ECE 637 Lab1 Report

Ruijie Song

Jan. 29. 2021

Section 3 Report:

1.

3 ① $h(m,n) = \begin{cases} 1/81 & |m| \le 4 \text{ and } |n| \le 4 \\ 0 & \text{otherwise} \end{cases}$

$$H(e^{j\mu}, e^{j\nu}) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h(m,n)\, e^{-j(\mu m + \nu n)}$$

$$= \sum_{n=-4}^{4} \sum_{m=-4}^{4} \frac{1}{81} e^{-j(\mu m + \nu n)}$$

Since $h(m,n)$ is separable.

$$\sum_{n=0}^{K-1} p^n = \frac{1-p^K}{1-p}$$

$$\therefore H = \sum_{n=-4}^{4} \frac{1}{9} e^{-j\nu n} \sum_{m=-4}^{4} \frac{1}{9} e^{-j\mu m} = \sum_{n=0}^{8} \frac{1}{9} e^{-j\nu(n-4)} \sum_{\cancel{m=-4}\ m=0}^{8} \frac{1}{9} e^{-j\mu(m-4)}$$

$$= \frac{1}{9} e^{j\nu 4} \sum_{n=0}^{8} e^{-j\nu n} \cdot \frac{1}{9} e^{j\mu 4} \sum_{m=0}^{8} e^{-j\mu m}$$

$$= \frac{1}{81} e^{j\nu 4} \frac{1-(e^{-j\nu})^9}{1-e^{-j\nu}} \cdot e^{j\mu 4} \frac{1-(e^{-j\mu})^9}{1-e^{-j\mu}}$$

Figure 1. A derivation of the analytical expression for H

2.

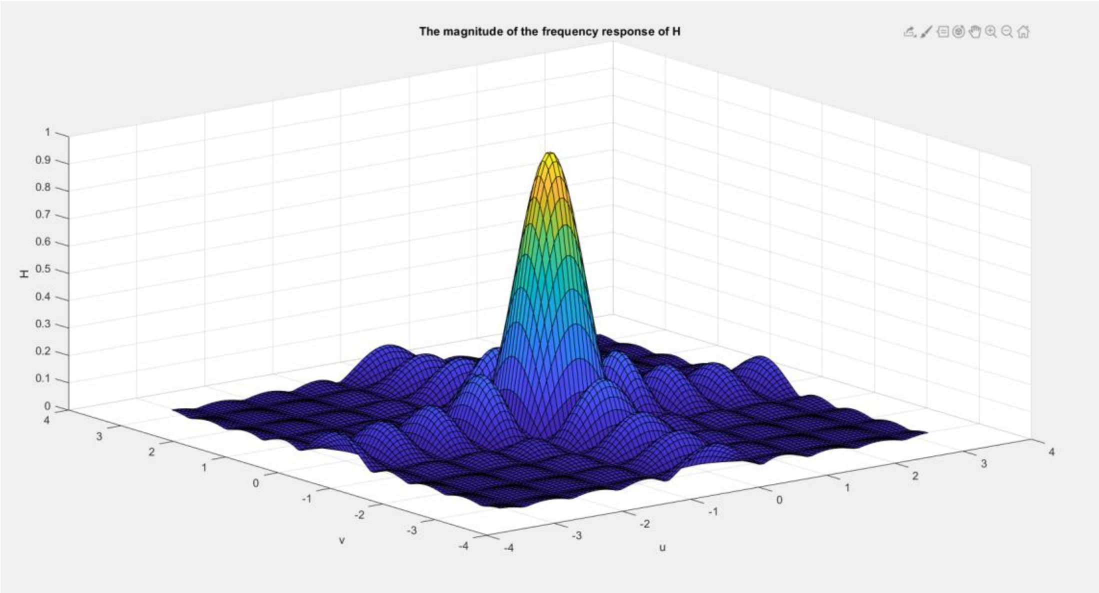


Figure 2. A plot of |H|

3.

Figure 3. img03.tif

Figure 4. color.tif

Figure 5. green.tif

4.

