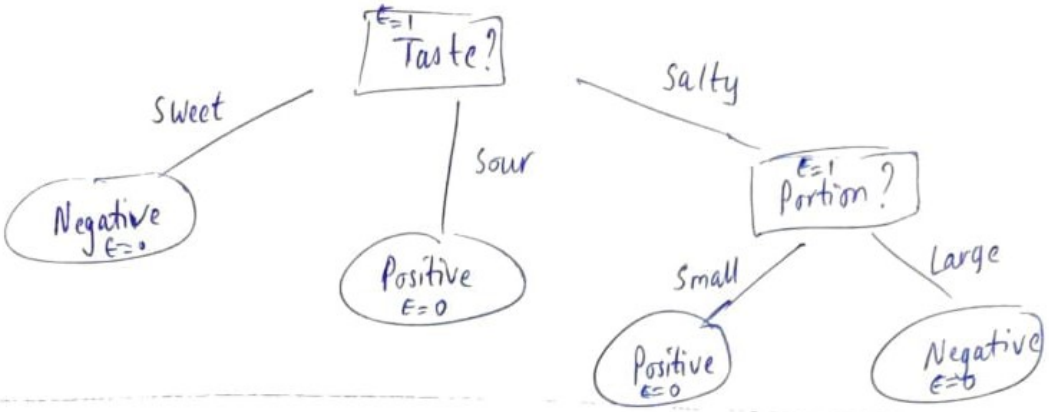
Review	Smell	Taste	Portion
Negative	Woody	Sweet	Small
Negative	Fruity	Salty	Large
Negative	Fruity	Salty	Large
Positive	Fruity	Sour	Small
Positive	Woody	Sour	Small
Negative	Woody	Sweet	Large
Positive	Woody	Sour	Large
Positive	Fruity	Salty	Small
Positive	Fruity	Salty	Small
Negative	Woody	Sweet	Large

Decision Tree



(0) Entropy total →

$p^+ = 5/10 = 0.5$, $p^- = 5/10 = 0.5$

$\therefore E_{Total} = -\sum p_i \log p_i = -(0.5\log_2 0.5 + 0.5\log_2 0.5) = -(\log_2 0.5) = 1.$

$\therefore \boxed{E_{Total} = 1}$

(1) For ~~After~~ taste partition →

Sweet → $p^{+ve} = 0/3 = 0$, $p^- = 3/3 = 1.$

$\therefore E_{sweet} = -(0\log_2 0 + 1\log_2 1) = 0.$ $\Rightarrow \boxed{E_{sweet} = 0}$

Sour → $p^+ = 3/3 = 1$, $p^- = 0/3 = 0.$

$E_{sour} = -(1\log_2 1 + 0\log_2 0) = 0.$ $\Rightarrow \boxed{E_{sour} = 0}$

Salty → $p^+ = \frac{2}{4} = 0.5$, $p^- = \frac{2}{4} = 0.5$

$E_{salty} = -(0.5\log_2 0.5 + 0.5\log_2 0.5) = 1 \Rightarrow \boxed{E_{salty} = 1}$

$E_{taste} = \frac{3}{10} E_{sweet} + \frac{3}{10} E_{sour} + \frac{4}{10} E_{salty} = \frac{3}{10}\times 0 + \frac{3}{10}\times 0 + \frac{4}{10}\times 1 = 0.4$

$\Rightarrow \Delta I_{taste} = E_{total} - E_{taste} = 1 - 0.4 = 0.6 \Rightarrow \boxed{\Delta I_{taste} = 0.6}$

For smell partition →

Woody → $p^{+ve} = 2/5$, $p^{-ve} = 3/5$

$E_{woody} = -\left(\frac{2}{5}\log_2 \frac{2}{5} + \frac{3}{5}\log_2 \frac{3}{5}\right) = 0.971 \Rightarrow \boxed{E_{woody} = 0.971}$

Fruity → $p^{+ve} = 3/5$, $p^{-ve} = 2/5$

$\Rightarrow \boxed{E_{fruity} = 0.971}$

$\Delta I_{smell} = E_{total} - E_{smell} = 1 - \frac{5}{10} \times E_{woody} - \frac{5}{10} E_{fruity} = 1 - \frac{1}{2} \times 0.971 - \frac{1}{2} \times 0.971$

$\Rightarrow$ **$\Delta I_{smell} = 0.029$**

For Portion partition →

Large → $p^+ = 1/5$, $p^- = 4/5$

$E_{large} = -\left(\frac{1}{5}\log_2\frac{1}{5} + \frac{4}{5}\log_2\frac{4}{5}\right)$ $\Rightarrow$ **$E_{large} = 0.7219$**

Small → $p^+ = 4/5$, $p^- = 1/5$

$\Rightarrow$ **$E_{small} = 0.7219$**

$\therefore \Delta I_{portion} = E_{total} - E_{portion} = 1 - \frac{5}{10} \times E_{large} - \frac{5}{10} E_{small} = 1 - \frac{1}{2} \times 0.7219 - \frac{1}{2} \times 0.7219$

$\Rightarrow$ **$\Delta I_{portion} = 0.2701$**

$\because$ In $\Delta I_{taste} > \Delta I_{portion}$ and ΔI_{smell} $\Rightarrow$ **Root node (decision) = Taste**

↓

(2) After taste partition → $\Delta I_{portion} > \Delta I_{smell}$ → We can take portion as the child node

For Sweet → All reviews are negative → We don't need to calculate E.

$E_{sweet} = 0$ } No need for further partitions.

For Sour → All reviews are positive. **$E_{sour} = 0$.**

For Salty → $p^+ = 2/4 = 0.5$, $p^- = 2/4 = 0.5$

$E_{salty} = -\left(0.5\log_2 0.5 + 0.5\log_2 0.5\right) = 1$ $\Rightarrow$ **$E_{salty} = 1$**

↓

Partition based on Portion →

Large → $p^+ = 0/2$, $p^- = 2/2 = 1$.

$\Rightarrow$ **$E_{large} = 0$**

Small → $p^+ = 2/2 = 1$, $p^- = 0$. $\Rightarrow$ **$E_{small} = 0$**

$\therefore \Delta I_{portion} = E_{salty} - \frac{2}{4} E_{large} - \frac{2}{4} E_{small} = 1 - \frac{1}{2} \times 0 - \frac{1}{2} \times 0 = 1$ $\Rightarrow$ **$\Delta I_{portion} = 1$**

$\therefore$ Now, $\Delta I = 1 = E_{salty}$ → No further portions will be required.

Refer Decision tree at the start.