

ECE 637 Lab1 Report

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Section 3 Report:

1.

Handwritten derivation of the analytical expression for H . The derivation starts with the definition of $h(m, n)$ as a 2D rectangular pulse. It then computes the 2D DFT $H(e^{ju}, e^{jv})$ by summing over m and n . The separability of $h(m, n)$ is noted, allowing the double sum to be split into two single sums. These sums are then evaluated using the geometric series formula $\sum_{n=0}^{K-1} p^n = \frac{1-p^K}{1-p}$. The final result is expressed as a product of two sinc-like functions in the frequency domain.

$$\begin{aligned} \textcircled{1} \quad h(m, n) &= \begin{cases} 1/81 & |m| \leq 4 \text{ and } |n| \leq 4 \\ 0 & \text{otherwise} \end{cases} \\ H(e^{ju}, e^{jv}) &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h(m, n) e^{-j(u m + v n)} \\ &= \sum_{n=-4}^4 \sum_{m=-4}^4 \frac{1}{81} e^{-j(u m + v n)} \\ \text{Since } h(m, n) &\text{ is separable,} \\ \therefore H &= \sum_{n=-4}^4 \frac{1}{9} e^{-jv n} \sum_{m=-4}^4 \frac{1}{9} e^{-j u m} = \sum_{n=0}^8 \frac{1}{9} e^{-jv(n-4)} \sum_{m=0}^8 \frac{1}{9} e^{-j u(m-4)} \\ &= \frac{1}{9} e^{jv4} \sum_{n=0}^8 e^{-jv n} \cdot \frac{1}{9} e^{ju4} \sum_{m=0}^8 e^{-j u m} \\ &= \frac{1}{81} e^{jv4} \frac{1 - (e^{-jv})^9}{1 - e^{-jv}} \cdot e^{ju4} \frac{1 - (e^{-ju})^9}{1 - e^{-ju}} \end{aligned}$$

Figure 1. A derivation of the analytical expression for H

2.

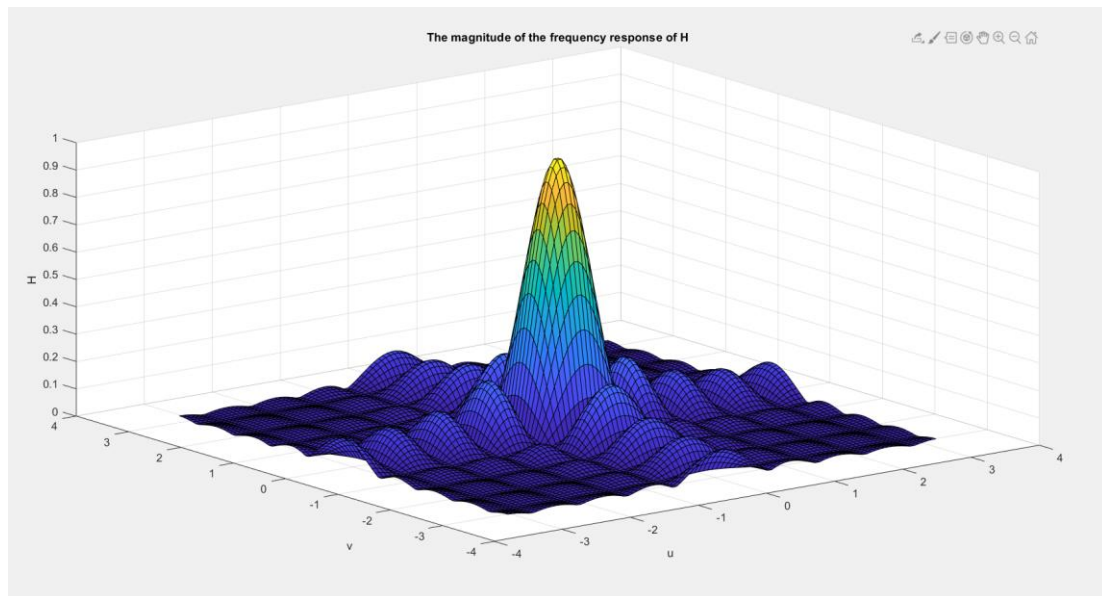


Figure 2. A plot of $|H|$

3.



Figure 3. img03.tif



Figure 4. color.tif



Figure 5. green.tif

4.