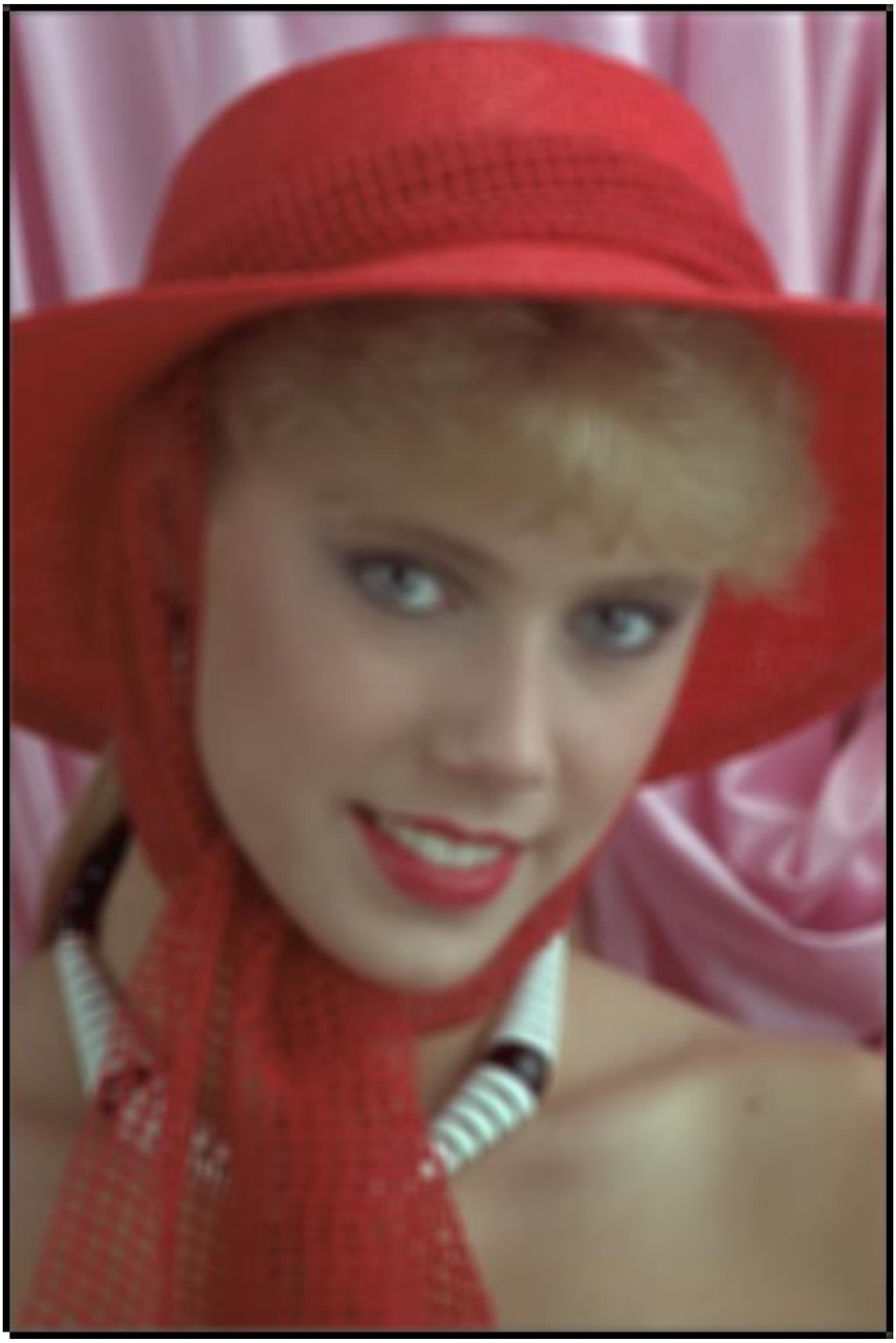


Figure 6. The filtered color image

Section 4 Report:

1. & 2.

