P8122 Homework 1

Due: 09/13/2022 at 5:00pm

Instructions

- Upload a single pdf file for your homework on Canvas.
- You may discuss these problems with each other verbally, but must write up the answers on your own, and may not share or show your answers to anyone else.
- Short and clear answers, please.
- No late homework are allowed.

Question 1

Consider the following population of individuals and suppose we knew all the potential outcomes:

Table 1: Counterfactuals

Individual	Y_0	Y_1
1	0	0
2	1	0
3	0	1
4	1	0
5	1	0
6	0	1
7	1	0
8	0	0
	4	2

(a) 5 points. For all individuals calculate the effect of treatment on the outcome and interpret.

(b) 5 points. Calculate the average causal effect of treatment on the outcome and interpret.

(c) 10 points. Calculate the association of the treatment with the outcome under the following treatment assignment for subjects $i = 1, .., 8$: $A_1 = 1, A_2 = 0, A_3 = 1, A_4 = 1, A_5 = 0, A_6 = 0, A_7 = 0, A_8 = 1$. Interpret the result, compare with the effect computed in question 2b.

(d) 10 points. Show a random assignment of the treatment for this population. Explain your work. Compute the association of the treatment with the outcome under the random assignment and compare with the treatment effect computed in question 1b.

(a)

Individual	Y_0	Y_1	$Y_1 - Y_0$
1	0	0	0
2	1	0	-1
3	0	1	1
4	1	0	-1
5	1	0	-1
6	0	1	1
7	1	0	-1
8	0	0	0

- For individuals with $Y_1 - Y_0 = 0$ (1, 8):
  There is no causal effect under different treatment assignment
- For individuals with $Y_1 - Y_0 = 1$ (3, 6)
  The treatment 1 has a beneficial causal effect on the outcome.
- For individuals with $Y_1 - Y_0 = -1$ (2, 4, 5, 7)
  The treatment 1 has a harmful causal effect on the outcome.

(b) $ACE = E[Y_1] - E[Y_0] = \frac{2}{8} - \frac{4}{8} = -\frac{1}{4}$

- The treatment has a causal relation with outcome.
  Since ACE is negative, treatment 0 is more beneficial to outcome than treatment 1 on average.

(c)

Individual	Y_0	Y_1	Treatment
1	?	0	A = 1
2	1	?	A = 0
3	?	1	A = 1
4	?	0	A = 1
5	1	?	A = 0
6	0	?	A = 0
7	1	?	A = 0
8	?	0	A = 1
Total	?	?	
Observed	3	1	

$$E[Y|A=1] - E[Y|A=0] = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

- The difference in observed treated and untreated group means (and apparent effect) is $-\frac{1}{2}$. The treatment and outcome are statistically associated.

- Compared to 2(b): The calculated association is compatible with the effect in 2(b), but has a bigger absolute value, which means that from observed data, people benefit more from treatment 0. (2c)

(d) Assign random treatment for subjects $i = 1, 2, \cdots, 8$:

$A_1 = 0$. $A_2 = 1$. $A_3 = 1$. $A_4 = 1$. $A_5 = 0$. $A_6 = 1$ $A_7 = 0$. $A_8 = 0$

Individual	Y_0	Y_1	Treatment
1	0	?	A = 0
2	?	0	A = 1
3	?	1	A = 1
4	?	0	A = 1
5	1	?	A = 0
6	?	1	A = 1
7	1	?	A = 0
8	0	?	A = 0
Total	?	?	
Observed	2	2	

$$E[Y_1 | A=1] - E[Y | A=0] = \frac{2}{4} - \frac{2}{4} = 0$$

- There is no difference of outcome between different treatment group under this random treatment assignment.
- Compared to 2(b): In 2(b), ACE is negative, which means individuals benefit more from treatment 0; while here the difference is 0, which means there is no difference between treatment 0 and 1.

Question 2

During a check-up, a physician finds that his patient's blood pressure levels are too low. He prescribes medication at a high dose and asks her to be re-tested in a month. At the second test, the patient's blood pressure levels are now too high, so the physician switches her to a low dose of medication and again asks her to be re-tested in a month. At the third test, the patient's blood pressure levels are perfect, and so the doctor decides that she should stay at the low dose indefinitely, with no further testing.

(a) 5 points. What are the units?

(b) 5 points. What is the treatment?

(c) 10 points. What are the potential outcomes?

(d) 10 points. Show the calculation that the physician conducts to conclude that the patient should remain on the low dose (compute the causal effect of the treatment).

(e) 10 points. Is SUTVA plausible? If so, explain why briefly. If not, offer an assumption that, if true, would make SUTVA plausible. (Remember that SUTVA has two parts).

(f) 10 points. Why must SUTVA be plausible in order for us to use the potential outcome framework?

(g) 10 points. Does the physician's assignment mechanism appear to be probabilistic? Individualistic? Unconfounded? and controlled? Explain in one sentence each.

(h) 10 points. How would you assign in this single subject study the treatment differently in order to validly compute the causal effect of treatment.

(a) The patient.

(b) High dose or low dose medication.

(c) Low blood pressure, high blood pressure, perfect (normal) blood pressure
OR: abnormal (low/high) bp, normal bp

(d)

Individual	Y_0 (A=0, high dose)	Y_1 (A=1, low dose)
1	0	1

$Y_1 - Y_0 = 1 - 0 = 1$

Y_0: normal bp
Y_1: abnormal bp

The average causal effect of low dose medication is 1. There is a beneficial causal effect of low dose medication, so the physician conclude that the patient should remain on low dose.

(e) • SUTVA is not plausible.

Two components of SUTVA:

1. Treatment applied to one unit does not affect the outcome of the other unit.
2. Potential outcomes must be well-defined.

- Since the high / low dose of medication is not well-defined, the component 1 is violated.
- If we specify the exact dose of high / low dose medication. for example, 20mg for high dose and 10 mg for low dose. SUTVA will be plausible.

f) To identify ACE using potential outcome framework, consisten assumption is required. The two components of SUTVA must be satisfied for consisteney assumption to hold. Therefore, SUTVA must be plausible for us to use the potential outcome framework.

g) • Probabilistic:
Yes. Each time before treatment the patient has some chance of being either treated with low dose or high dose medication.

- Individualistic:
No. Since the doctor gave treatment based on the result of last trail, the probability a unit is assigned to a certain treatment is dependent on other covariates.

- Unconfounded:

  No. The assignment mechanism is dependent on potential outcome — high / low blood pressure. The doctor adjusted the treatment upon the potential outcome.

- Controlled:

  Yes. The assignment of treatment — high/ low dose of medication is controlled by the doctor.

(h) 1. Specify the exact dose for high / low medication.
e.g. 20mg for high and 10 mg for low

2. Expand the interval of each test, to make sure the patients BP level stays at the same level (as the BP level at the begining of the 1st test) before each treatment.

3. Random assign low (10mg) / high (20mg) medication to each treatment.

These 3 points together would make the calculation of causal effect validate.