Figure 6. The filtered color image

Section 4 Report:

1. & 2.

$$\begin{aligned}
 4 \quad h(m, n) &= \begin{cases} 1/25 & |m| \leq 2, |n| \leq 2 \\ 0 & \text{otherwise} \end{cases} & \sum_{n=0}^{K-1} p^n = \frac{1-p^K}{1-p} \\
 g(m, n) &= s(m, n) + \lambda (s(m, n) - h(m, n)) \\
 \textcircled{1} \quad H(e^{ju}, e^{jv}) &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h(m, n) e^{-j(um+vn)} \\
 &= \frac{1}{25} \sum_{n=-2}^2 \sum_{m=-2}^2 e^{-j(um+vn)} \\
 &= \frac{1}{5} \sum_{n=-2}^2 e^{-jvn} \cdot \frac{1}{5} \sum_{m=-2}^2 e^{-jum} = \frac{1}{5} \sum_{n=0}^4 e^{-jv(n-2)} \cdot \frac{1}{5} \sum_{m=0}^4 e^{-ju(m-2)} \\
 &= \frac{1}{5} e^{jv2} \sum_{n=0}^4 e^{-jvn} \cdot \frac{1}{5} e^{ju2} \sum_{m=0}^4 e^{-jum} \\
 &= \frac{1}{25} e^{jv2} \frac{1-(e^{-jv})^5}{1-e^{-jv}} \cdot e^{ju2} \frac{1-(e^{-ju})^5}{1-e^{-ju}} \\
 \textcircled{2} \quad \underline{s(x, y)} = \underline{s(m, n) = s(m)s(n)} & \quad s(m) \leftrightarrow 1 \\
 \text{DSFT}(s(m, n)) &= \text{DSFT}(s(m)s(n)) = \text{DTFT}(s(m)) \cdot \text{DTFT}(s(n)) \\
 &= 1 \\
 \therefore G &= 1 + \lambda(1 - H)
 \end{aligned}$$

Figure 7. A derivation of the analytical expression for H & G

3.

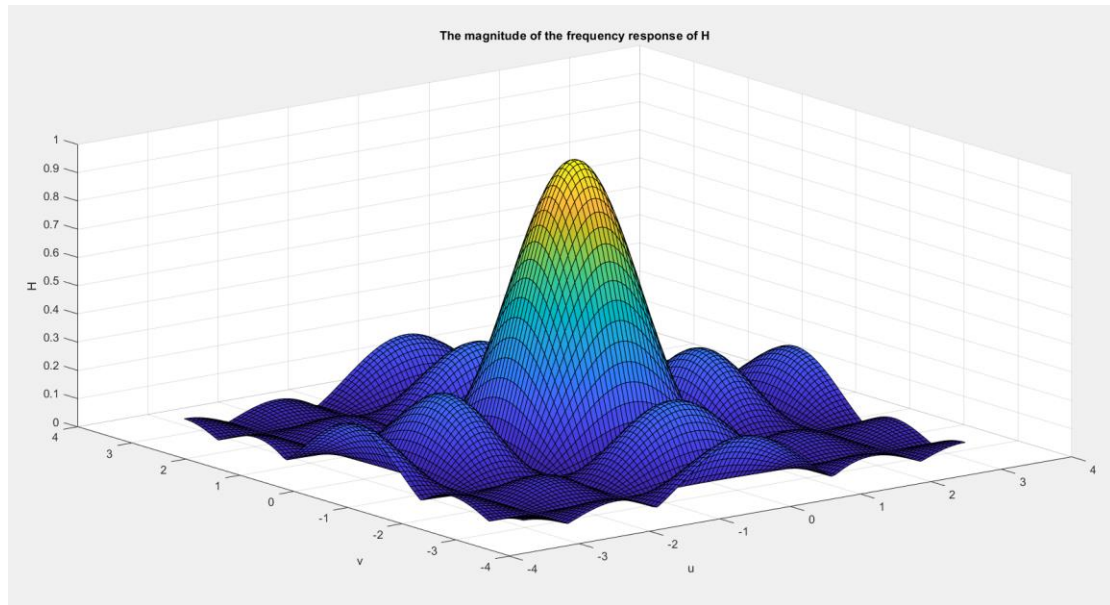


Figure 8. A plot of $|H|$

4.