4. $h(m,n) = \begin{cases} 1/25 & |m| \le 2,\ |n| \le 2 \\ 0 & \text{otherwise} \end{cases}$ $\qquad \sum_{n=0}^{K-1} p^n = \frac{1-p^K}{1-p}$

$g(m,n) = \delta(m,n) + \lambda(\delta(m,n) - h(m,n))$

① $H(e^{ju}, e^{jv}) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h(m,n)\, e^{-j(um+vn)}$

$$= \frac{1}{25} \sum_{n=-2}^{2} \sum_{m=-2}^{2} e^{-j(um+vn)}$$

$$= \frac{1}{5} \sum_{n=-2}^{2} e^{-jvn} \cdot \frac{1}{5} \sum_{m=-2}^{2} e^{-jum} = \frac{1}{5} \sum_{n=0}^{4} e^{-jv(n-2)} \cdot \frac{1}{5} \sum_{m=0}^{4} e^{-ju(m-2)}$$

$$= \frac{1}{5} e^{jv2} \sum_{n=0}^{4} e^{-jvn} \cdot \frac{1}{5} e^{ju2} \sum_{m=0}^{4} e^{-jum}$$

$$= \frac{1}{25} e^{jv2} \frac{1-(e^{-jv})^5}{1-e^{-jv}} \cdot e^{-ju2} \cdot \frac{1-(e^{-ju})^5}{1-e^{-ju}}$$

② ~~$\delta(x,y)$~~ $\delta(m,n) = \delta(m)\delta(n)$ $\qquad \delta(m) \longleftrightarrow 1$

$\updownarrow$

1

$DSFT(\delta(m,n)) = DSFT(\delta(m)\delta(n)) = DTFT(\delta(m)) \cdot DTFT(\delta(n))$

$= 1$

$\therefore G = 1 + \lambda(1 - H)$

Figure 7. A derivation of the analytical expression for H & G

3.

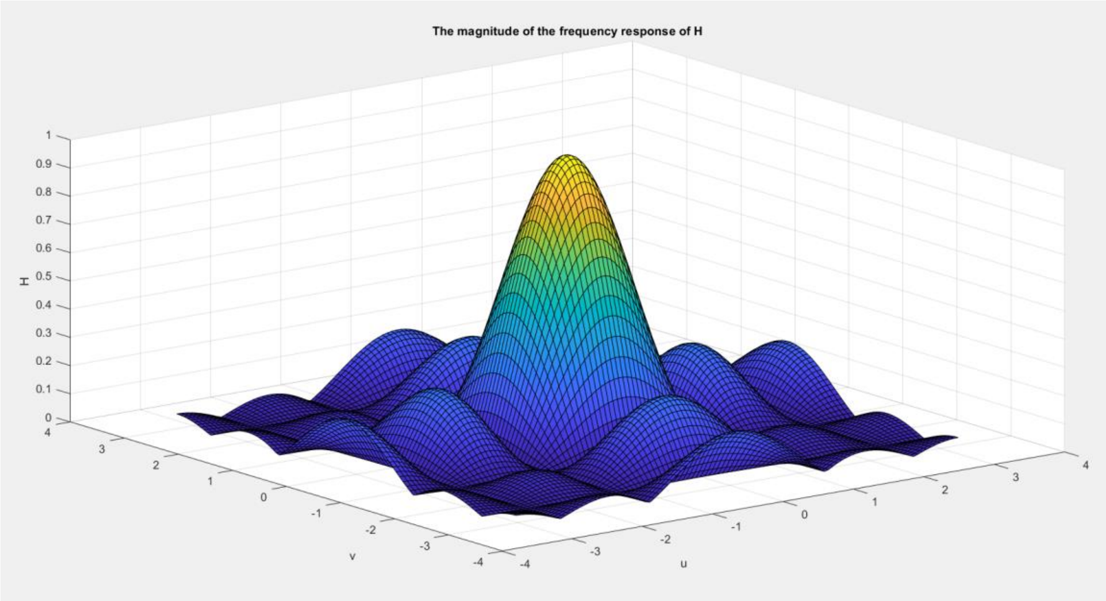


Figure 8. A plot of |H|

4.

