval_legendre(2,t2) + beta[3]*eval_legendre(3,t2) + \
        beta[4]*eval_legendre(4,t2)
plt.plot(t2,yhat2,'r', linewidth=2)
plt.show()
```

Implementing Noise2Noise for Dynamic Scenes

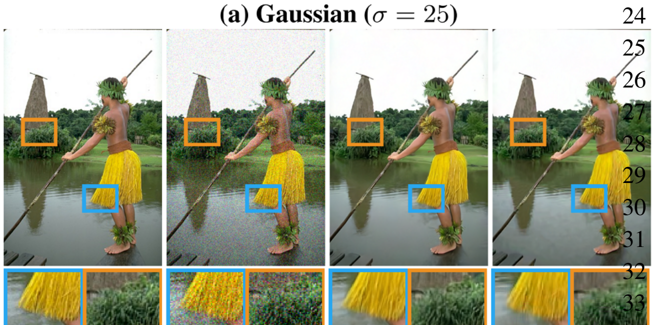
Ruijie Song M.S. [*1]

Abstract

Implementing Noise2Noise for Dynamic Scenes

1. Dataset

(a) Gaussian ($\sigma = 25$)



Ground truth Input Our Comparison BM3D

There's the dataset.

1.1. Codes

```
# Copyright (c) 2018, NVIDIA CORPORATION
#
# This work is licensed under the Creative
# 4.0 International License. To view a cop
# http://creativecommons.org/licenses/by-n
# Creative Commons, PO Box 1866, Mountain

import tensorflow as tf

def parse_tfrecord_tf(record):
    features = tf.parse_single_example(rec
        'shape': tf.FixedLenFeature([3], t
        'data': tf.FixedLenFeature([], tf.
    data = tf.decode_raw(features['data'],
    return tf.reshape(data, features['shape'

# [c,h,w] -> [h,w,c]
def chw_to_hwc(x):
    return tf.transpose(x, perm=[1, 2, 0])

# [h,w,c] -> [c,h,w]
def hwc_to_chw(x):
    return tf.transpose(x, perm=[2, 0, 1])

def resize_small_image(x):
    shape = tf.shape(x)
    return tf.cond(
        tf.logical_or(
            tf.less(shape[2], 256),
            tf.less(shape[1], 256)
        ),
        true_fn=lambda: hwc_to_chw(tf.image.resize_i
        false_fn=lambda: tf.cast(x, tf.float32)
    )

def random_crop_noised_clean(x, add_noise):
    cropped = tf.random_crop(resize_small_image(x),
    return (add_noise(cropped), add_noise(cropped),

def create_dataset(train_tfrecords, minibatch_size,
    print ('Setting up dataset source from', train_
    buffer_mb   = 256
    num_threads = 2
    dset = tf.data.TFRecordDataset(train_tfrecords,
    dset = dset.repeat()
    buf_size = 1000
    dset = dset.prefetch(buf_size)
    dset = dset.map(parse_tfrecord_tf, num_paralle
    dset = dset.shuffle(buffer_size=buf_size)
    dset = dset.map(lambda x: random_crop_noised_cl
    dset = dset.batch(minibatch_size)
    it = dset.make_one_shot_iterator()
    return it
```

A. Do *not* have an appendix here

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[*]Equal contribution [1]Department of Electrical and Computer Engineering, Purdue University, West Lafayette, USA. Correspondence to: Cieua Vvvvv <c.vvvvv@googol.com>, Eee Pppp <ep@eden.co.uk>.

Proceedings of the 38th International Conference on Machine Learning, PMLR 139, 2021.

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