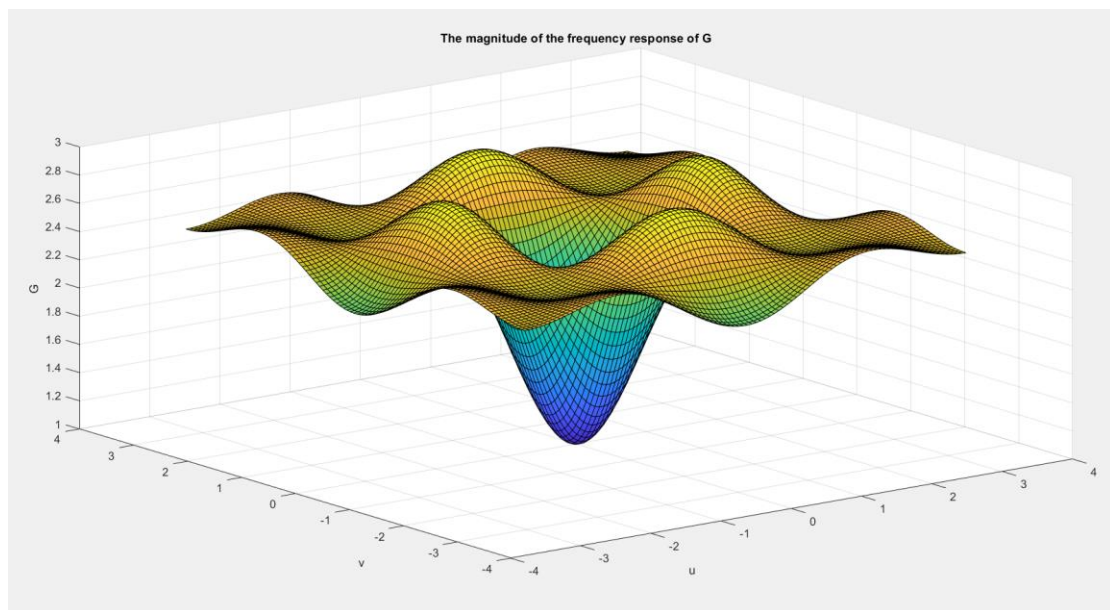


Figure 9. A plot of $|G|$

5.



Figure 10. imgblur.tif.

6.



Figure 11. The output sharpened color image for $\lambda = 1.5$

5. IIR Filter

1.

$$\begin{aligned}
 5. \quad y(m, n) &= 0.01x(m, n) + 0.9(y(m-1, n) + y(m, n-1)) - 0.81y(m-1, n-1) \\
 \cancel{y(m, n)} &\longleftrightarrow \cancel{Y(z_1, z_2)} & \cancel{x(m, n)} &\longleftrightarrow \cancel{X(z_1, z_2)} \\
 y(m, n) &\longleftrightarrow Y(e^{ju}, e^{jv}) & y(m-1, n) &\longleftrightarrow e^{-ju} Y(e^{ju}, e^{jv}) \\
 y(m, n-1) &\longleftrightarrow Y(e^{ju}, e^{jv}) \cdot e^{-jv} & y(m-1, n-1) &\longleftrightarrow e^{-ju} e^{-jv} Y(e^{ju}, e^{jv}) \\
 H = \frac{Y}{X} & & Y &= 0.01X + 0.9(e^{-ju}Y + e^{-jv}Y) - 0.81Y \cdot e^{-ju} e^{-jv} \\
 & & &= 0.01X + (0.9e^{-ju} + 0.9e^{-jv})Y - 0.81e^{-ju} e^{-jv} Y \\
 0.01X &= Y + 0.81e^{-jv} e^{-ju} Y - (0.9e^{-ju} + 0.9e^{-jv})Y \\
 &= Y(1 + 0.81e^{-jv} e^{-ju} - (0.9e^{-ju} + 0.9e^{-jv})) \\
 H = \frac{Y}{X} &= \frac{0.01}{1 + 0.81e^{-jv} e^{-ju} - (0.9e^{-ju} + 0.9e^{-jv})}
 \end{aligned}$$

Figure 12. A derivation of the analytical expression for H

2.