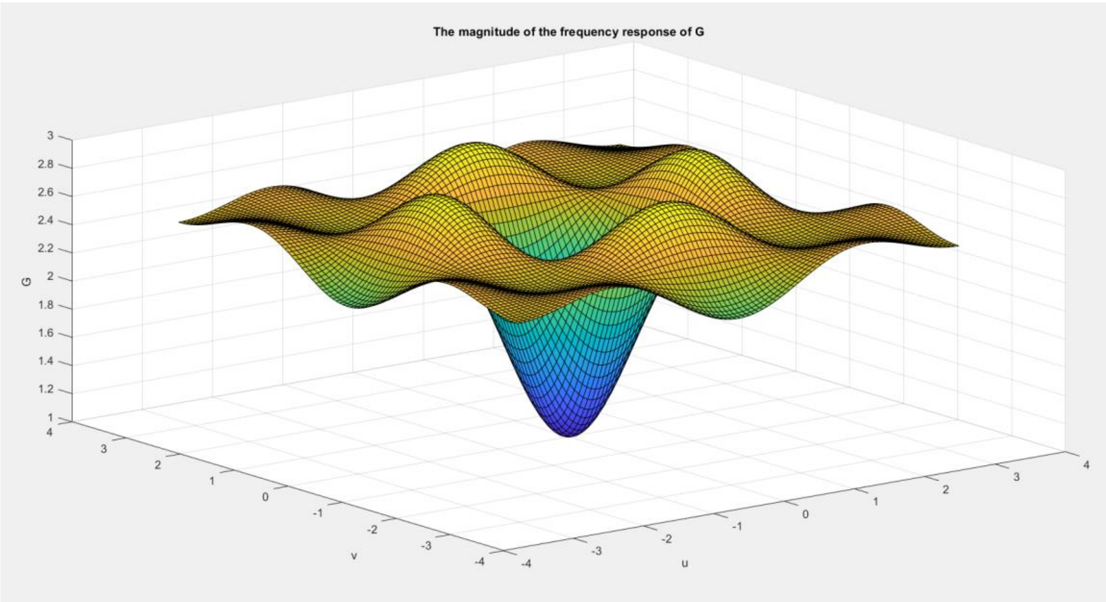


Figure 9. A plot of |G|

5.

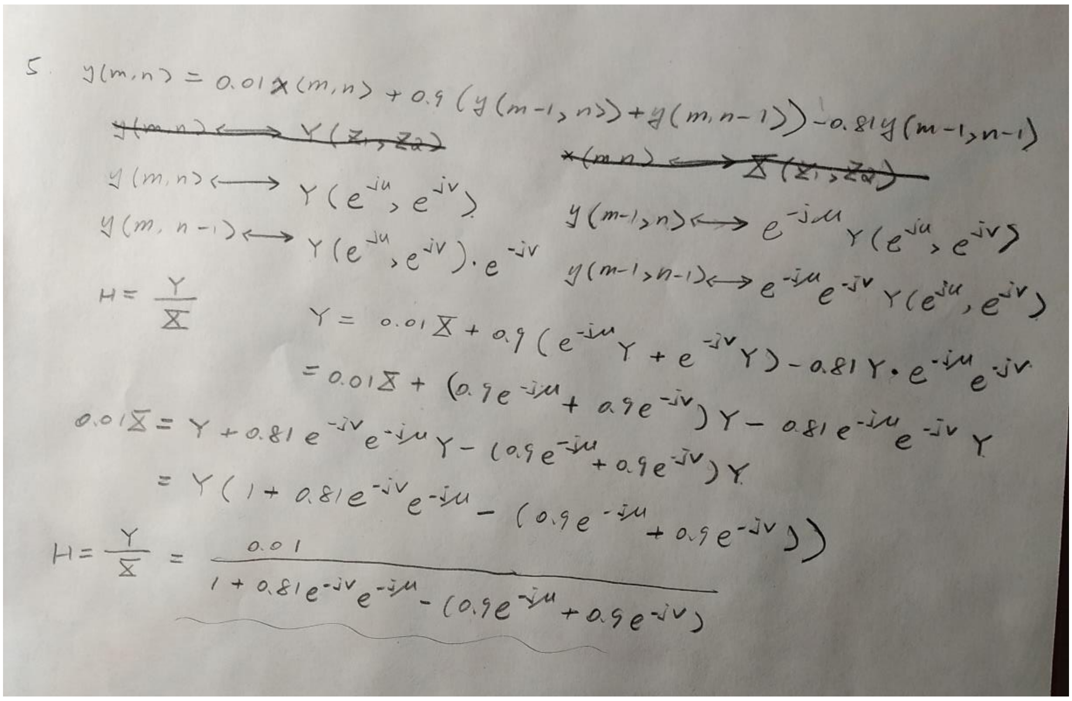
Figure 10. imgblur.tif.

