A Nonparametric Test for Instantaneous Causality with Time-varying Variances

Jilin Wu*[a], Ruike Wu[a], and Zhijie Xiao[b]

[a]WISE & School of Economics, Xiamen University
[b]Department of Economics, Boston College, Chestnut Hill, MA, USA.

April 7, 2023

Abstract

This manual introduces the main functions of paper "A Nonparametric Test for Instantaneous Causality with Time-varying Variances" and provide a simple example to show how to use them.

JEL Codes: C12, C22.
Keywords: Instantaneous Causality; Time-varying Variances; U-statistic; Wild Bootstrap.

Contents

*Corresponding author at: Department of Finance, School of Economics, and Wang Yanan Institute for Studies in Economics (WISE), Xiamen University, Xiamen, China. Email: rainforest1061@126.com. Jilin Wu acknowledges support from Ministry of Education, Humanities and Social Science Foundation (20YJA790067), and the Fundamental Research Funds for the Central Universities (20720201042).

1 Main Functions

1.1 $Test\_inst\_causal\_by\_nonpara.m$

Description: This function compute proposed test statistic J_T in section 3.1 of main paper.
Usage:

$$[p\_val, J\_T] = Test\_inst\_causal\_by\_nonpara(Y, p, d1, h)$$

Input:

(1). Y is $d \times T$ input data, d is the dimension, T is sample size.

(2). p is lag length of vector. autoregressive model.

(3). $d1$ is the dimension of first group u_{1t}.

(4). h is bandwidth coefficient, the used bandwidth is $hT^{-1/5}$.

Output:

(1). p_val is p-value of test statistic using standard normal critical values.

(2). J_T is the level of proposed test statistic.

1.2 $Test\_inst\_causal\_by\_npbs.m$

Description: This function compute p-value of proposed test statistic J_T using wild bootstrapped critical values developed in section 4.
Usage:

$$[pval, J\_T, ifa] = Test\_inst\_causal\_by\_npbs(Y, p, B, d_1, cf, h)$$

Input:

(1). Y is $d \times T$ input data, d is the dimension, T is sample size.

(2). p is lag length of vector autoregressive model.

(3). B is the replication of bootstrap algorithm.

(4). $d1$ is the dimension of first group u_{1t}.

(5). cf is confidence level.

(6). h is bandwidth coefficient, the used bandwidth is $hT^{-1/5}$.

Output:

(1). p_val is p-value of test statistic using wild bootstrapped critical values.

(2). J_T is the level of proposed test statistic.

(3). ifa is a logistic variable, it takes 1 if the result rejects the null hypothesis, and takes 0 if not.

2 Example

In this section, we plot the monotonic power curve of J_T^B with bandwidth $T^{-1/5}$. We generate the virtual data based on following data generating process used in section 4 of main paper with sample size $T = 200$. Specifically,

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \begin{pmatrix} 0.64 & -1 \\ -0.01 & 0.44 \end{pmatrix} \begin{pmatrix} Y_{1t-1} \\ Y_{2t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix},$$

and

$$\Sigma(r) = \begin{pmatrix} \Sigma^{11}(r) & \Sigma^{12}(r) \\ \Sigma^{12}(r) & \Sigma^{22}(r) \end{pmatrix}, \forall r \in [0,1],$$

where $\Sigma^{11}(r) = 1.1 - \cos(11r)$, $\Sigma^{12}(r) = c\sin(2\pi r)$ and $\Sigma^{22}(r) = 1.1 + \sin(11r)$. The replication is set to 299, total replication is 1000, confidence level is 0.95.

```
clear; clc;
T = 200; cfl = 0.95; B = 299; LOOP = 1000; h = 1;

% generate data
c = 0.5 % We firstly take c = 0.5 as example
rng('default');
rng(1);
Y_set = cell(LOOP, 1);
for count0 = 1 : LOOP
    x_0 = [0; 0];
    Y = [];
    for r = 1 : T
        sigma = [1.1 - cos(11 * r/T), c * sin(2 * pi * r/T); c * sin(2 * pi * r/T), 1.1 + sin(11 * r/T)];
        u_1 = (sigma^0.5) * randn([2, 1]);
        x_1 = [0.64, -1; -0.01, 0.44] * x_0 + u_1;
        Y(:, r) = x_1;
        x_0 = x_1;
    end
% Y is 2 × T matrix
    Y = Y(1 : 2, 1 : T);
    Y_set{count0} = Y;
end
```

With data Y in hand, we conduct proposed test with bootstrapped critical values to make statistical inference at significance level 0.05.

```
size_count = [];
for count1 = 1 : LOOP
    tic; % set timer
    Y = Y_set{count1};
    [~, J_T, ifa] = Test_inst_causal_by_npbs(Y, 1, B, 1, cfl, h);
    size_count(count1) = ifa;
    toc;
end
powerb = sum(size_count)/LOOP;
```

Simple input "*powerb*" into command window and we can see the final result as below.

```
>> powerb
powerb = 0.873
```

Repeat above procedure with $c = 0.1, 0.2, \ldots, 0.9, 1$ iteratively, we can naturally get the corresponding empirical powers $0.058, 0.106, 0.226, 0.455, 0.683, 0.873, 0.965, 1, 1, 1, 1$.

```
local_power = [0.058, 0.106, 0.226, 0.455, 0.683, 0.873, 0.965, 1, 1, 1, 1];
c_set = 0 : 0.1 : 1;
plot(c_set, local_power, 'm-.', 'MarkerSize', 4, 'LineWidth', 1);
xlabel('The deviation c from the null');
ylabel('Power');
```

The plot of power curve of J_T^B with $h = T^{-1/5}$ and sample size $T = 200$ is shown in Figure 1.

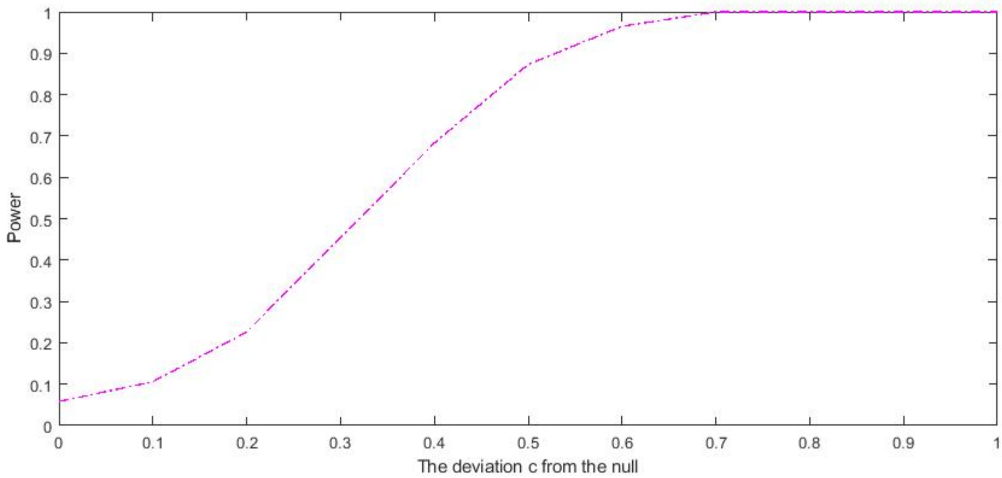

Figure 1: Empirical powers of J_T^B.