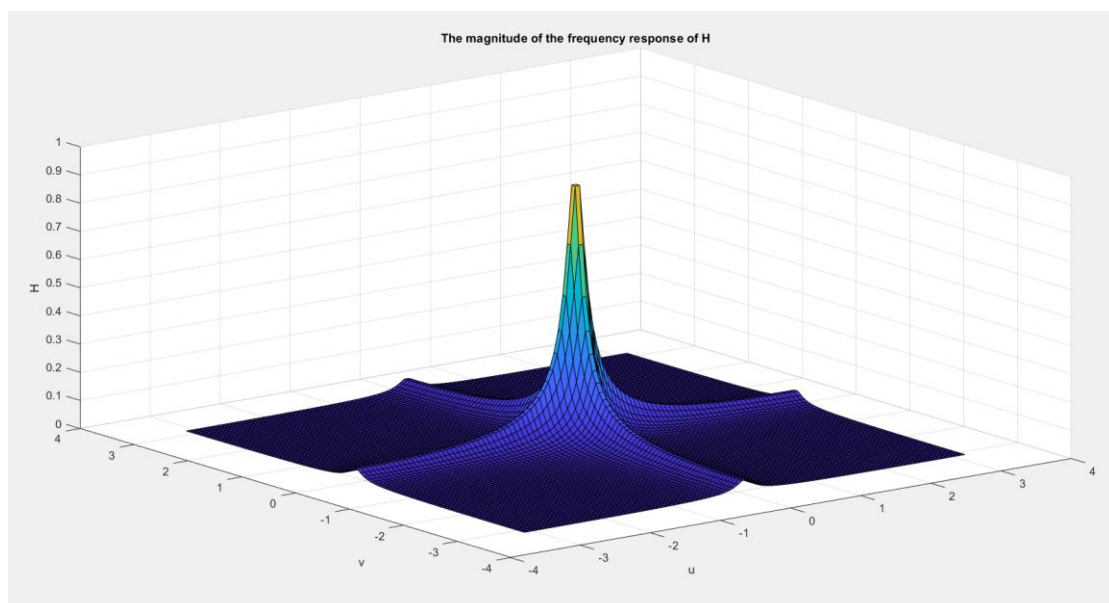


Figure 13. A plot of |H|

3.

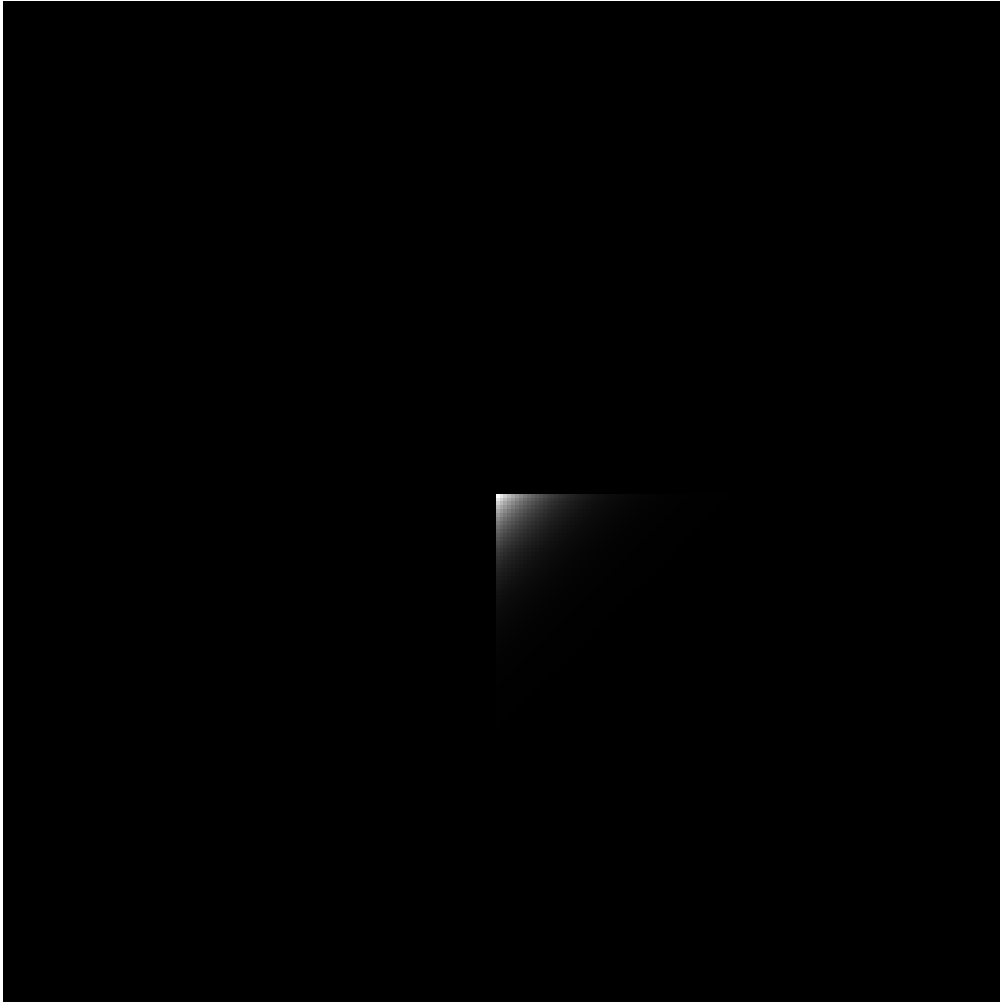


Figure 14. An image of the point spread function

4.



Figure 15. The filtered output color image.