6.

Figure 11. The output sharpened color image for $\lambda = 1.5$

5. IIR Filter

1.



5. $y(m,n) = 0.01x(m,n) + 0.9\,(y(m-1,n) + y(m,n-1)) - 0.81\,y(m-1,n-1)$

$\cancel{y(m,n) \longleftrightarrow Y(z_1,z_2)}$ $\cancel{x(m,n) \longleftrightarrow X(z_1,z_2)}$

$y(m,n) \longleftrightarrow Y(e^{ju}, e^{jv})$

$y(m,n-1) \longleftrightarrow Y(e^{ju}, e^{jv}) \cdot e^{-jv}$

$y(m-1,n) \longleftrightarrow e^{-ju}\,Y(e^{ju}, e^{jv})$

$y(m-1,n-1) \longleftrightarrow e^{-ju}e^{-jv}\,Y(e^{ju}, e^{jv})$

$H = \frac{Y}{X}$

$$Y = 0.01X + 0.9\,(e^{-ju}Y + e^{-jv}Y) - 0.81\,Y \cdot e^{-ju}e^{-jv}$$

$$= 0.01X + (0.9e^{-ju} + 0.9e^{-jv})Y - 0.81e^{-ju}e^{-jv}Y$$

$$0.01X = Y + 0.81e^{-jv}e^{-ju}Y - (0.9e^{-ju} + 0.9e^{-jv})Y$$

$$= Y(1 + 0.81e^{-jv}e^{-ju} - (0.9e^{-ju} + 0.9e^{-jv}))$$

$$H = \frac{Y}{X} = \frac{0.01}{1 + 0.81e^{-jv}e^{-ju} - (0.9e^{-ju} + 0.9e^{-jv})}$$

Figure 12. A derivation of the analytical expression for H

2.

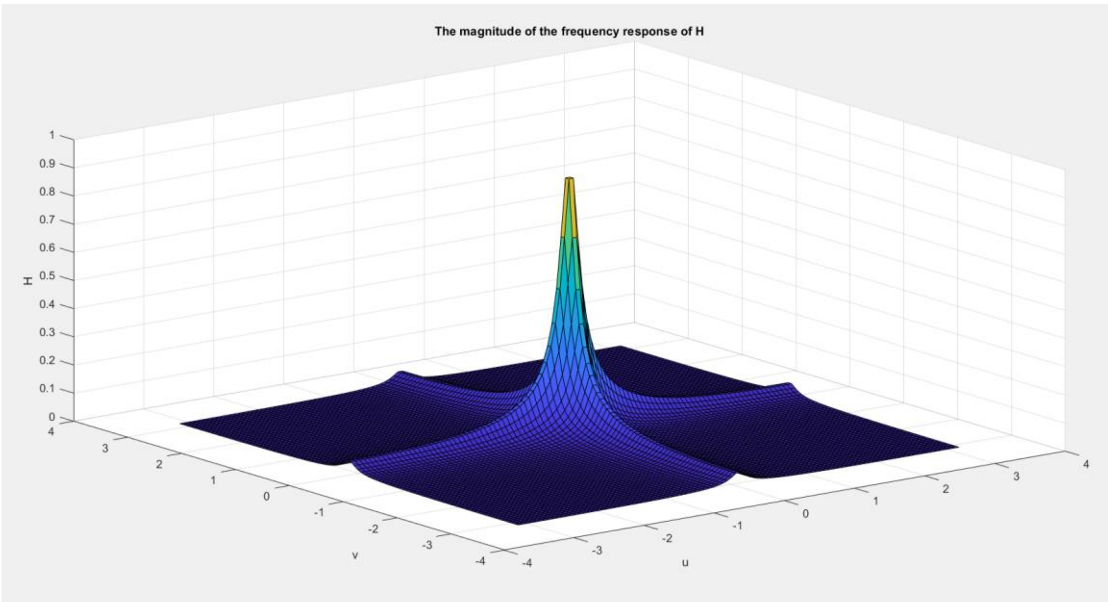


Figure 13. A plot of |H|

3.

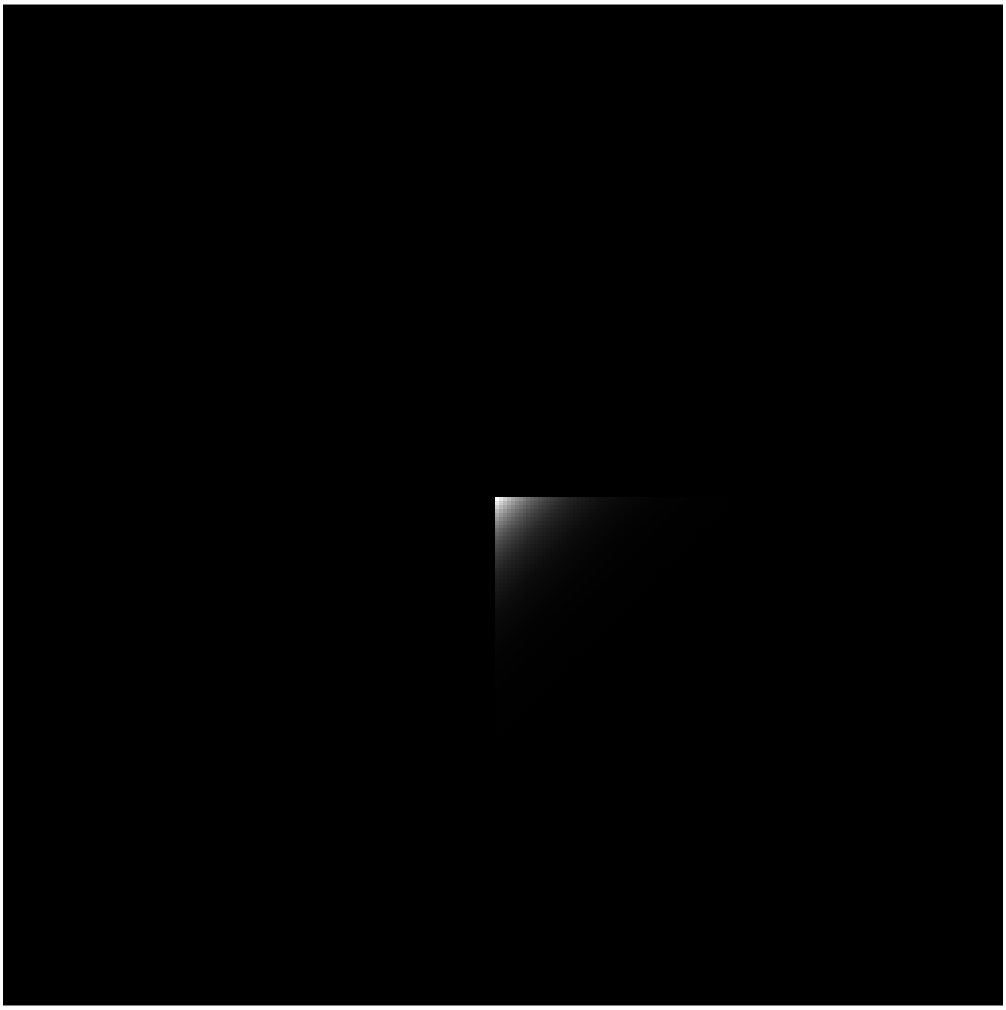


Figure 14. An image of the point spread function

4.

Figure 15. The filtered output color image.